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Numerical Method of Heat Transfer for Fin Profiles

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***Abstract*— In the this work a numerical method is proposed in order to solve the thermal performance problems of heat transfer of fins surfaces. The bidimensional temperature distribution on the longitudinal section of the fin is calculated by restoring to the finite volumes method. The heat flux dissipated by a generic profile fin is compared with the heat flux removed by the rectangular profile fin with the same length and volume. In this study it is shown that a finite volume method for quadrilaterals unstructured mesh is developed to predict the two dimensional steady-state solutions of conduction equation, in order to determine the sinusoidal parameter values which optimize the fin effectiveness. In this scheme, based on the integration around the polygonal control volume, the derivatives of conduction equation must be converted into closed line integrals using same formulation of the Stokes theorem. The numerical results show good agreement with analytical results. To demonstrate the accuracy of the method, the absolute and root-mean square errors versus the grid size are examined quantitatively.**

***Keywords*—Stokes theorem, unstructured grid, heat transfer, complex geometry, effectiveness.**

I. INTRODUCTION

In many engineering sectors, where high thermal fluxes must be transferred, the finned surface power removers are today an usual tool. Since finned surfaces allow evident improvements in heat transfer effectiveness, the heat exchangers field is one of the most interested in their applications. Moreover new industrial sectors present an increasing interest in the introduction of extended surfaces for heat flux removal. In particular, the electronics industry has promoted a new interest in developing heat removers, aimed at transferring heat from electronic components to the environment, in order to reduce the working temperature and to improve the characteristics and the reliability [1],[2]. The optimization of heat remover longitudinal profile, in order to transfer the highest power with the smallest volume, is a problem that is not yet completely solved. Such a problem was talked for the first time in the 1920s [3], when Schmidt proposed a parabolic longitudinal profile. Afterwards authors contested Schmidt's conclusion, correct from the point of view of utilized model, but scarcely corresponding to the real phenomenon characteristics. Since then many fin profiles have been proposed, mainly parabolic or triangular, but without giving a final solution to the optimization problem [4] and leaving perplexedness regarding the structural integrity of heat removers with an excessively have been proposed [5],[6] and a parabolic-undulated fin has been demonstrated as having a noticeably improved effectiveness. In 1996 "Giampierto Fabbri" consider polynomial profile heat removers and he propose a genetic algorithm in order to determine the polynomial parameter values [7].

This study is motivated by the need for a numerical approach that is not only capable of performing accurate computations but that also provides an easier way to implement these computations. Our objectives is to develop a simple and accurate procedure to deal with curved boundaries, which capable of achieving second order accuracy with relative economy, for heat transfer and flow problem, employing unstructured mesh. For this objective, we develop a method based on same formulation of the Stokes theorem. For testing our method, we consider the computation of passive scalar in fin profile, the numerical method calculates the heat flux dissipated by the sinusoidal profile heat remover on the basis of the bidimensional temperature distribution on its longitudinal section, which is obtained with the help of our method.

II. THE FIN MODEL

In the orthogonal coordinate system we will refer to a heat remover with longitudinal section symmetrical with respect to the x axis and with a rectangular profile, as shown in Fig. 1, then with the proposed model that described by the sinusoidal function *y(x)*, as shown in Fig. 2. The fin width and length L, is immersed in a fluid with a constant bulk temperature T_F.

Morever, the fin base temperature T_0 is known.

In order to calculate the heat flux removed by such a fin it is necessary to determine the temperature distribution in the longitudinal section (plane *xy*). This distribution must satisfy the Laplace's equation:

$$\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right) - 0 \quad (1)$$

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With the boundary conditions:

$$T(0,y) = T_0 \tag{2}$$

$$\left[\frac{\partial T}{\partial x}\right]_{L,y} = -\frac{h_f}{k}\left[T(L,y) - T_F\right] \tag{3}$$

$$\left[\frac{\partial T}{\partial x}\right]_{L,y} = -\frac{h_f}{k}\left[T(L,y) - T_F\right] \tag{4}$$

$$\left[\frac{\partial T}{\partial x}\right]_{x,f(x)} - \left[\frac{df}{dx}\frac{\partial T}{\partial x}\right]_{x,f(x)} = -\frac{h\sqrt{\left(\frac{df}{dx}\right)^2_x}}{k}\left[T(x,f(x)) - T_F\right] \tag{5}$$

h and h_f being the convective heat transfer coefficients for the longitudinal fin surface and for the final surface and for the final transversal one, respectively, k being the thermal conductivity of the fin. Due to the complexity of the problem it is convenient to determine the temperature distribution numerically using for example our method.

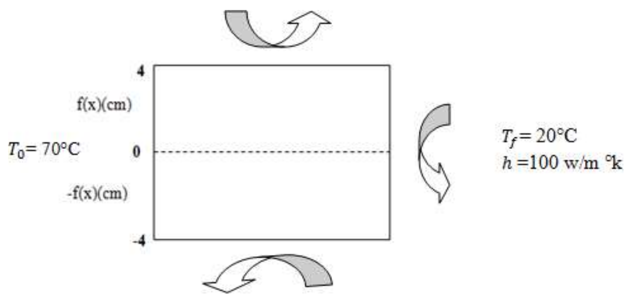

Fig.1. Longitudinal section of a symmetrical profile for a rectangular profile.

III. EFFECTIVENESS OF THE FIN

The fin performance can be evaluated on the basis of the compared effectiveness, i.e. the ratio between the heat flux (Q_d) dissipated by the heat remover with a generic profile and the heat flux (Q_r) removed by a fin of the same volume and length and with rectangular profile:

$$E = \frac{Q_r}{Q_r} \tag{6}$$

Let us then consider a rectangular fin of width 2 $\bar{f}$, $\bar{f}$ being the average value of *f(x):*

$$\bar{f} = \frac{1}{L}\int_0^L f(x)\,dx \tag{7}$$

The temperature distribution on the longitudinal section of such a fin must satisfy equation (1), the boundary conditions (2)-(4) and the following:

$$\left[\frac{\partial T}{\partial x}\right]_{x,f(x)} = -\frac{h}{k}\left[T(x,f(x)) - T_F\right] \tag{8}$$

Since both longitudinal and final transversal surfaces are plane we can assume equal to h_f. By integrating equation (1) with the above boundary conditions the following solutions is obtained [8]:

$$T(x,y) = T_F + \frac{2h(T_0 - T_F)}{k}\sum_{n=1}^{\infty}\left[\frac{1}{\left(\alpha_n^2 + \frac{h^2}{k^2}\right)\bar{f} + \frac{h}{k}}\frac{\cos(\alpha_n y)}{\cos(\alpha_n \bar{f})} \times \frac{\alpha_n \cosh\left[\alpha_n(L-x)\right] + \frac{h}{k}\sinh\left[\alpha_n(L-x)\right]}{\alpha_n \cosh(\alpha_n L) + \frac{h}{k}\sinh(\alpha_n L)}\right] \tag{9}$$

Being α_n the solution of the equation:

$$\alpha \tan(\alpha \bar{f}) = \frac{h}{k} \tag{10}$$

The heat flux dissipated for unit of length is then:

$$Q_r = 4h(T_0 - T_F)\sum_{n=1}^{\infty}\left|\frac{\tan(\alpha_n \bar{f})}{\left(\alpha_n^2 + \frac{h^2}{k^2}\right)\bar{f} + \frac{h}{k}}\frac{\alpha_n \sinh(\alpha_n L) + \frac{h}{k}\cosh(\alpha_n L)}{\alpha_n \cosh(\alpha_n L) + \frac{h}{k}\sinh(\alpha_n L)}\right| \tag{11}$$

We can calculate the heat flux dissipated by the remover for unit of width in the following way:

$$Q_d = 2\left(\sum_1 g_{oi}(T_0 - T_i) + g_{h0}(T_0 - T_F)\right) \tag{12}$$

g_{oi} being the thermal conductance between the fin base and the *ith* element, where it is zero for all the elements which are not adjacent to the fin base. While g_{h0} being the thermal conductance between the fin base and the coolant fluid.

IV. NUMERICAL PROCEDURE

We now propose the numerical method which is able to determine the values of the fin profile describing parameters which allow the highest compared effectiveness. We will consider heat removers for which the profile function $f(x)$ has a sinusoidal form and we have 2 cases:

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$$f_1(x) = w - a_0 \sin\left(\frac{2\pi x}{\lambda_0}\right) \quad (13)$$

$$f_2(x) = w + a_0 \sin\left(\frac{2\pi x}{\lambda_0}\right) \quad (14)$$

a_0, λ_0 being the amplitude and length of the sinusoid respectively.

At the beginning we have to fix the fin profile on one undulation to compare the performance of the two geometries and changing the amplitudes from 0.01 to 0.035 cm. in this way the fin profile which have best performance is then selected and reproduced in all the study.

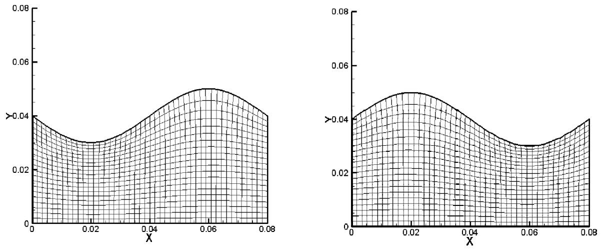

Fig. 2. Longitudinal section of the two cases $f_1(x)$ and $f_2(x)$.

The Laplace's equation is integrated in space using a finite volume method that is developed for an unstructured grid made up of quadrilaterals [9],[10],[11],[12].

For the integration around finite volume, the derivations of the flow equation must be converted into closed line integrals using same formulation of the Stokes theorem, which is described by the following equation:

$$\oint_E \vec{T}\cdot\vec{dr} = \iint_E rot\,\vec{T}\cdot\vec{n}dS \quad (15)$$

Where $\vec{dr}$ is the elementary arc dS is the elementary surface and is the normal vector to this surface. The computational domain is discredited on a quadrilateral unstructured grid where each node is the centre of polygonal cell constituted of four elements; all computed variables are stored at the centres of the polygonal as:

A. Approximation of the first derives

The convective terms are calculated at the node P (fig.2). The nodal finite volume descritization scheme is used for the discretization of the convective terms that appear in the governing equation. The first differences are calculated as:

$$\left(\frac{\partial T}{\partial x}\right)_c = \frac{1}{A_C}\int_{Sc} T.dy = \frac{1}{A_C}\sum_{i=1}^{NC}\frac{T_{i+1}+T_i}{2}(y_{i+1}-y_i) \quad (16)$$

$$\left(\frac{\partial T}{\partial y}\right)_c = \frac{1}{A_C}\int_{Sc} T.dx = \frac{1}{A_C}\sum_{i=1}^{NC}\frac{T_{i+1}+T_i}{2}(x_{i+1}-x_i) \quad (17)$$

Where A_C is the area of the polygonal control volume (1,2,3,…NE), T the temperature and $x,y$are the coordinate of the polygonal vertices, and I refers to the vertices number of external polygonal control volume.

B. Approximation of the second derives

This terms must be calculated at the node P and this achieved by computing the second order derivatives at the same point. The required second differences may be computed as:

$$\left(\frac{\partial^2 T}{\partial x^2}\right)_c = \left[\frac{\partial}{\partial x}\left(\frac{\partial T}{\partial x}\right)\right]_c = \frac{1}{A_{CI}}\int_{SCI} T.dy = \frac{1}{A_{CI}}\sum_{i=1}^{NC}\left(\frac{\partial T}{\partial x}\right)_E (y_{i+1}-y_i) \quad (18)$$

$$\left(\frac{\partial^2 T}{\partial y^2}\right)_c = \left[\frac{\partial}{\partial y}\left(\frac{\partial T}{\partial y}\right)\right]_c = \frac{1}{A_{CI}}\int_{SCI} T.dx = \frac{1}{A_{CI}}\sum_{i=1}^{NC}\left(\frac{\partial T}{\partial y}\right)_E (x_{i+1}-x_i) \quad (19)$$

$A_{CI}$is the area of polygonal control volume (2,4,…NE) (fig.2) and I refer to the vertices number of internal polygonal control volume. Where, the first differences at the middle of the edge are defined as:

$$\left(\frac{\partial T}{\partial x}\right)_E = \frac{1}{A_E}\int_{SE} T.dy = \frac{1}{A_E}\sum_{i=1}^{4}\frac{T_{i+1}+T_i}{2}(y_{i+1}-y_i) \quad (20)$$

$$\left(\frac{\partial T}{\partial y}\right)_E = -\frac{1}{A_E}\int_{SE} T.dx = -\frac{1}{A_E}\sum_{i=1}^{4}\frac{T_{i+1}+T_i}{2}(x_{i+1}-x_i) \quad (21)$$

A_E is the area of the quadrilateral control volume ((1),(2),(3),(4))(Fig.2.) and the four vertices of quadrilateral control volume.

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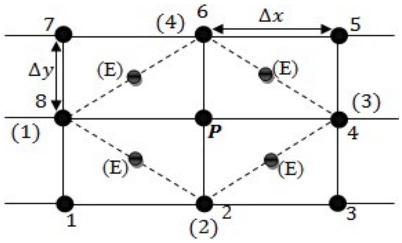


Fig. 3.The computational control volume structure.

V. RESULTS

Examination of errors and accuracy:

The numerical errors are calculated to show how the errors are improved by refining meshes and the order of accuracy is achieved. The maximum (***E****max*) and root – min – square (RMS) errors were calculated to compare with the analytical solution of the rectangular fin. These errors are defined by:

$$E_{max} = |(T_{analytic} - T_{numeric}| \quad (22)$$

$$RMS = \sqrt{\frac{\sum_{i=1}^{n}(T_{analytic} - T_{numeric})^2}{n}} \quad (23)$$

To examine the accuracy quantitatively, the maximum and RMS errors depending on grid sizes (nodal number) are presented in fig. the results are obtained when the thermal conductivity diffusion is equal to 200 w/m °k and the coefficient of heat transfer is equal to 100 w/m² °k .

Therefore, the error decreases at the same rate of root grid size. The solid lines in (fig.3.) shows how the root grid size decrease as the grid number increase (grid size decreases).

The maximum and RMS errors decrease at the same rate this verifies that the order accuracy of the method is achieved.

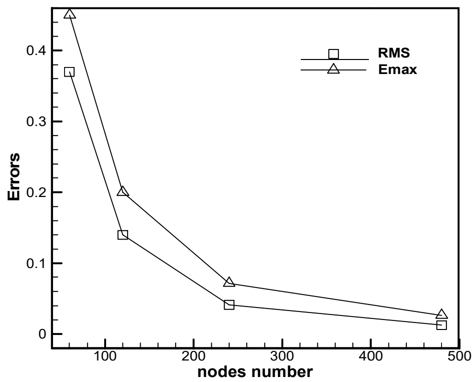


Fig. 4. The maximum and RMS error according to the grid size.

The proposed numerical method has been utilized in order to optimize the sinusoidal profile of aluminum fins. For the finite volumes model parameters.

The coefficient *h* has been assumed constant and equal to 100 w /m²°k.

The numerical method was utilized by choosing first of all the beginning of the function *f(x),* once from the top and in the second time from the lower (fig.2), then we compared the effectiveness of the sinusoidal profile with the rectangular.

The compare effectiveness, in fact, always grows with the fin that started from the lower (first undulation down then sublimate) (fig.5).

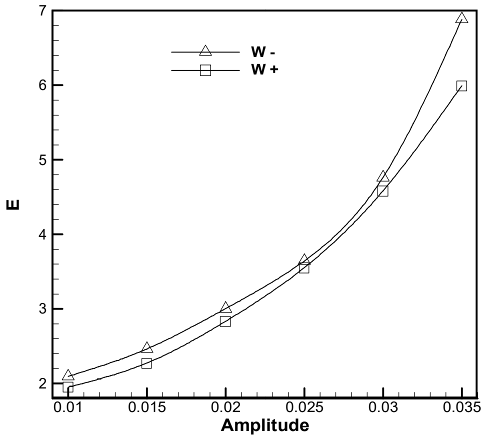
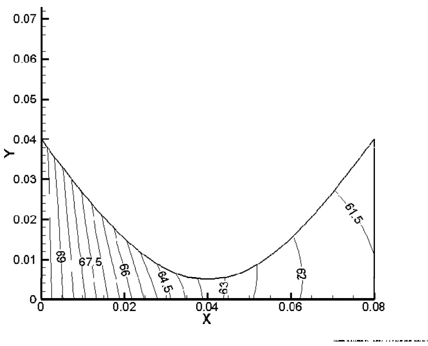


Fig. 5. The compare effectiveness.

The sinusoidal profiles obtained with our numerical method reported in (fig.6) (for the amplitude a_0=0.035):



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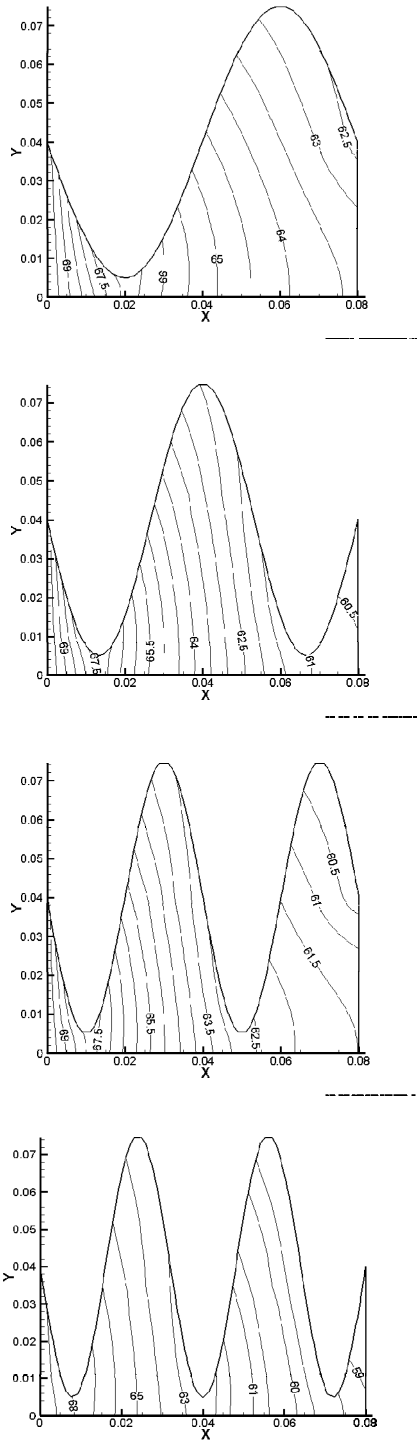

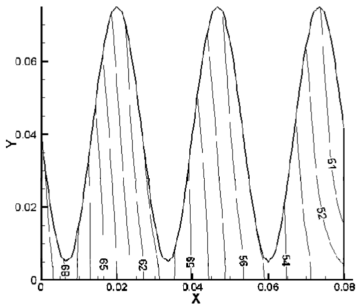

Fig. 7. Temperature distribution in different undulations with a_0= 0.035 cm and *k* =200 w/m.°k.

In order to better understand the compromise between the requirement of extending the heat transfer surfaces as much as possible and that of making the longitudinal thermal conductivity easier, the temperature distribution on the longitudinal section of the fin drop between the base and final section is higher.

In fig(f) the highest values obtained for the compared flux are shown in the order of the sinusoidal with different undulation in different amplitude . It's evident that the more we increasing the number of the undulation the more we have a higher flux evacuations.

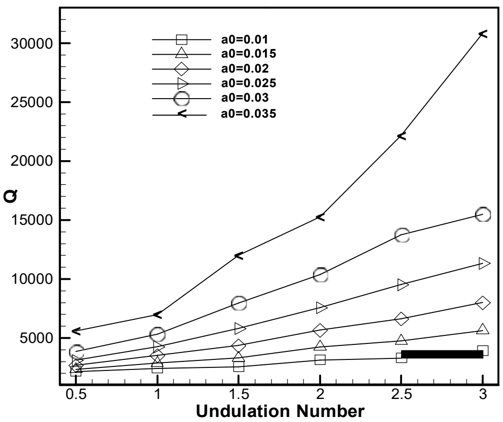

Fig. 8. The heat flux in different sinusoidal profiles with *k* =200 w/m.°k and *h* =100 w/m².°k.

VI. CONCLUSION

The numerical method proposed seems able to resolvedifferent problem of heat transfer especially when we have the geometries pretty complexes and the problem of optimizing the longitudinal profile of a fin, in order to improve its performances compared with those of a rectangular longitudinal section of the same volume and length.

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The optimizations examples shown in the article demonstrate that is possible to noticeably increase the compared effectiveness and flux of a fin by introducing undulations in its profile when the convective heat transfer coefficient is not very high. In such a condition. For example, a fin with three sinusoidal profile, even though it is not yet too difficult to build for the procedure, can remove even early twice as much heat flux as that dissipated by a rectangular fin of the same volume.

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