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Singular Perturbed Uncertain Multivariable System controlled by Internal Model Control in Discrete-Time

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***Abstract*—this paper present a singular perturbation approximation method for order reduction of uncertain multivariable linear system in discrete case. Then an Internal Model Control is applied for the reduced model to verify the effectiveness of this control.**

***Keywords*—Singular perturbations method, Order reduction, Gershgorine's circles, Internal Model Control, Uncertain multivariable systems.**

I. Introduction

Generally physical processes involve interacting dynamic phenomena of widely different speeds. The study of this class of processes using the techniques of singular perturbations [1],[2],[3],[4],[5], makes it possible to define a model of reduced order, thus leading to a reduction in the dimension of the associated regulators and to a simplification of the elaboration of the command.

In the case of certain systems, the discovery of the separability of slow and fast dynamics has been the object of various works using matrix norms [5], Gershgorine's circles [1],[6], lines [4], and defining a positive parameter in the associated state representation.

In this paper we present a condition to conserve the property of the separability of an uncertain linear discrete system using the Gershgorine's circles then, an IMC approach was extended to the reduced system [7],[8].

The remainder of this paper is organized as follows: Section II presents the basic IMC structure. Section III describes the proposed IMC structure, and finally an application of state-transition and bilinear discretization methods are applied for two-input-two-output system with specific IMC approach [7], [8], [9], [13], [14], [15].

II. Singular perturbed linear systems

The use of singular perturbations may allow the decomposition of the global system into many dynamic subsystems

A. Problem statement

Consider the uncertain linear multivariable discrete-time system

$$\begin{cases} X(k+1) = AX(k) + BV(k) \\ Y(k) = CX(k) \\ x, x(k) \in \mathbb{R}^n, V(k) \in \mathbb{R}^m \end{cases} \quad (1)$$

It is assumed that the uncertainty of the system lies only in the characteristic matrix A.

<u>Gershgorin's Theorem:</u>

Each eigenvalue δ of a matrix A of dimension (n×n) satisfies at least one of the following conditions:

$$\left|\delta\text{-}A_{ij}\right| \le \sum_{\substack{j=1 \\ j\neq i}}^{n} \left|A_{ij}\right| \qquad i = 1, 2, \ldots n \quad (1.1)$$

This condition means that all eigenvalues lie within the union U (C) of the circles Ci of (ci,Ri) such that:

$$c_i = A_{ii} \text{ and } R_i = \sum_{\substack{j=1 \\ j\neq i}}^{n} \left|A_{ij}\right|, i = 1, 2, \ldots n \quad (1.2)$$

The rays are taken on the elements of each row of the matrix A, we shall have the same conclusion if these are taken on the columns of A. If the set of circles U(C) can be partitioned into two disjoint sets V(C) and W(C) then the system has two different separable dynamics with:

$$V(C) = \{C_i, (c_i, R_i), R_i = 1, 2, \ldots n_1\} \quad (1.3)$$

$$W(C) = \{C_k, (c_k, R_k), R_k, k = n_1 + 1, \ldots n\} \quad (1.4)$$

As a consequence, the spectrum of the system is divided into a set of slow eigenvalues in V (C) and a set of fast eigenvalues in W (C) with: $V(C) \cap W(C) = \varnothing$. The separability factor μ can then be approximated by the following expression: $\mu = \max_{i,k} \dfrac{\beta(i,k)}{\alpha(i,k)}$ (1.5)

Our problem consists to search a condition which conserve the property of the separability using the method of Gershgorine circles for our system.

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B. Conservation of the separation of dynamics

When the system has the property of double time scale, it is written in the following singularly perturbed form (2):

$$\begin{bmatrix} X_1(k+1) \\ X_2(k+1) \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12}* \\ A_{21} & A_{22}* \end{bmatrix} \begin{bmatrix} X_1(k) \\ \mu X_2(k) \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} V(k) \qquad (2.1)$$

$$Y = \begin{bmatrix} C_1 & C_2* \end{bmatrix} \begin{bmatrix} X_1(k) \\ \mu X_2(k) \end{bmatrix} \qquad (2.2)$$

When $X_1, X_1(k) \in \mathbb{R}^{n_1}$ are the slow vectors of the considered system and $X_2, X_2(k) \in \mathbb{R}^{n_2}$ are the fast vectors of the considered system.

$$A_{ij}^* = \frac{A_{ij}}{\mu}, \quad C_2^* = \frac{C_2}{\mu}, \quad n_1 + n_2 = n$$

The change in μ=0 to μ>0 is called a singular perturbation [2],[3],[4],[5], in this section we derive the fast and slow subsystems comprising a high order system.

C. Decoupling slow part -fast part

The fast part is expressed by the equations (3)

$$X_r(k+1) = (A_{22} - A_{21}A_{11}^{-1}A_{12})X_r(k) + (B_2 - A_{21}A_{11}^{-1}B_1)V_r(k) \qquad (3.1)$$

$$Y_r(k) = (C_2 - C_1A_{11}^{-1}A_{12})X_r(k) + C_1A_{11}^{-1}B_1V_r(k) \qquad (3.2)$$

$$X_{r0} = X_2(0) - A_{21}A_{11}^{-1}X_1(0) \qquad (3.3)$$

If system (2) is asymptotically stable, the fast modes are important only during a short transient period, and decay rapidly. After that period, they are negligible and the behaviour of the full system (2) can be described by its slow modes.

The slow part is obtained by cancelling μ in both system (4)

$$X_1(k+1) = A_{11}X_1(k) + B_1V_1(k) \qquad (4.1)$$

$$Y_1(k) = C_1X_1(k) \qquad (4.2)$$

$$X_{10} = X_0 \qquad (4.3)$$

It should be emphasized that letting $\mu \longrightarrow 0$ has reduced the $n_1 + n_2$ dimensional system (3.1), (3.2) to the n_1 dimensional system (4).

III. PROPOSED IMC FOR MULTIVARIABLE UNCERTAIN LINEAR SYSTEMS IN DISCRETE CASE

The IMC structure use explicitly the model [16], as a controller algorithm of the plant that is stable in open loop. In this case, the inverse model can obtain the controller [17], [18], [19], [20]. The IMC structure for multivariable discrete-time system is shown in Figure 1.

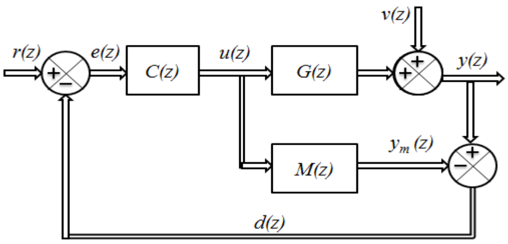

Fig. 1 Structure for inverse model

G(z): the process	e(z): the reference vector
y(z): the Output vector of the process	u(z): the control vector
	d(z) :difference between outputs model and outputs process
v(z): the disturbance vector	
y_m (z): the model output vector	r(z): the reference vector

The expression of the proposed controller *C(z)* is given by the following equation.

$$C(z) = A_1/(I + A_1M(z)) = I/(A_1^{-1} + M(z)) \qquad (5)$$

A_1 is a gain matrix and *M(z)* is the transfer matrix of the model. *A_1* is expressed by the following expression $A_1 = \alpha \times I$ where *I* is the identity matrix and α is a chosen coefficient, *A_1^{-1}* is the inverse of the gain *A_1*.

A. Stability

The system is stable if and only if each block of the IMC structure is stable in open loop. To ensure the process's stability we are interested in the determination of the non-localized extreme models using the indirect method based on the algebraic Kharitonov's approach [10], [11], [12].

B. Static Error

The precision of the system is evaluated by the difference between the output *y(z)* and the reference signal *r(z).* If the reference and the disturbances are chosen as vector of steps of amplitude equal to 1,

$$E = \lim_{z \to 1} (1 - z^{-1}) y(z) \qquad (6)$$

E is a vector of dimension *(m).* We can conclude that the static error between the real and the desired output is asymptotically zero.

IV. APPLICATION

Let's consider the interconnected multivariable system defined by the following transfer function $H_i(p)$.

$$H_1(p) = (p+\lambda)/(1+\tau_1 p)(1+\tau_2 p) \qquad (7.1)$$

$$H_2(p) = (p+\psi)/(1+T_1 p)(1+T_2 p) \qquad (7.2)$$

The global system S composed of transfer function can then be represented by the following differential equations:

$$G(s) = \begin{cases} \dot{X} = AX + BU \\ Y = CX \end{cases} \qquad (8)$$

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With:

$$A=\begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{-(1+k_1\lambda)}{\tau_1\tau_2} & \frac{-(k_1+\tau_1+\tau_2)}{\tau_1\tau_2} & \frac{-b_{12}\lambda}{\tau_1\tau_2} & \frac{-b_{12}}{\tau_1\tau_2} \\ 0 & 0 & 0 & 1 \\ \frac{-b_{21}\delta}{T_1T_2} & \frac{-b_{21}}{T_1T_2} & \frac{(1+k_2\delta)}{T_1T_2} & \frac{-(k_2+T_1+T_2)}{T_1T_2} \end{bmatrix}$$

$$B=\begin{bmatrix} 0 & 0 \\ k_1 & b_{21} \\ 0 & 0 \\ b_{12} & k_2 \end{bmatrix};\ C=\begin{bmatrix} \frac{\lambda}{\tau_1\tau_2} & \frac{1}{\tau_1\tau_2} & 0 & 0 \\ 0 & 0 & \frac{\delta}{T_1T_2} & \frac{1}{T_1T_2} \end{bmatrix}$$

The step response of the system is figured in the following figure.

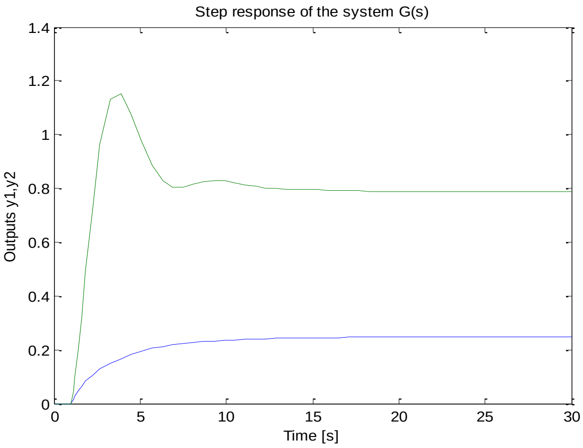

Fig. 2 Step Response for the system G(s)

The application of the Gershgorine circles for the last system is presented by the following simulations.

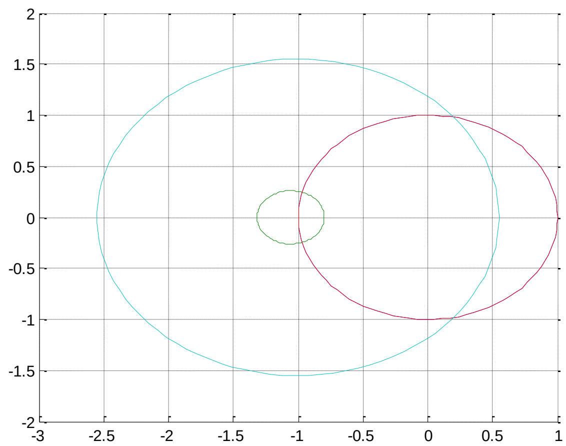

Fig. 3 Gershgorine's circles for the matrix A

$C_1(0,1)$, $C_2(-1.05,0.26)$, $C_3(0,1)$, $C_4(-1,1.5)$

We note that the separability is not preserved in this case, which leads us to make a calibration to appear this property of separability. It is necessary to choose a calibration matrix Ca such that:

$$Ca=\begin{bmatrix} c_1 & 0 & 0 & 0 \\ 0 & c_2 & 0 & 0 \\ 0 & 0 & c_3 & 0 \\ 0 & 0 & 0 & c_4 \end{bmatrix}$$

It is assumed that the system parameters are:

$\tau_1 = 1.3s; \tau_2 4s; k_1 = 0.2; k_2 3$

$\delta = 4; \lambda = 0.4; T_1 = 2s; T_2 = 5s$

$$A=\begin{bmatrix} 0 & 1 & 0 & 0 \\ -3.32 & -16.92 & -0.246 & -1.538 \\ 0 & 0 & 0 & 1 \\ -5 & -1.25 & -32.5 & -25 \end{bmatrix}$$

In our case the calibration matrix is equal to:

$$Ca=\begin{bmatrix} 0 & 1 & 0 & 5 \\ 1 & -16.92 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ -1 & 1 & -16 & -25 \end{bmatrix}$$

A. Case of uncertain parameter b_{12}

Let's consider that the uncertain parameter b_{12} =0.2 and the uncertainty interval is [-5, 5]. The application of the Gershgorine's theorem insure that the system have two-time scale.

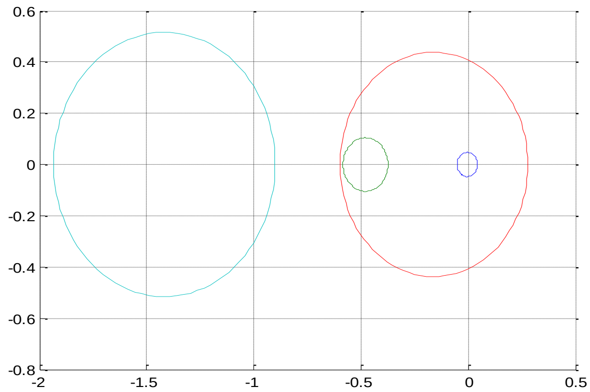

Fig. 4 Gershgorine's circles for uncertain parameter b_{12}

b_{12}	V(C)	W(C)	Diff(V,W) =αi/αk	µ	µ max
-5	0.77	1.032	-0.59/-0.9	0.655	0.827
-1	0.318	1.032	-0.596/-0.901	0.661	0.760
-0.5	0.318	1.032	-0.596/-0.901	0.661	0.760
-0.2	0.318	1.032	-0.596/-0.901	0.661	0.760
0	0.86	1.03	-0.59/-0.9	0.655	0.723
0.5	0.32	1.032	-0.597/-0.901	0.662	0.752
5	0.86	1.03	-0.59/-0.9	0.655	0.718

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B. *Case of uncertain parameter* b_{21}

Now let's consider an uncertain parameter b_{21} =0.5 and the uncertainty interval is included between ϵ [-30, 30]. The application of the Gershgorine's theorem show us that the system have two-time scale.

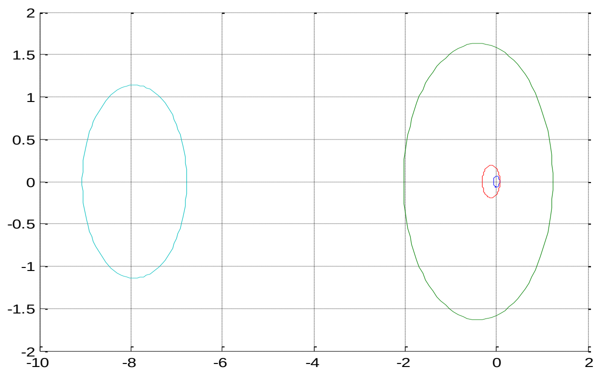

Fig. 5 Gershgorine's circles for uncertain parameter b_{21}

b_{21}	**W(C)**	**V(C)**	**Diff(V,W)=αi/αk**	**μ max**
-30	---	---	--	No conclusion
-20	---	---	--	No conclusion
-10	0.35	4.048	-2.591/-2.701	0.949
-5	0.308	2.215	-1.635/-2.027	0.761
-2	1.392	0.703	-1.062/-1.413	0.722
-1.5	1.286	0.769	-0.966/-1.31	0.708
-1	1.18	0.832	-0.87/-1.208	0.687
-0.5	1.074	0.9	-0.77/-1.106	0.684
0	0.969	0.967	-0.67/-1.003	0.672
1	0.84	1.1	-0.58/-0.798	0.814
2	---	---	--	No conclusion
5	---	---	--	No conclusion
10	2.29	3.259	-2.029/-6.778	0.3684
20	---	---	--	No conclusion
30	---	---	--	No conclusion

C. *Case of IMC control for reduced model*

Let' s consider the imperfect modeling characterized by the absence of disturbances, such that v(z) =0 where the model is chosen diffrent to the plant M(z)≠G(z) and the sampling time is equal to T=0.1 s. The chosen matrix A1 is equal to A1=70×I. The reduced systems are associated by applying the change of variable $P=\begin{bmatrix}1&0&0&0\\ \alpha_1&1&0&0\\ 0&0&1&0\\ 0&0&\alpha_2&1\end{bmatrix}$

α_1, α_2 are two distinct negative constant parameters which can be chosen arbitrarily. We obtained an estimated matrix included the subsystems

The fast subsystem and the slow subsystem has been computed with the following matrix.

$$A_{11}=\begin{bmatrix}-0.0039&-0.0203&0.0290\\ -0.0205&-0.1595&0.0352\\ 0.0285&0.0278&-0.4790\end{bmatrix} A_{12}=\begin{bmatrix}-0.0562\\ -0.4525\\ 0.0488\end{bmatrix}$$

$$A_{21}=\begin{bmatrix}-0.0605&-0.4976&0.0419\end{bmatrix} A_{22}=-1.4172$$

The reference signals r_1, r_2 are chosen as vector of steps of amplitude equal to 1. The following simulations present the two outputs y1 and y2 of the slow subsystem.

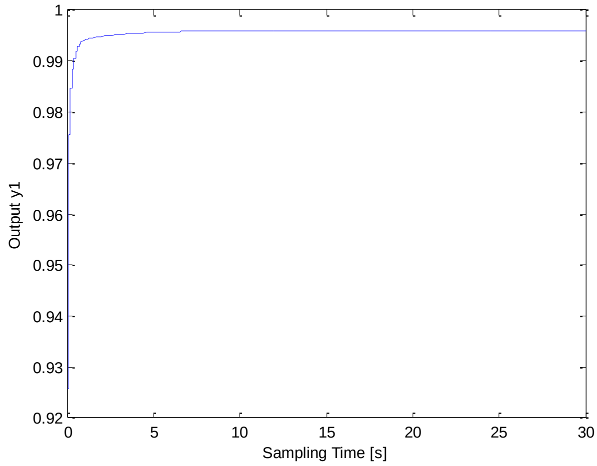

Fig. 6 Output signal y_1 for slow subsystem

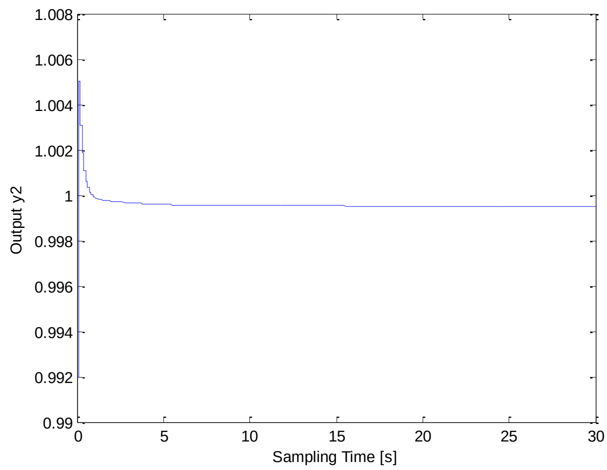

Fig. 7 Output signal y_2 for slow subsystem

We remark that the reduced uncertain multivariable system controlled by IMC is able to conserve stability in the discrete case and to reach the reference signal with a small difference of amplitude.

D. *Case of disturbed system*

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Now let's consider the presence of a disturbance vector and let's show its effect in the case of the IMC proposed control. The disturbances are applied at the time T=15s. $A1=10 \times I$. The simulations results of the slow subsystem are given by the following plots.

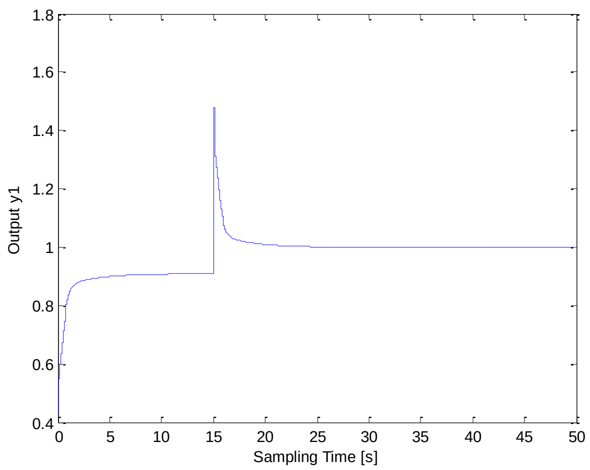

Fig. 8 Output signal y_1 for disturbed slow subsystem

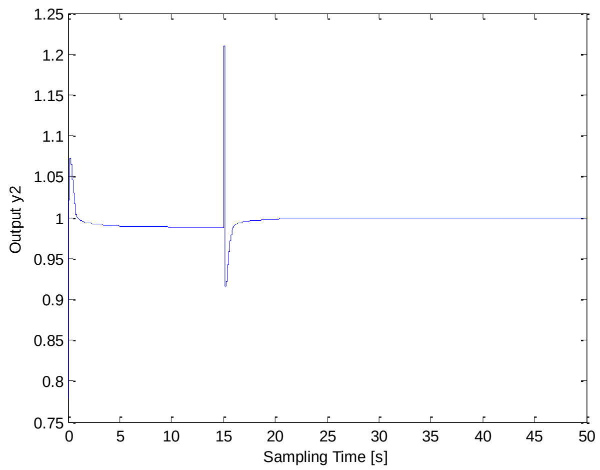

Fig. 9 Output signal y_2 for disturbed slow subsystem

It is clear that the system outputs reach perfectly the input reference and the simulations show a robust behavior even on the presence of disturbances affecting directly the process outputs. We conclude that the proposed IMC approach rejects disturbances and ensure again its robustness.

V. Conclusions

In this paper a new approach for IMC of linear multivariable uncertain singular perturbed systems is developed in discrete case. The realized research is an extension of the IMC concept defined for continuous monovariable uncertain linear systems. An application of this proposed structure is proposed to test the effectiveness of this control to conserve stability and robustness despite the presence of both the reduced model and the perturbation. The simulation results show the proposed approach capability to preserve the system performances, to maintain the stability and to reject the external disturbances.

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