

# Technique for estimating the variation of asynchronous machine parameters using the extended Kalman filter

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**Abstract**—Precise knowledge of the asynchronous machine model parameters is a prerequisite for implementing efficient control algorithms. What's more, in variable speed drives for induction machines, the sensitivity of the control is relatively high in relation to variations in rotor resistance, so it seemed only natural to look at its on-line identification. The aim of this paper is to estimate the machine parameters needed to ensure the robustness of the control. In this context, the Kalman filter method, which takes the form of a set of differential or recurrent equations, can give good results if the initial conditions are carefully chosen to ensure that the filter operates correctly. The results obtained demonstrate the effectiveness of the extended Kalman filter. They are characterized by a very small estimation error.

**Keywords**—SPIM, Extended Kalman filter, parameter estimation, rotor and stator resistance, error, s-function.

## I. INTRODUCTION

In the low-power sector, the single-phase asynchronous machine is by far the most familiar electrical machine, so widely used are they in mass-market applications. What's more, it was invented over a hundred years ago. This was the first rotating electrical machine to introduce the phenomenon of induction. Unfortunately, as this is an asymmetrical electromagnetic structure, most modern electromagnetic textbooks give it very little space, and the result, in our opinion, is a profound misunderstanding of this particular electrotechnical system. Studies on single-phase motors go back a long way, to the beginning of the century. The idea that led us to tackle the theme of parametric estimation of asynchronous machines is currently a particularly interesting area of research, contributing to the improvement and universality of control algorithms. In addition, techniques for estimating variable parameters are justified by the growing interest of industry in the design of estimating controllers. The emphasis placed by research on improving the quality of this machine's technological performance, the development of power electronics components and the emergence of different static converter structures mean that variable-frequency energy sources are now available. As a result, the combination of single-phase induction motors and static converters is the domestic actuator of the future. The present work implements the extended Kalman filter estimation method to estimate the variable parameters of a device equipped with an asynchronous machine.

## II. MODELING THE SINGLE-PHASE ASYNCHRONOUS MACHINE WITH CAPACITOR

The single-phase asynchronous machine can be modeled in a reference frame linked to the stator ( $d$ - $q$ ) by two fictitious transformers, one along the  $[d]$  axis, and the other along the  $q$  axis.

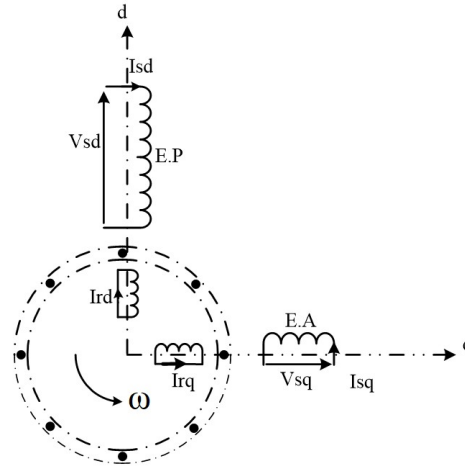


Fig. 1 Model of a single-phase asynchronous machine in a stator reference frame.

In a reference frame linked to the stator, the electrical equations of the single-phase asynchronous machine are given by:

$$V_{sd} = R_{sd}i_{sd} + L_{sd} \frac{di_{sd}}{dt} + M_{srd} \frac{di_{rd}}{dt} \quad (1)$$

$$V_{sq} = R_{sq}i_{sq} + L_{sq} \frac{di_{sq}}{dt} + M_{srq} \frac{di_{rq}}{dt} \quad (2)$$

$$0 = R_r i_{rd} + L_r \frac{di_{rd}}{dt} + M_{srd} \frac{di_{sd}}{dt} + \omega_r (L_r i_{rq} + M_{srq} i_{sq}) \quad (3)$$

$$0 = R_r i_{rq} + L_r \frac{di_{rq}}{dt} + M_{srq} \frac{di_{sq}}{dt} - \omega_r (L_r i_{rd} + M_{srd} i_{sd}) \quad (4)$$

## III. KALMAN FILTER METHOD

Several estimation techniques are used in the literature, including online recursive methods based essentially on the least-squares technique, online adaptive methods with reference model (MRAS) and, more recently, Kalman filter-based estimation techniques. All these methods seem to be promising, especially when it comes to adapting them to non-linear processes. [11].

The historical overview of these techniques can be described by Carl Friedrich Gauss during 1777-1855, the least squares method by the first Optimal estimation method) between 1894-1964 by Norbert Wiener and the Wiener filter by the use of correlation and spectral density functions then in 1960 by the Kalman filter using state representation..

The applications of this filter are numerous and widespread. It is used to estimate unknown initial conditions (ballistics), to predict mobile trajectories, to locate a device (navigation, radar, etc.) and also to implement control laws based on a state estimator and state feedback.

## IV. MODELING THE SYSTEM UNDER STUDY

In an ideal context, the measurements made on a process are error-free and the working hypotheses set are perfect. However, this is not always in line with reality. In fact, measured process outputs are accompanied by

random disturbances, even though the essential working conditions are kept constant. To remedy this problem, it is obvious to take into account the stochastic environment in which the process evolves. This can be justified by the noise [11].

$$\begin{cases} X(k+1) = f[X(k), U(k), k] + b_{rs}(k) \\ Y(k+1) = C(k)X(k) + b_{rm}(k) \end{cases} \quad (5)$$

$b_{rs}$  : Noise vector on the system.

$b_{rm}$  : Noise vector measurements.

These two vectors are defined by the covariance matrices assumed to be independent, white uncorrelated and of zero mean [11].

$$\begin{cases} E[b_{rs}] = 0 & E[b_{rm}] = 0 & \text{cov}(b_{rs}) = E\{b_{rs}b_{rs}^T\} = Q \geq 0 \\ E[b_{rs}b_{rm}^T] = 0 & E[b_{rm}b_{rm}^T] = R & \text{cov}(b_{rm}) = E\{b_{rm}b_{rm}^T\} = R \geq 0 \end{cases}$$

- $E[\ ]$  Represents the mathematical expectation operator.
- $Q$  is the covariance of the model error reflecting the disturbance on the state.
- $R$  is the covariance of the measurement error, which reflects the measurement noise.
- - The covariance matrices  $Q$  and  $R$  are diagonal.

Development of the Kalman filter algorithm requires the dynamic state model, relying solely on the machine's electrical equations, for joint estimation of the state of the parameters. The technique consists in considering a higher-dimensional state vector including the parameters to be estimated.

The state-space representation of the machine is recognized by the following general form:

$$\frac{d}{dt} \begin{bmatrix} i_{sd} \\ i_{sq} \\ i_{rd} \\ i_{rq} \\ R_r \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12}\omega_r & a_{13}R_r & a_{14}\omega_r & 0 \\ a_{21}\omega_r & a_{22} & a_{23}\omega_r & a_{24}R_r & 0 \\ a_{31} & a_{32}\omega_r & a_{33}R_r & a_{34}\omega_r & 0 \\ a_{41}\omega_r & a_{42} & a_{43}\omega_r & a_{44}R_r & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} i_{sd} \\ i_{sq} \\ i_{rd} \\ i_{rq} \\ R_r \end{bmatrix} + \begin{bmatrix} b_{11} & 0 \\ 0 & b_{22} \\ b_{31} & 0 \\ 0 & b_{42} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} v_{sd} \\ v_{sq} \end{bmatrix} \quad (6)$$

$$\begin{bmatrix} i_{sd} \\ i_{sq} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} i_{sd} \\ i_{sq} \\ i_{rd} \\ i_{rq} \\ R_r \end{bmatrix} \quad (7)$$

With  $[X] = [i_{sd} \ i_{sq} \ i_{rd} \ i_{rq} \ R_r]^T$ ,  $[U] = [v_{sd} \ v_{sq}]^T$  et  $[Y] = [i_{sd} \ i_{sq}]^T$ ;

$$\begin{aligned} a_{11} &= -\frac{1}{\tau_d \sigma_d}; a_{12} = \frac{M_{srd} M_{srq}}{L_r L_{sd} \sigma_d}; a_{13} = \frac{M_{srd}}{L_r L_{sd} \sigma_d}; a_{14} = \frac{M_{srd}}{L_{sd} \sigma_d} \\ a_{21} &= -\frac{M_{srd} M_{srq}}{L_r L_{sq} \sigma_q}; a_{22} = -\frac{1}{\tau_q \sigma_q}; a_{23} = -\frac{M_{srq}}{L_{sq} \sigma_q}; a_{24} = \frac{M_{srq}}{L_r L_{sq} \sigma_q}; \\ a_{31} &= \frac{M_{srd}}{L_r \tau_d \sigma_d}; a_{32} = -\frac{M_{srq}}{L_r \sigma_d}; a_{33} = -\frac{1}{L_r \sigma_d}; a_{34} = -\frac{1}{\sigma_d}; a_{41} = \frac{M_{srd}}{L_r \sigma_q}; a_{42} = \frac{M_{srq}}{L_r \tau_q \sigma_q}; a_{43} = \frac{1}{\sigma_q}; \\ a_{44} &= -\frac{1}{L_r \sigma_q}; b_{11} = \frac{1}{L_{sd} \sigma_d}; b_{22} = \frac{1}{L_{sq} \sigma_q}; b_{31} = -\frac{M_{srd}}{L_r L_{sd} \sigma_d} \text{ et } b_{42} = -\frac{M_{srq}}{L_r L_{sq} \sigma_q} \end{aligned}$$

## V. KALMAN FILTER ALGORITHM

Implementation of the Kalman algorithm requires two phases, the first of which is a prediction phase that consists of determining the prediction vector from the state equations.

In this case, the aim is to predict an estimate of the state given the information available at time  $k$  [10].

$$\hat{X}(k+1/k) = f(\hat{X}(k/k), U(k), k) \quad (8)$$

We can see that this prediction phase allows us to know the estimated quantities, as well as the output measurements.

$$\hat{Y}(k + 1/k) = C(k)\hat{X}(k + 1/k) \tag{9}$$

Calculation of the gain matrix requires definition of the covariance matrices of the prediction and state estimation.

$$\tilde{P}(k + 1) = P(k + 1/k) = F(k)P(k/k)F^T(k) + Q(k) \tag{10}$$

This matrix is chosen to minimize the variance of the state vector estimation error. This minimization concerns the diagonal elements of the estimation matrix

The second phase is a correction step. It consists in correcting the prediction vector by the measurement vector, in order to obtain the estimate of the state vector at the present time (k+1).

The prediction phase produces a difference between the measured output and the predicted output. To improve the state, this deviation must be taken into account and corrected via the Kalman gain. By minimizing the variance of the error, we obtain the new expression for the state vector at time k+1[10]. The update of the filter covariance matrix constitutes the a posteriori variance at time k+1.

The description of the Kalman filter algorithm can be illustrated by the following block diagram:

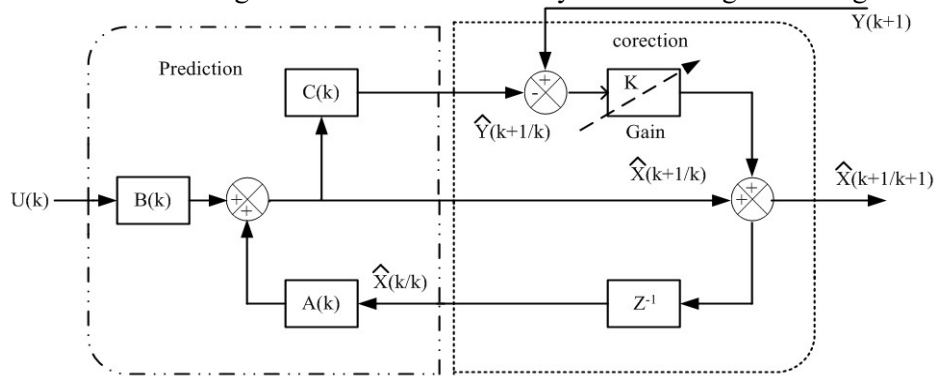


Fig. 2 Representation of the Kalman filter algorithm

*A. Discretization of the SPIM model*

The Kalman filter is based on discrete mathematical equations, whereas the machine model is continuous. The transition from the continuous to the discrete model is therefore made by discretization using Euler's method, applying a development of order 1 for a constant sampling period. [5], [10] [6].

*B. Extended state model of the single-phase asynchronous machine corresponding to the estimation of the rotor resistance Rr*

The model (6) used to estimate the rotor resistance Rr is a non-linear model. [1], [2].

$$\begin{cases} X(k + 1) = f(X(k), U(k), k) \\ Y(k + 1) = C(k)X(k) \end{cases}$$

(11)

For:

$$f = [f_1 \quad f_2 \quad f_3 \quad f_4 \quad f_5]^T$$

(12)

That we can still write:

$$f = \begin{cases} f_1 = i_{sd}(k) + T[a_{11}i_{sd}(k) + a_{12}\omega_r(k)i_{sq}(k) + a_{13}R_r(k)i_{rd}(k) + \\ \quad a_{14}\omega_r(k)i_{rq}(k) + b_{11}v_{sd}(k)] \\ f_2 = i_{sq}(k) + T[a_{21}\omega_r(k)i_{sd}(k) + a_{22}i_{sq}(k) + a_{23}\omega_r(k)i_{rd}(k) + \\ \quad a_{24}R_r(k)i_{rq}(k) + b_{22}v_{sq}(k)] \\ f_3 = i_{rd}(k) + T[a_{31}i_{sd}(k) + a_{32}\omega_r(k)i_{sq}(k) + a_{33}R_r(k)i_{rd}(k) + \\ \quad a_{34}\omega_r(k)i_{rq}(k) + b_{31}v_{sd}(k)] \\ f_4 = i_{rq}(k) + T[a_{41}\omega_r(k)i_{sd}(k) + a_{42}i_{sq}(k) + a_{43}\omega_r(k)i_{rd}(k) + \\ \quad a_{44}R_r(k)i_{rq}(k) + b_{42}v_{sd}(k)] \\ f_5 = R_r(k) \end{cases}$$

### C. Linearization procedure

Applying the Kalman filter to the asynchronous machine results in a system modeled by non-linear equations of state, so that the transition matrix  $F(k)$  and the observation are non-linear functions. For linearization, the extended Kalman filter algorithm uses the Jacobians [8], [3], [4], and [9].

$$\begin{cases} F(k) = \frac{\partial f(x)}{\partial x} / X=\hat{x}(k) \\ H(k) = \frac{\partial C(x)}{\partial x} / X=\hat{x}(k) \end{cases} \quad (13)$$

The linearization matrix:

$$\frac{\partial f(x)}{\partial x} / X=\hat{x}(k) = \begin{bmatrix} \frac{\partial f_1}{\partial i_{sd}(k)} & \frac{\partial f_1}{\partial i_{sq}(k)} & \frac{\partial f_1}{\partial i_{rd}(k)} & \frac{\partial f_1}{\partial i_{rq}(k)} & \frac{\partial f_1}{\partial R_r(k)} \\ \frac{\partial f_2}{\partial i_{sd}(k)} & \frac{\partial f_2}{\partial i_{sq}(k)} & \frac{\partial f_2}{\partial i_{rd}(k)} & \frac{\partial f_2}{\partial i_{rq}(k)} & \frac{\partial f_2}{\partial R_r(k)} \\ \frac{\partial f_3}{\partial i_{sd}(k)} & \frac{\partial f_3}{\partial i_{sq}(k)} & \frac{\partial f_3}{\partial i_{rd}(k)} & \frac{\partial f_3}{\partial i_{rq}(k)} & \frac{\partial f_3}{\partial R_r(k)} \\ \frac{\partial f_4}{\partial i_{sd}(k)} & \frac{\partial f_4}{\partial i_{sq}(k)} & \frac{\partial f_4}{\partial i_{rd}(k)} & \frac{\partial f_4}{\partial i_{rq}(k)} & \frac{\partial f_4}{\partial R_r(k)} \\ \frac{\partial f_5}{\partial i_{sd}(k)} & \frac{\partial f_5}{\partial i_{sq}(k)} & \frac{\partial f_5}{\partial i_{rd}(k)} & \frac{\partial f_5}{\partial i_{rq}(k)} & \frac{\partial f_5}{\partial R_r(k)} \end{bmatrix}$$

### D. Stochastic machine model

In an ideal context, the measurements made on a process are error-free and the working hypotheses set are perfect. However, this is not always in line with reality. In fact, the measured outputs of the process are accompanied by random disturbances, even though the essential working conditions are kept constant. To remedy this problem, it is obvious to take into account the stochastic environment in which the process evolves.

### E. Implementation of the extended Kalman filter algorithm

As far as implementation is concerned, the extended Kalman filter is recognized by an algorithm that relies on a very complicated and generally cumbersome matrix calculation. It is therefore very difficult to implement using Simulink. So it's obvious to use it in the form of an s-function, represented by a Simulink block, inserted into the global system simulation diagram [3].

VI. STATE ESTIMATION RESULTS IN THE CASE OF ROTOR RESISTANCE ESTIMATION  $R_r$

The simulation results obtained are shown in the following figures:

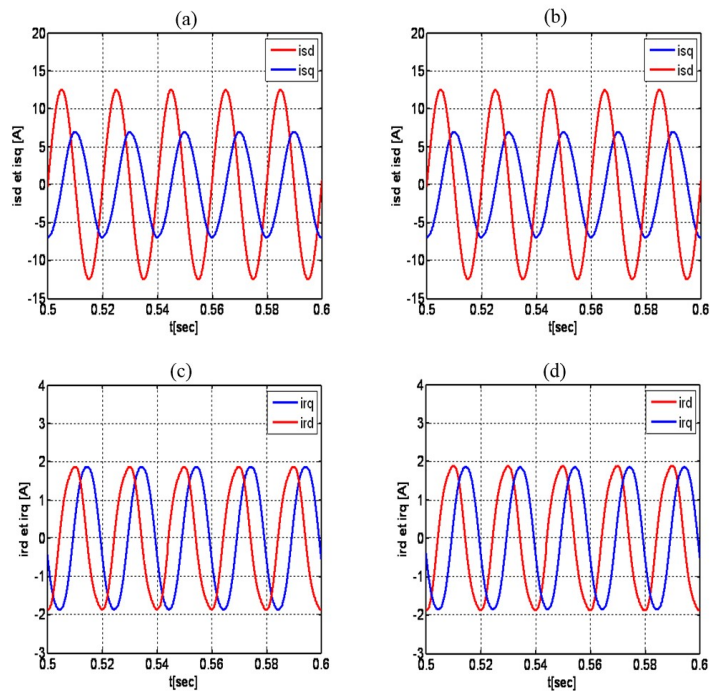


Fig. 3 Simulated (a-c) and estimated (b-d) stator and rotor currents

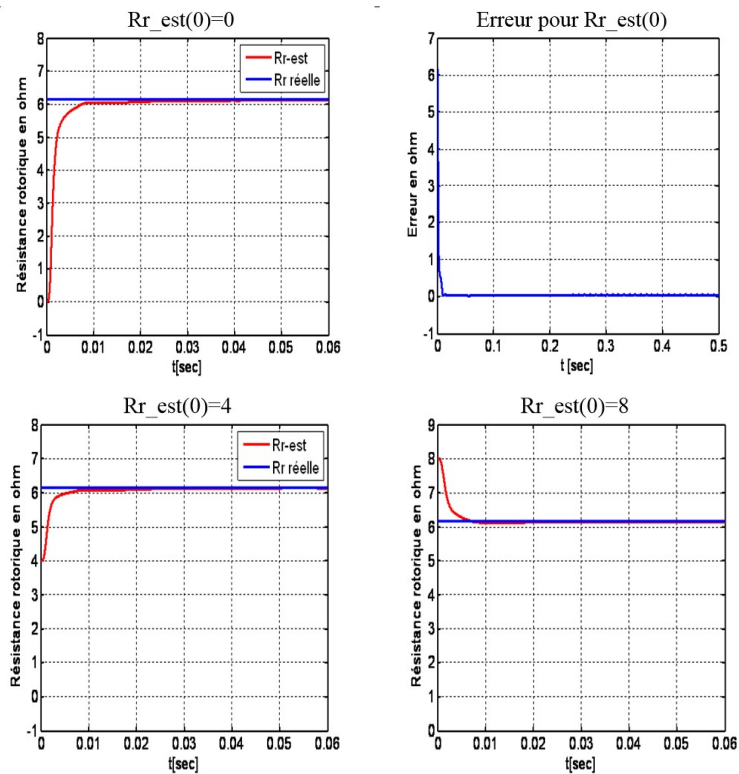


Fig. 4 Estimation results for SPIM stator resistance  $R_r$  by the extended Kalman filter for different initial conditions.

For the  $R_{sd}$  resistivity, we do the same work as for the previous model.

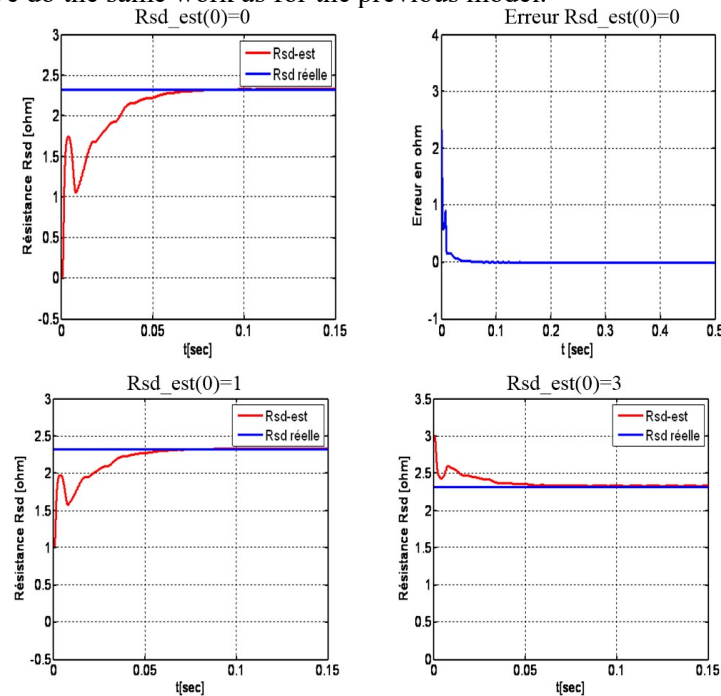


Fig 5. Results of estimating SPIM's stator resistance  $R_{sd}$  by the extended Kalman filter for different initial conditions

Analysis of the estimation results shows that:

- The robustness of the estimator can be tested by varying the initial conditions.
- Under transient conditions, the quantities estimated by the extended Kalman filter are slightly different from those simulated.
- However, under steady-state conditions, the error between the various estimated and simulated quantities is approximately negligible.
- For stator and rotor currents, the error between the various estimated and simulated quantities is approximately zero.
- One of the main difficulties in implementing the Kalman filter lies in the choice of initial conditions. This choice is often very delicate, as the critical point in Kalman filter design is to obtain a good numerical evaluation of the filter parameters from knowledge of the initial conditions.

## VII. CONCLUSION

We conclude by pointing out the vital importance of parameter identification in electrical machine control studies. Indeed, most of these studies are carried out using numerical simulation. And a good knowledge of the machine's parameters reduces design costs and avoids the hazardous trial-and-error encountered in the “black box” case.

Kalman filter methods are very interesting for estimating the state and parameters of a knowledge model. They are adopted for both linear and non-linear systems. The advantages of this method lie in its ability to identify parameters and status.

The results obtained demonstrate the effectiveness of the extended Kalman filter. They are characterized by a very small estimation error.

Although the present work is a very interesting start to estimating the parameters of a single-phase in-line asynchronous machine. On the one hand, this study needs to be completed on a device equipped with a machine powered by a static converter, in order to estimate the parameters in real time. On the other hand, the innovative



aspect of this theme leads us to several avenues of research which we feel should be explored to ensure proper control and guarantee correct system behavior.

Motor characteristics are given in the following table:

TABLE I MACHINE PARAMETERS

$U_n=220$ V	$L_{sd} = 0.0908$ Ω
$I_n=7.58$ A	$L_{sq} = 0.1156$ Ω
$C=35$ μF	$L_r = 0.0915$ Ω
$R_{sd} = 2.40$ Ω	$M_{srd} = 0.0828$ Ω
$R_{sq} = 6.06$ Ω	$M_{srq} = 0.0986$ Ω
$R_r = 6.0738$ Ω	$f=2,026.10^{-4}$ N.m.rad s <sup>-1</sup>

#### NOMENCLATURE AND ABBREVIATIONS

SPIM: Single Phase Induction Motor.

EP: Main winding.

EA: Auxiliary winding.

$V_{sd}, V_{sq}$  : Stator voltages along the d-axis and q.

$i_{sd}, i_{sq}$  : Stator currents along the d-axis and q.

$i_{rd}, i_{rq}$  : Rotor currents along the d-axis and q.

$R_{sd}, R_{sq}, R_r$  : Stator and rotor resistors.

$L_{sd}, L_{sq}, L_r$  : stator and rotor inductances.

$M_{srd}$  et  $M_{srq}$  : Mutual d- and q-axis inductances.

$\sigma_d = 1 - \frac{M_{srd}^2}{L_{sd}L_r}$ : Blondel coefficient along the direct axis d.

$\sigma_q = 1 - \frac{M_{srq}^2}{L_{sq}L_r}$ : Blondel coefficient along the q axis.

$\tau_d = \frac{L_{sd}}{R_{sd}}$ : Stator time constant along the d axis.

$\tau_q = \frac{L_{sq}}{R_{sq}}$ : Stator time constant along the q axis.

$\tau_r = \frac{L_r}{R_r}$ : Rotor time constant.

$\omega_r$  : Electrical rotor speed.

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