

# High gain observer based visual servoing for depth estimation in robotics

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**Abstract**—This paper proposes a mono-vision depth estimation method for mobile robots, based on high gain observers. The proposed method switches between multiple observers in order to gauge the distance between the camera and the target object. Simulations and experimental results illustrate the practical applicability and benefits of this method for visual servoing.

**Key words:** Range estimation, depth measurement, mono-vision, non-linear observer, mobile robot.

## I. INTRODUCTION

Depth estimation is one of the most discussed problems in robotics and computer vision. It is of key importance for range finding in mobile robots as their environment can vary, through which a definite trajectory is impractical due to detecting static and dynamic obstacles [1]. In presence of dynamic obstacles, the depth or range estimation algorithm needs to calculate the position as well as the relative velocity of an obstacle in order for the robot to effectively replan its trajectory. This problem has led to the rise of mono-vision range estimation algorithms in contemporary research. Contrary to conventional range estimation methods such as laser reflectometry, stereovision etc, the reliance of single camera based depth calculation through the velocity of robots and/or objects inherently provides both pieces of information required for trajectory replanning.

The objective of the vision problem is to derive the 3D motion variables from the 2D image produced by a camera. Due to the loss of depth information, the use of computer vision has mostly remained limited to feature and shape identification of obstacles, not their distance. In [2], the authors have used mobile camera network for localization problem in cooperative tasks for multi non-holonomic mobile robots. In this case, each robot is equipped with vision sensor and provides information to other robots. The trajectory tracking control for arm manipulator [3] consists of using a homography-based camera technique, and the visual control is designed to track the desired trajectory of the end-effector equipped with fixed camera. In this work, two important parameters are lacking: depth measurement and 3D object model, and a

Lyapunov-based adaptive control strategy is designed to track the desired trajectory via reference and sequence images provided by the configured camera. However, if the robot and/or the obstacle are in motion, this missing information can be reconstructed by analyzing the difference in obstacle features in consecutive images. Some researchers have used a technique called Image Based Visual Servoing (IBVS) to deal with the control of robot or camera motion [4]. With this technique, there is some drawback concerning the estimation of the Interaction matrix and stability condition which involve the 3D parameters information. In IBVS control [5], the non-linear observer is designed to recover the unmeasurable parameter of the system i.e. depth measurement through the motion of the robot or camera. [6] presents their works on the non-linear observer to get the feedback information on range estimation from a moving camera with a predefined 3D motion project on 2D image space. In this case, the observer is based on the method called Immersion and Invariance.

In this paper, we will introduce an observer based method to recover the depth value during the motion of robot equipped with a single on-board camera. This method is based on nonlinear high gain observer [7], [8], [9], constructed using a model based on 2D camera projection of a fixed point located in space. High gain observers provide better performance as compared to contemporary methods employing extended kalman filters [10]. The performance of the proposed high gain observer based visual servoing method has been evaluated using simulations, which provide promising results for further continuation of the research.

The rest of the paper is organized as follows: Section II presents the perspective of camera model, Section III contains the observer design and Section IV presents simulation results. Concluding remarks are presented in Section V.

## II. PROBLEM FORMULATION

### A. Mobile robot model

The work space of a mobile 2D robot is presented in Fig. 1 and its kinematic equations are given as follows:

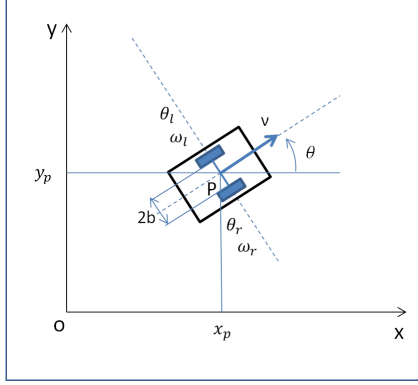


Fig. 1. Non-holonomic mobile robot model

$$\begin{cases} \dot{x} = v \cos \theta \\ \dot{y} = v \sin \theta \\ \dot{\theta} = \omega \end{cases} \quad (1)$$

The plane of motion is  $(x, y)$  and  $\theta$  represents the orientation of the mobile robot in terms of its angle w.r.t. the  $x$  axis. In this model, the control inputs are the linear velocity  $v$  in the direction of the motion and the angular velocity  $\omega$  about the  $z$  axis.

### B. Image model

The camera perspective model provides a relationship between the physical position of a point in the visual space of a camera and its position in the image plane. The image is formed by intersecting points with the image plane from the envisioned rays of the point or object through the center of the projection or pinhole of the camera [11].

Let us consider Fig. 2, which depicts a camera 'c' fixed

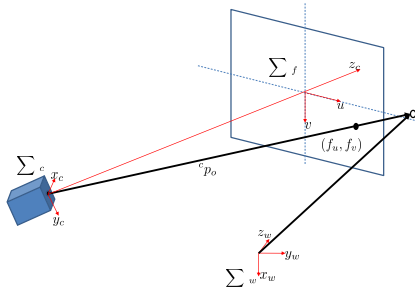


Fig. 2. Camera and image coordinate frames

on a robot, whose position is identified by a reference frame  $\sum c$ , moving in the universal world frame  $\sum w$ . The origin of  $\sum c$  is situated at the aperture of the camera. The image plane is situated at a distance  $\lambda$  from the camera's origin, which is equivalent to the focal

length of the camera's lens. The dynamics of a fixed point  $(x_\omega, y_\omega, z_\omega)$  can be determined in the moving frame 'c' as:

$$\begin{cases} \dot{x}_c = \omega_3 x_c - \omega_2 z_c - v_1 \\ \dot{y}_c = -\omega_3 x_c + \omega_1 z_c - v_2 \\ \dot{z}_c = \omega_2 x_c - \omega_1 y_c - v_3 \end{cases} \quad (2)$$

The image plane is 2D, identified by the frame of reference  $\sum f(u, v)$ , where  $(f_u, f_v)$  mark the projection of a physical point  $O(x_c, y_c, z_c)$  on the image. The  $z$  axis of  $\sum c$  passes through the origin of  $\sum f$ . The relationship between  $\sum f$  and  $\sum w$  can be established as

$$f_u = \lambda \frac{x_c}{Z}, f_v = \lambda \frac{y_c}{Z} \quad (3)$$

Let suppose that the camera (the entire robot) moves with linear velocities  $(v_1, v_2, v_3)$  and angular velocities  $(\omega_1, \omega_2, \omega_3)$ . Then we can derive the velocity of the image projection of the point from (3) as:

$$\begin{cases} \dot{f}_u = -\frac{\lambda}{z_c} v_1 + \frac{f_u}{z_c} v_3 + \frac{f_u f_v}{\lambda} \omega_1 - \left(\lambda + \frac{f_u^2}{\lambda}\right) \omega_2 + f_v \omega_3 \\ \dot{f}_v = -\frac{\lambda}{z_c} v_2 + \frac{f_v}{z_c} v_3 + \left(\lambda + \frac{f_v^2}{\lambda}\right) \omega_1 - \frac{f_u f_v}{\lambda} \omega_2 - f_u \omega_3 \end{cases} \quad (4)$$

The image coordinates  $f_u$  and  $f_v$  are our measurements, obtained from image processing. Our aim in this paper is to design a nonlinear observer to estimate the value of  $z_c$  i.e. the missing depth information. From Eq. (4), we will rewrite the system in the usual mathematical form:

$$\begin{aligned} x &= (x_1 \ x_2 \ x_3)^T = (f_u \ f_v \ 1/z_c)^T \\ y &= (x_1 \ x_2)^T \\ u &= (u_1, \dots, u_6)^T = (v_c^T \ \omega_c^T)^T \end{aligned}$$

Then the state space representation can be given by

$$\begin{cases} \dot{x} = \alpha(x_{12}, x_3)u \\ y = [x_1 \ x_2] \end{cases}, \quad (5)$$

where  $\alpha(x_{12}, 3)$  is the matrix

$$\begin{bmatrix} -\lambda x_3 & 0 & x_1 x_3 & \frac{x_1 x_2}{\lambda} & -\left(\lambda + \frac{x_1^2}{\lambda}\right) & x_2 \\ 0 & -\lambda x_3 & x_2 x_3 & \left(\lambda + \frac{x_2^2}{\lambda}\right) & -\frac{x_1 x_2}{\lambda} & -x_1 \\ 0 & 0 & x_3^2 & \frac{x_2 x_3}{\lambda} & -\frac{x_1 x_3}{\lambda} & 0 \end{bmatrix} \quad (6)$$

The objective now is to estimate the state variable  $x_3$  from the measured outputs  $x_1, x_2$ , to find  $z_c$ .

### III. OBSERVER DESIGN

From Eq. (6), we can formulate the following sub systems :

$$\begin{cases} \dot{x}_1 = a_1(u, y)x_3 + \phi_1^1(u, y) \\ \dot{x}_3 = \phi_2^1(u, y, x_3) \\ y = x_1 \end{cases} \quad (7)$$

$$\begin{cases} \dot{x}_2 &= a_1(u, y)x_3 + \phi_1^1(u, y) \\ \dot{x}_3 &= \phi_2^1(u, y, x_3) \\ y &= x_1 \end{cases} \quad (8)$$

where

$$\begin{aligned} a_1(u, y) &= \dots \\ a_2(u, y) &= \dots \end{aligned}$$

From observability point of view, (7) is observable whenever  $a_1(u, y) \neq 0$ , and likewise, (7) is observable whenever  $a_2(u, y) \neq 0$ . Let us now set

$$A_1(t) = \begin{bmatrix} 0 & a_1(t) \\ 0 & 0 \end{bmatrix}, \quad A_2(t) = \begin{bmatrix} 0 & a_2(t) \\ 0 & 0 \end{bmatrix} \quad (9)$$

Then, (7) and (8) can be rewritten as

$$\begin{cases} \dot{x}^1 &= A_1(t)x^1 + \phi^1(u, x^1, y_2) \\ y_1(t) &= C_1x^1(t) \end{cases} \quad (10)$$

$$\begin{cases} \dot{x}^2 &= A_2(t)x^2 + \phi^1(u, x^2, y_1) \\ y_2(t) &= C_2x^2(t) \end{cases} \quad (11)$$

where

$$\begin{aligned} \phi_1 &= \begin{pmatrix} \phi_1^1(u, y) \\ \phi_1^2(u, y) \end{pmatrix}, \quad \phi_2 = \begin{pmatrix} \phi_2^1(u, y) \\ \phi_2^2(u, y) \end{pmatrix} \\ C_1 &= C_2 = [1 \ 0] \end{aligned}$$

Now, considering the following hypotheses :

- $\mathcal{H}1 \exists t_0 \geq 0 | a_i(t) \geq \epsilon \forall t \geq t_0$
- $\mathcal{H}2 \exists t_0 \geq 0 | a_i(t) \leq -\epsilon \forall t \geq t_0$

If one of these hypotheses holds true, then an observer for the System (10) and (11) can be constructed in the following form:

$$\dot{\hat{x}}^i = A_i(t)\hat{x}^i(t) + \phi^i(u, y, \hat{x}_3) + a_i(t)K_\theta(\hat{x}_1 - y_i) \quad (12)$$

for  $a_i \geq \epsilon$

$$\dot{\hat{x}}^i = A_i(t)\hat{x}^i(t) + \phi^i(u, y, \hat{x}_3) + a_i(t)EK_\theta(\hat{x}_1 - y_i) \quad (13)$$

for  $a_i \leq -\epsilon$ . Here

$$K_\theta = \begin{pmatrix} \theta K_1 \\ \theta^2 K_2 \end{pmatrix} \text{ and } E = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

The parameter  $\theta > 0$  is the tuning gain, also called high gain as its value is generally large. The matrix

$$K = \begin{pmatrix} K_1 \\ K_2 \end{pmatrix}$$

is chosen such that  $\begin{pmatrix} K_1 & 1 \\ K_2 & 0 \end{pmatrix}$  is Hurwitz.

In practice,  $a_i(t)$  may vanish and change the sign. In this case, the alternative solution is to switch between observers (12) and (13). If the average dwell time is assumed to be larger than a positive constant (which is generally true in our application), then the following derivation will demonstrate that this presented switching observer can be used to estimate  $x_3(t)$ .

### A. Proof of the convergence

For the sake of simplicity, we will give the proof of the switching observer in the case where only one switch is required. In fact, four switches are required in order to estimate  $x_3(t)$ .

Consider the following system:

$$\begin{cases} \dot{\xi}(t) &= a(t)A\xi(t) + \phi(\xi(t), t) \\ y(t) &= C\xi(t) \end{cases} \quad (14)$$

where  $\xi$  is a column vector of  $\mathbb{R}^n$ ,  $a(t)$  is a bounded continuous scalar function,  $a(\cdot)$  may depend on the output  $y(t)$  and other known signals.  $\phi$  is a Lipschitz function;  $A$  and  $C$  are constant matrices defined by:

for  $i = 1, \dots, n$ ,  $\phi_i(\xi, t) = \phi_i(\xi_1, \dots, \xi_i, t)$ ;  $C = \begin{pmatrix} 1 & 0 & \dots & 0 \end{pmatrix}$ , and,

$$A = \begin{pmatrix} 0 & 1 & \dots & 0 \\ 0 & \ddots & \dots & 0 \\ 0 & & 0 & 1 \\ 0 & 0 & \dots & 0 \end{pmatrix};$$

The following assumptions are stated:

- **Assumption 1:** there exists a constant  $c > 0$  such that for every  $\xi, \xi'$ ,  $\|\phi_i(\xi, t) - \phi_i(\xi', t)\| \leq c\|\xi - \xi'\|$ .
- **Assumption 2:** The real positive line  $\mathbb{R}^+$  is partitioned into intervals  $[t_i, t_{i+1}]$  on which  $|a(t)| \geq \epsilon$  or  $|a(t)| \leq \epsilon$ , for some fixed  $\epsilon$ . For instance, we assume that  $|a(t)| \geq \epsilon$  for  $t \in ]t_{2i-1}, t_{2i}[$  and  $|a(t)| \leq \epsilon$  for  $t \in ]t_{2i}, t_{2i+1}[$ , with the additional condition: there exist  $\delta_1 > 0$ ,  $\delta_2 > 0$ , such that  $t_{2i} - t_{2i-1} \geq \delta_1$  and  $t_{2i+1} - t_{2i} \leq \delta_2$ . In particular if  $a(t) \geq \epsilon$ , for every  $t \geq 0$  (resp.  $a(t) \leq -\epsilon$ , for every  $t \geq 0$ ), the **Assumption 2** holds.

Set  $\Delta_\theta = \text{diag}(\theta, \theta^2, \dots, \theta^n)$  to be the  $n \times n$  diagonal matrix. Let  $K$  be a constant column vector such that  $A + KC$  is an  $n \times n$  Hurwitz matrix.

$$\text{Finally, set } E = \begin{pmatrix} -1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ 0 & \ddots & (-1)^{n-1} & 0 \\ 0 & 0 & \dots & (-1)^n \end{pmatrix}.$$

The candidate observer for system (14) takes the following form:

$$\begin{cases} \text{If } a(t) > 0, \text{ and if } t_{2i} \leq t < t_{2i+1}, \\ \dot{\hat{\xi}}(t) &= a(t)A\hat{\xi}(t) + \phi(\hat{\xi}(t), t) + a(t)\Delta_\theta K(C\hat{\xi}(t) - y(t)), \\ \text{If } a(t) < 0, \text{ and if } t_{2i} \leq t < t_{2i+1}, \\ \dot{\hat{\xi}}(t) &= a(t)A\hat{\xi}(t) + \phi(\hat{\xi}(t), t) + a(t)E\Delta_\theta K(C\hat{\xi}(t) - y(t)), \\ \text{If } t_{2i-1} \leq t < t_{2i}, \\ \dot{\hat{\xi}}(t) &= a(t)A\hat{\xi}(t) + \phi(\hat{\xi}(t), t) \end{cases} \quad (15)$$

Now we can state the following theorem:

**Theorem 1.** If  $\tau_0 = t_{2i_0-1}$  is the smallest initial time and  $\tau_0 \leq \delta_2$ , then under **Assumption 1** and **Assumption**

2, there exists  $\theta_0 \geq 1$ , such that

$$\|\widehat{\xi}(t) - \xi(t)\| \leq \alpha_0 \theta^N e^{-\beta(\theta)t} \|\widehat{\xi}(0) - \xi(0)\| \quad (16)$$

for every  $\theta \geq \theta_0$ , and for every  $t \geq \tau_0$ , where  $\alpha_0$  is a constant depending only on  $\theta_0$  and the upper bound of  $|a(\cdot)|$ , and the constant  $\beta(\theta)$  is such that  $\lim_{\theta \rightarrow \infty} \frac{\beta(\theta)}{\theta} = \beta_0$ , where  $\beta_0 > 0$  is a constant.

**Remark 1.** The estimation (17) implies that for any  $\tau > \tau_0$  that is sufficiently close  $\tau_0$ , and for every  $\eta > 0$  sufficiently small; there exists  $\theta_0 > 0$  (sufficiently large) such that  $\|\widehat{\xi}(t) - \xi(t)\| \leq \eta$ , for every  $t \geq \tau$ . This means that the convergence rate of the above observer may be chosen arbitrary from the initial time  $\tau_0$ .

**Proof** The proof of the theorem can be obtain using classical high gain observer technique (see for instance [7], [8], [9], [12], [13], [14], [15]).

Set  $e(t) = \widehat{\xi}(t) - \xi(t)$  to be the error estimation between the state of the observer (15) and that of the system (6). Our aim is to prove that  $e(t)$  satisfies the estimation proposed in the theorem. To do so, let us make the following linear change of coordinates  $\bar{e}(t) = \Delta_{\theta^{-1}} e(t)$ , and consider the dynamical equation associated to  $\bar{e}(t)$ , which is of the form:

$$\begin{cases} \text{If } t_{2i-1} \leq t < t_{2i}, \\ \dot{\bar{e}}(t) = a(t)\theta(A + KC)\bar{e}(t) \\ \quad + \Delta_{\theta^{-1}}(\phi(\widehat{\xi}(t)) - \phi(\xi(t))), \text{ if } a(t) > 0 \\ \dot{\bar{e}}(t) = -a(t)\theta(A + KC)\bar{e}(t) \\ \quad + E\Delta_{\theta^{-1}}(\phi(\widehat{\xi}(t)) - \phi(\xi(t))), \text{ if } a(t) < 0 \\ \text{If } t_{2i} \leq t < t_{2i+1}, \\ \dot{\bar{e}}(t) = \theta A\bar{e}(t) + \Delta_{\theta^{-1}}(\phi(\widehat{\xi}(t)) \\ \quad - \phi(\xi(t))), \text{ if } a(t) < 0 \end{cases} \quad (17)$$

Since  $A + KC$  is Hurwitz, let  $P$  be a symmetric positive definite matrix such that  $P(A + KC) + (A + KC)^T = -I$ , where  $I$  is the  $n \times n$  identity matrix, and set  $V(\bar{e}) = \bar{e}^T P \bar{e}$ .

Without lost of generality, assuming that at  $a(t) \geq \epsilon$  in some  $[t_{2i-1}, t_{2i}[$ , a simple computation gives:

$$\dot{V} = a(t)\theta \bar{e}^T [(A + KC)^T P + P(A + KC)] \bar{e} + 2\bar{e}^T P \Delta_{\theta^{-1}}(\phi(\widehat{\xi}) - \phi(\xi)) \quad (18)$$

The  $j$ th component of  $\Delta_{\theta^{-1}}(\phi(\widehat{\xi}) - \phi(\xi))$  is equal to  $\theta^{-j}(\phi_j(\widehat{\xi}_1, \dots, \widehat{\xi}_j) - \phi_j(\xi_1, \dots, \xi_j))$ , and that  $\theta^{-j}|\phi_j(\widehat{\xi}_1, \dots, \widehat{\xi}_j) - \phi_j(\xi_1, \dots, \xi_j)| \leq \theta^{-j} c \sqrt{(\widehat{\xi}_1 - \xi_1)^2 + \dots + (\widehat{\xi}_j - \xi_j)^2}$ , where  $c$  is the Lipschitz constant of  $\phi$  given in **Assumption 1**. Hence, for  $\theta \geq 1$ , we have  $\theta^{-j}|\phi_j(\widehat{\xi}_1, \dots, \widehat{\xi}_j) - \phi_j(\xi_1, \dots, \xi_j)| \leq c \sqrt{\bar{e}_1^2 + \dots + \bar{e}_j^2} \leq c \|\bar{e}\|$ . Thus,

$$\|\Delta_{\theta^{-1}}(\phi(\widehat{\xi}) - \phi(\xi))\| \leq \lambda \sqrt{\bar{e}_1^2 + \dots + \bar{e}_j^2} \leq \lambda \|\bar{e}\| \quad (19)$$

where  $\lambda = c\sqrt{n}$ .

Combining (18), (19), and the fact that  $(A + KC)^T P + P(A + KC) = -I$ , and that  $a(t) \geq \epsilon$  for  $t_{2i-1} \leq t \leq t_{2i}$ , we deduce that:

$$\dot{V} \leq -(a(t)\theta - 2\|P\|\lambda)\|\bar{e}\|^2 \leq -\frac{(\epsilon\theta - 2\|P\|\lambda)}{\mu_{\max}} V \quad (20)$$

where  $\mu_{\max}$  is the largest eigenvalue of  $P$ . Hence,

$$\begin{cases} \text{for } t_{2i-1} \leq t < t_{2i} \\ V(t) \leq e^{-(t-t_{2i-1})\frac{(\epsilon\theta - 2\|P\|\lambda)}{\mu_{\max}}} V(\bar{e}(t_{2i-1})) \end{cases} \quad (21)$$

But  $e(t) = \Delta_{\theta} \bar{e}(t)$ , hence for  $\theta \geq 1$ ,  $\|e(t)\| \leq \theta^N \|\bar{e}(t)\|$ , and from (21), we deduce that:

$$\|e(t)\| \leq \theta^n e^{-(t-t_{2i-1})\frac{(\epsilon\theta - 2\|P\|\lambda)}{2\mu_{\max}}} \sqrt{\mu_{\max}} \|e((t_{2i-1}))\|, \quad \text{for } t_{2i-1} \leq t < t_{2i} \quad (22)$$

#### IV. SIMULATION

We will now demonstrate the application of the proposed controller through simulation, carried out in Matlab. The application takes into account the time constraint imposed by the image processing capability of our camera and acquisition system (data available every 3s). Three types of trajectories are chosen for evaluating the performance of the observer as shown in the following :

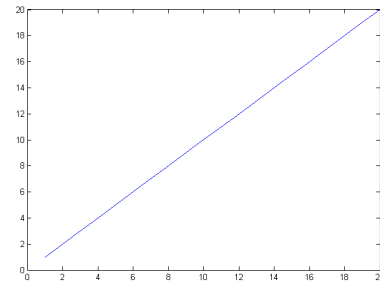


Fig. 3. Straight line trajectory

a) *Simulation: straight line trajectory with linear velocity  $v=0.016\text{cm/s}$ : Fig. 3 shows the trajectory and Fig. 4 represents the error signals by simulation between theoretical and observer model which compare two signals  $e_1=x_1-\hat{x}_1$  in blue color and  $e_3=x_3-\hat{x}_3$  in green color.*

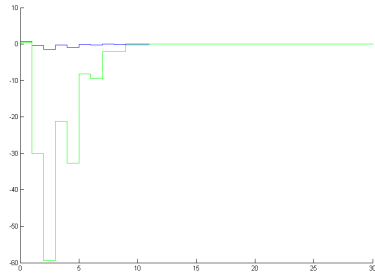


Fig. 4. Error between  $(x_1, \hat{x}_1)$  and  $(x_3, \hat{x}_3)$

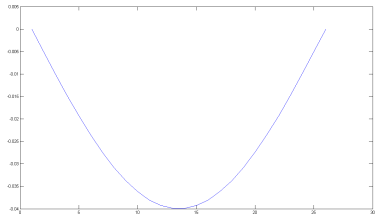


Fig. 5. Half-circle trajectory

b) *Simulation: half-circle trajectory with  $t=25s$  and velocity  $v=0.016*\sin(\pi/25*t)$  cm/s:* The Fig. 6 represents the error signals by simulation between theoretical and observer model which compare two signals  $e_1 = x_1 - \hat{x}_1$  in blue color and  $e_3 = x_3 - \hat{x}_3$  in green color.

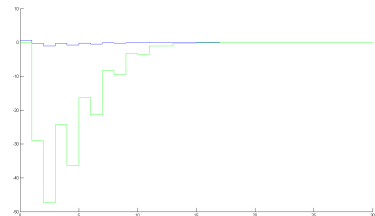


Fig. 6. Half-circle error signals

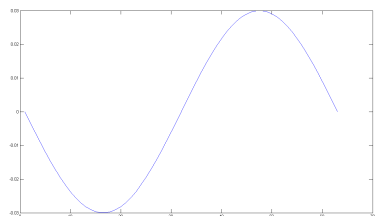


Fig. 7. S curve trajectory

c) *Simulation: Scurve trajectory with  $t=50s$  and velocity  $v = 0.016 * \sin(\pi/31 * t)$ cm/s:* The Fig. 8

represents the error signals by simulation between theoretical and observer model which compare two signals  $e_1 = x_1 - \hat{x}_1$  in blue color and  $e_3 = x_3 - \hat{x}_3$  in green color.

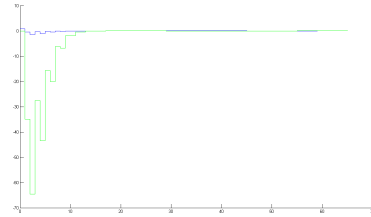


Fig. 8. S curve error signals

In all cases, we can see that the observer converges to the actual state  $x_3$ , i.e. the distance between the camera and the reference point, in 10 secs, which corresponds to five images. Indeed if a faster image processing computer is employed to reduce the time required to measure the image coordinates, then the delay will reduce proportionally. The fact that it takes only 5 images for the observer to estimate the depth through a single camera shows great improvement with respect to similar techniques using Kalman filters.

## V. CONCLUSION

This work presents an original solution to the depth estimation problem for such nonlinear system and some nonlinear constraints. The depth point (target or obstacle) is observed via the projection of 3D coordinates into 2D coordinates from the robot camera (fixed on-board), which provides the output measurements as image coordinates. The designed observer is based on High Gain observer and employs a switch between multiple observers in order to obtain the best estimate from output measurements. The simulations show the convergence and effectiveness of the proposed observer. In the future works, we will implement the designed observer in the mobile robot and conduct online-tests to demonstrate the effectiveness of the proposed observer to estimate the depth of the static point. We will also investigate the problem of using the observed variable as feedback for mobile robot control.

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