

# Robust Diagnosis and Analysis of Residue Sensitivity by Bond Graph Model for a DC Motor

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**Abstract**— In this paper, a Bond Graph (BG) approach is used for modeling, simulation, robust diagnosis, and analysis of residue sensitivity for a DC motor. The design and modeling of the DC motor are performed using graphic methods thanks to the structural properties of the link graph model. The simulation results are used to show the dynamic behavior of the system variables. The bond graph model using fractional linear transformations (BG-LFT) to generate a redundant analytical relation (ARR) consisting of two perfectly separate parts: A nominal portion designates the residual part and an uncertain part, which serves both for calculation Adaptive thresholds for normal operation and sensitivity analysis.

**Keywords**— Bond Graph, Robust Diagnosis, DC Motor, Analysis of Residue Sensitivity, Linear Fractional Transformations, Analytical Redundant Relationship

## I. INTRODUCTION

The diagnostic system is primarily intended to issue alarms which aims to draw attention of the supervising operator of the occurrence of one or more events that could affect the proper functioning of the installation.

Given the complexity of the processes, the generation of alarms is the most used way to alert the operator of the occurrence of an "abnormal" event. Alarms are related to malfunctions that may appear on the production system. It is important to clarify the meaning given to the words used to evoke the malfunctions that may occur in the system. We retain, for this, the definitions in [1, 2 and 3].

These industrial systems are governed by multiple physical phenomena and various technology components, so the Bond Graph approach, based on an energy analysis and multi-physics, is well suited. The Bond Graph modeling tool was defined by Paynter since 1961 [4]. This approach allows energy to highlight the analogies between the different areas of physics (mechanics, electricity, hydraulics, thermodynamics, acoustics, etc. ...) and represent in uniform multidisciplinary physical systems [5 and 6]. The diagnosis of uncertain systems has been the focus of much research work in recent years [7and 8].

This interest is reflected in the fact that natural systems are complex and non-stationary and manufacturers seek greater safety and efficiency. The Bond Graph approach proposed in this article allows, for its energy structure and multi physics, to use a single tool for modeling, structural analysis and generation of uncertain ARR.

In this study we try to show how the Bond Graph model can be used for modeling, simulation and construction of observers of linear and nonlinear systems (next section) on the one hand and on the other hand the construction of the system elements to be analyzed by bond graph elements as LFT to generate RRAS consist of two parts perfectly separated: A face portion, which is the residue and an uncertain part, which serves both to the calculation of adaptive thresholds for normal operation and sensitivity analysis.

## II. DIAGNOSIS BY BOND GRAPH MODEL

### A. Bond graph Model

Two methods are proposed by Sueur [9] to build parametric uncertainty by BG. The first is to represent uncertainty on bond graph element as another element of the same type, causally linked to the nominal element (figure 1) or the rest of the model. These uncertainties are kept in derivative causality when the model is preferred in integral causality not change the order of the model. The second method is the LFT form (Linear Fractional Transformations) introduced on mathematical models Redheffer since 1994 [10].

The physical aspect of the multi-hop graphs comes from the fact that from any physical system, it is possible to obtain an independent graphical representation of the studied physical realm. Building a bond graph model can be done in three levels:

- The technological level

- The physical level
- The structural and mathematical

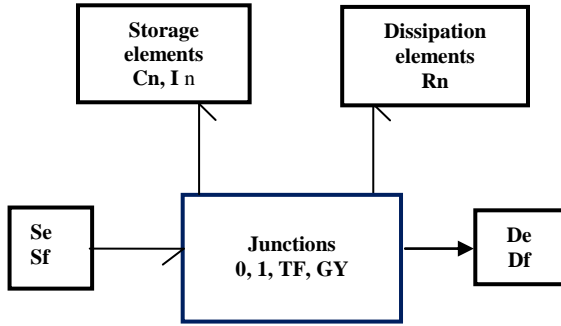


Fig. 1 Representation BG with the nominal element

- Storage elements: potential (C) or inertial (I);
- Dissipation elements: R;
- Junction elements: parallel (0), serial (1), transformation and gyrator;
- Sources elements: Sources effort or sources flow;
- Detectors elements: Detectors effort or detectors flow.

### B. LFT Representation

Linear Fractional Transformations (LFTs) are very generic objects used in the modeling of uncertain systems. The universality of LFT is due to the fact that any regular expression can be written in this form after Oustaloup (1994) [11] and Alazard *et al.* (1999) [12]. This form of representation is used for the synthesis of control laws of uncertain systems using the principle of the  $\mu$ -analysis. It involves separating the nominal part of a model of its uncertain part as shown in figure 2.

Ratings are aggregated into an augmented matrix denoted P, supposedly clean and uncertainties regardless of their type (structured and unstructured parametric uncertainties, modeling uncertainty, measurement noise ...) are combined in a matrix structure  $\Delta$  diagonal.

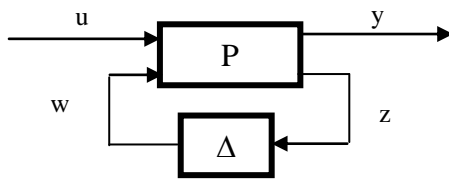


Fig. 2 Representation LFT for physical system

With:

- $x \in R^n$ : System state vector;
- $u \in R^m$ : Vector grouping system control inputs;
- $y \in R^p$ : Vector grouping the measured outputs of the system;
- $w \in R^l$  et  $z \in R^l$ : Respectively include inputs and auxiliary outputs.  $n, m, l$  and  $p$  are positive integers

### C. BG Modeling Elements by LFT Representation

Modeling linear systems with uncertain parameters was developed in C. Sie Kam, we invite the reader to view the references for details on the modeling of uncertain BG components (R, I, C, TF and GY) figure 3.

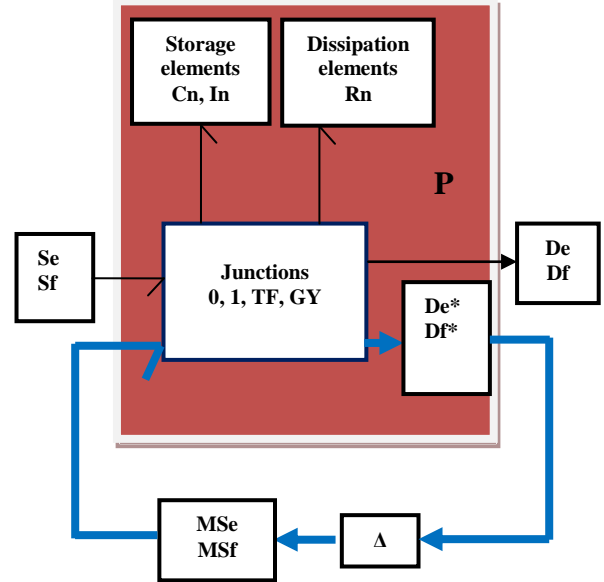


Fig. 3 Representation of BG-LFT

We therefore limit this part to show the two methods of modeling uncertain BG elements and the advantages of BG-LFT for robust diagnosis.

Full BG-LFT can then be represented by the diagram in figure 3.

### D. Generate Robust Residues

The generation of robust analytical redundancy relations from a clean bond graph model, observable and overdetermined be summarized by the following steps:

- 1<sup>st</sup> step: Checking the status of the coupling on the bond graph model deterministic preferential derived causality; if the system is over determined, then continue the following steps
- 2<sup>nd</sup> step: The bond graph model is made into LFT
- 3<sup>rd</sup> step: The symbolic expression of the RRA is derived from equations junctions. This first form will be expressed by:

- For a junction 0:

$$\sum b_i f_{inc} + \sum Sf + \sum w_i \quad (1)$$

- For a junction 1:

$$\sum b_i e_{inc} + \sum Se + \sum w_i \quad (2)$$

With the sum  $\Sigma Sf$  of sources flows due to the junction 0, the sum  $\Sigma Se$  of the sources of stress related to junction 1,  $b = \pm 1$  at the half-arrow into or out of the junction and  $e_{in}$  and  $f_{in}$  purpose are unknown variables, the sum  $\Sigma w_i$  of modulated inputs corresponding to the uncertainties on the junction-related items:

- 4<sup>th</sup> Step: The unknowns are eliminated by browsing the causal paths between the sensors or sources and unknown variables  
 5<sup>th</sup> step: After eliminating the unknown variables, are uncertain as  $RRA_S$ :

$$RRA: \Phi \left( \begin{array}{c} \sum Se, \sum Sf, De, Df, \tilde{D}e, \tilde{D}f, \\ \sum w_i, R_n, I_n, C_n, TF_n, GY_n \end{array} \right) \quad (3)$$

- $TF_n$  and  $GY_n$  are the nominal values of the elements and modules, respectively  $TF$  and  $GY$
- $R_n$ ,  $C_n$  and  $I_n$  are the nominal values of the elements R, C and I

### III. ANALYSIS OF RESIDUE SENSITIVITY BY BOND GRAPH

The sensitivity analysis of residues has been developed in recent years. Indeed, methods are proposed to evaluate these residues. When residues are assumed to be normally distributed around known average, statistical methods to generate normal operating thresholds are well adapted [13]. In the case where the uncertainties do not occur at the same frequency as the defects, the filtering methods are well adapted [14]. While the actuator and sensor defects are determined using the parity space [15]. Unfortunately, these residue generation methods are not effective since they neglect parametric inter-correlation (the thresholds are often overstated and may diverge).

The Bond Graph tool provides an efficient solution to the parametric dependency problem since BG-LFT generation automatically separates residuals and adaptive thresholds [Djeziri, 2007]. In this paper, we will use the BG-LFT model to generate residuals and adaptive thresholds for normal operation.

#### A. Generation of Performance Indices

To improve diagnostic performance, it is necessary to determine the performance indices [Djeziri, 2007] which are the index of sensitivity and the index of detectability of defects.

#### B. Sensitivity Index (SI)

The parameterized parametric sensitivity index explains the evaluation of the energy contributed to the residue by the uncertainty on each parameter by comparing it with the total energy contributed by all the uncertainties.

$$IS_{ai} = \frac{|a_i|}{d} \frac{\partial d}{\partial |a_i|} = \frac{|w_i|}{d} \quad (4)$$

- $a_i$ : uncertainty on the  $i^{\text{th}}$  parameter
- $w_i$ :  $i^{\text{th}}$  modulated input corresponding to Uncertainty on the  $i^{\text{th}}$  parameter
- $i \in \{R, C, I, TF, GY\}$

#### C. Defect detectability index ID

The defect detectability index represents the difference between the effort (or flux) provided by the defects in absolute value and that contributed by the set of uncertainties in absolute value.

- Junction 1:

$$ID = |Y_i| |e_{in}| + |Y_s| - d \quad (5)$$

- Junction 0:

$$ID = |Y_i| |f_{in}| + |Y_s| - d \quad (6)$$

Then the conditions for detectability of defects will be as follows:

- Undetectable fault :  $ID \leq 0$
- Detectable fault :  $ID > 0$

### IV. ROBUST DIAGNOSIS OF DC MOTOR BY BOND GRAPH APPROACH

#### C. Bond Graph Model of DC Motor

Consider the circuit diagram of a DC motor and its bond graph model given in figure 4. On this system, we will detect and locate defects in the flow sensors (current by sensor  $Df_1$  and speed by sensor  $Df_2$ ).

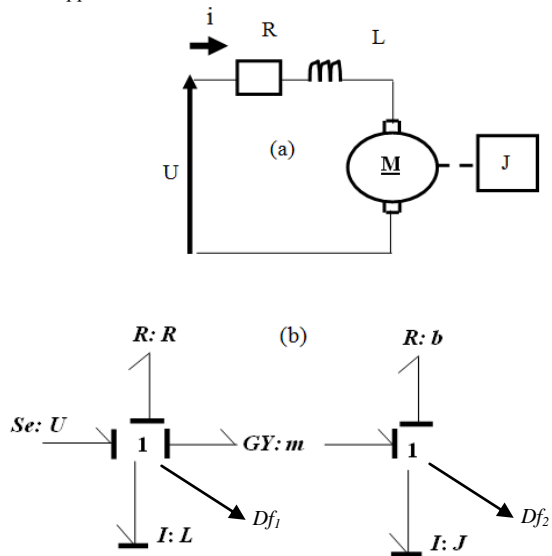


Fig 4. (a) DC motor, (b) Bond Graph model of DC motor

**D. Simulation of the DC Motor**

The simulation of the current  $i(t)$ , the speed  $n(t)$  and the residuals  $r_1(t)$  and  $r_2(t)$  of the DC motor by the software 20-sim intended for industrial systems modeled by the bond graph approach in figure 4.

The figures 5a), 5b) and 5c) show the current pattern, the speed of rotation and the torque in the normal operation of the DC motor.

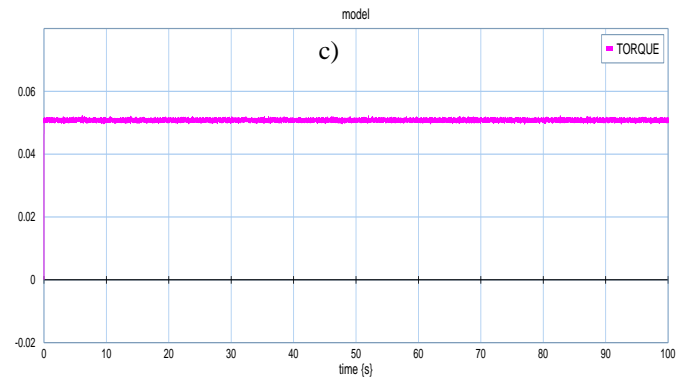
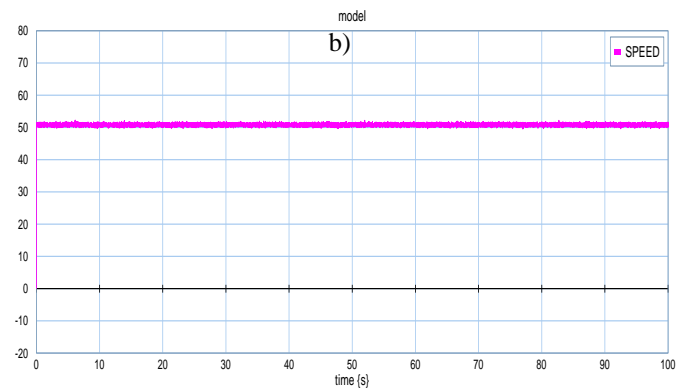
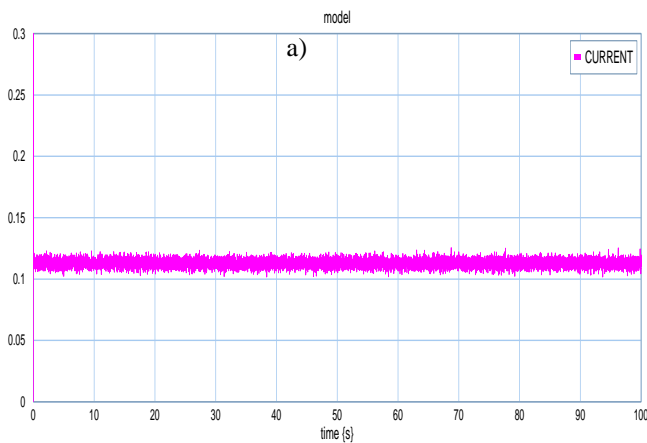


Fig.5 a) Current of the DC motor, (b) Speed of the DC motor, (c) torque of the DC motor

The figure 6a) and 6b) show the residues in the normal operation of the DC motor.

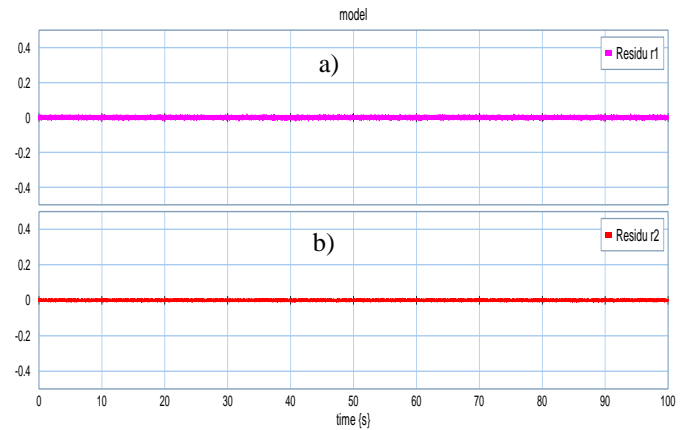


Fig 6 a) Residual  $r_1(t)$ , b) Residual  $r_2(t)$  in the normal operation

Figure 6a) and 6b) shows that in the case of normal operation, the values average residues are almost zero.

The figures 7a), 7b) and 7c) show the current pattern, the speed of rotation and the torque with electrical fault and mechanical fault of the DC motor.

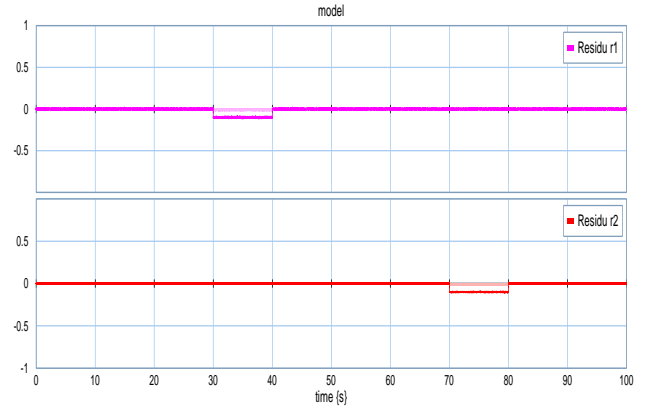
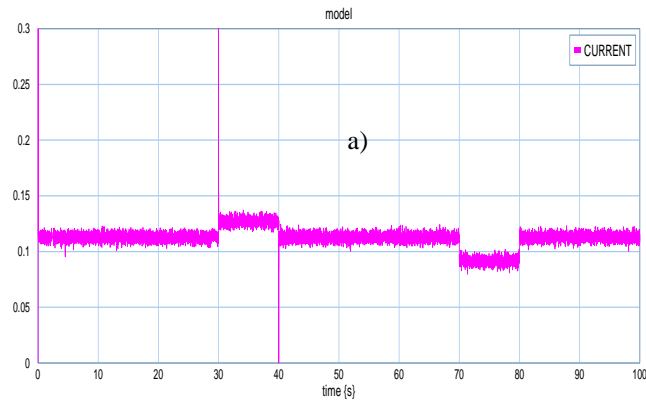


Fig 8 a) Residual  $r_1(t)$ , b) Residual  $r_2(t)$  with electrical fault and mechanical fault of the DC motor.

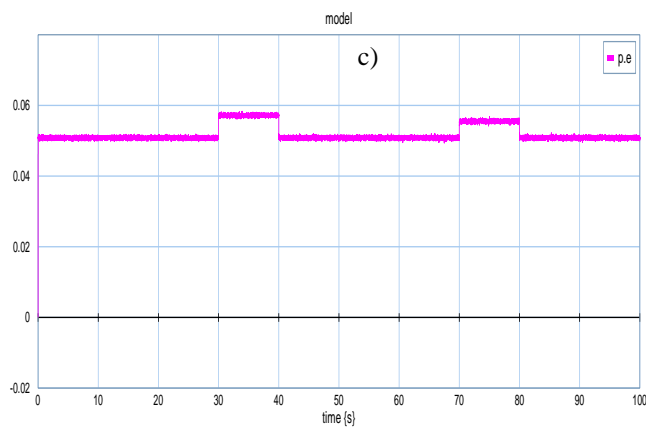
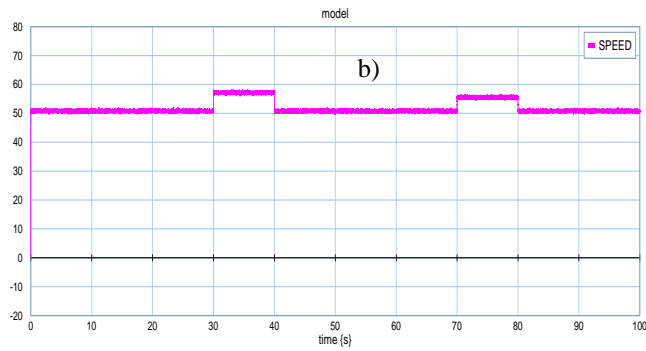


Fig.7 a) Current, (b) Speed, c) torque with electrical fault and mechanical fault of the DC motor.

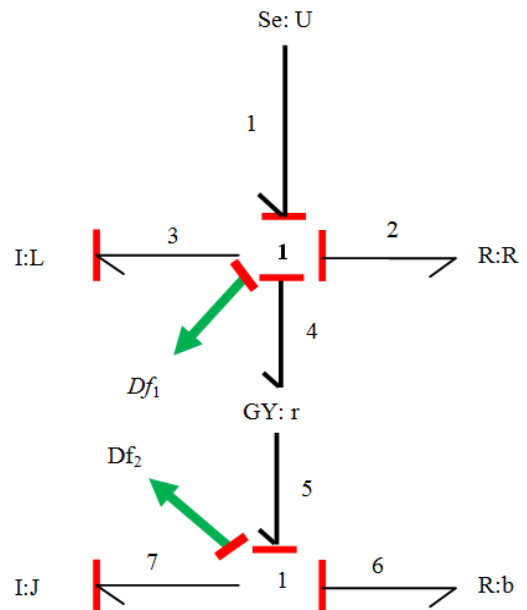


Fig. 9 BG Model of DC motor and sensors

The figure 8a) and 8b) show the residues with electrical fault and mechanical fault of the DC motor.

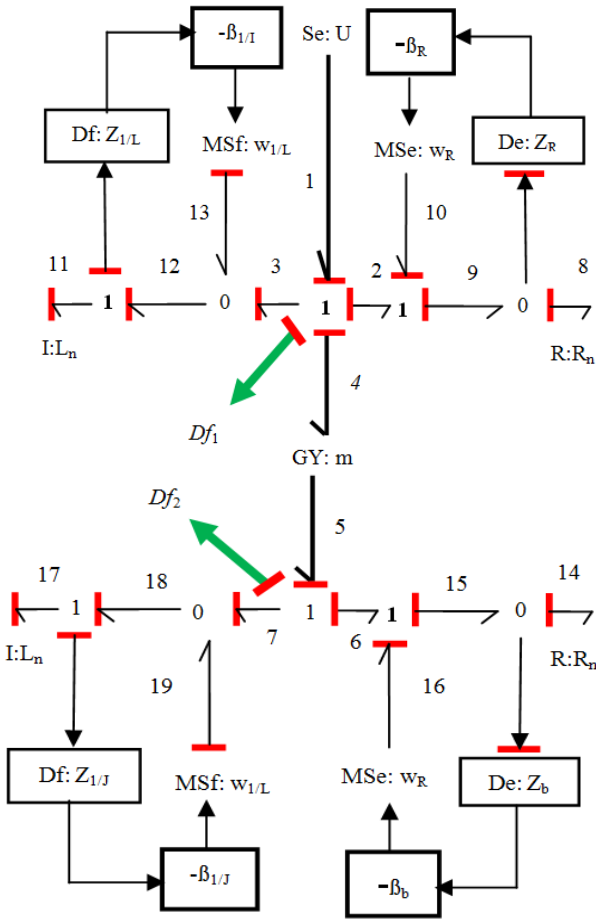


Fig. 10 BG-LFT Model integral causality of DC motor and sensors

To determine the residues, we must put the system in the form derivative and also put sensors under dualized form (figure 11).

We have introduced two four parametric defects ( $Y_L$ ,  $Y_R$ ,  $Y_J$  and  $Y_b$ ) and structural defects ( $Y_{s1}$  and  $Y_{s2}$ )

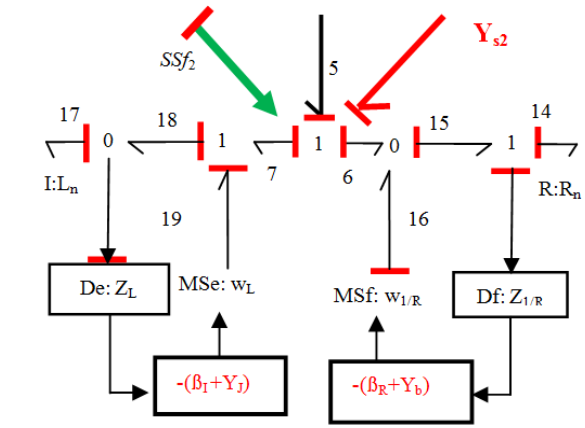
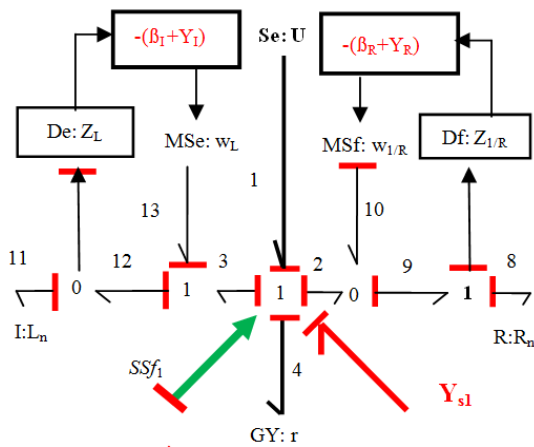


Fig. 11 BG-LFT Model derived causality of DC motor and sensors dualisation

### E. The Equations BG Model before Default

Junction 1<sub>1</sub>:

- $e_2: SSf_1 \rightarrow \Psi_{R_n}(f_8, e_8) \rightarrow e_2 = R_n \cdot SSf_1$
- $e_3: SSf_1 \rightarrow \Psi_{L_n}(f_{11}, e_{11}) \rightarrow e_3 = L_n \cdot SSf_1$
- $e_4: SSf_2 \rightarrow \Psi_{GY}(f_5, e_4) \rightarrow e_4 = m \cdot SSf_2$

The ARR1 equation before default can be written:

$$\begin{cases} ARR_1 = r_n + d_1 \\ RRA : U R_n SSf_1 L_n \frac{dSSf_1}{dt} - m \cdot SSf_2 + w_R + w_L = 0 \\ r_n = U R_n SSf_1 L_n \frac{dSSf_1}{dt} \\ d_1 = |w_R| + |w_L| \\ d_1 = |\delta_R R_n SSf_1| + \left| \delta_L L_n L_n \frac{dSSf_1}{dt} \right| \end{cases}$$

Junction 1<sub>2</sub>:

- $e_5: SSf_1 \rightarrow \Psi_{GY}(f_4, e_5) \rightarrow e_5 = m \cdot SSf_1$
- $e_6: SSf_2 \rightarrow \Psi_{R_n}(f_{14}, e_{14}) \rightarrow e_{14} = b_n \cdot SSf_2$
- $e_3: SSf_1 \rightarrow \Psi_{L_n}(f_{17}, e_{17}) \rightarrow e_{17} = J_n \cdot SSf_2$

The ARR<sub>2</sub> equation before default can be written:

$$\begin{cases} ARR_2 = r_{2n} + d_2 \\ ARR_2 = m \cdot SSf_2 b_n SSf_2 J_n \frac{dSSf_2}{dt} \\ + w_b + w_j = 0 \\ r_{2n} = m \cdot SSf_2 b_n SSf_2 J_n \frac{dSSf_2}{dt} \\ d_2 = |w_{1/b}| + |w_j| \end{cases}$$

The  $ARR_1$  equation after default can be written:

$$\begin{cases} ARR_1 = r_{1n} + d_1 \\ ARR_1 = Y_{s1} + U R_n SSf L_n \frac{dSSf_1}{dt} m .SSf_2 \\ + w_{1/R} + w_L = 0 \\ r_{1n} = Y_{s1} + U R_n SSf_1 L_n \frac{dSSf_1}{dt} \\ d_1 = |w_{1/R}| + |w_L| \\ d_1 = |\delta_R R_n SSf_1| + \left| \delta_L L_n L_n \frac{dSSf_1}{dt} \right| \\ + |Y_R e_{Rn}| + |Y_L e_{Ln}| \end{cases}$$

The  $ARR_2$  equation after default can be written:

$$\begin{cases} ARR_2 = r_{2n} + d_2 \\ ARR_2 = Y_{s2} + m .SSf_2 b_n SSf_2 J_n \frac{dSSf_2}{dt} \\ + w_b + w_j = 0 \\ r_{2n} = Y_{s2} + m .SSf_2 b_n SSf_2 J_n \frac{dSSf_2}{dt} \\ d_2 = |w_{1/b}| + |w_j| \\ d_2 = |\delta_R b_n SSf_1| + \left| \delta_L j_n j_n \frac{dSSf_1}{dt} \right| \\ + |Y_b e_{bn}| + |Y_j e_{jn}| \end{cases}$$

## V. CONCLUSIONS

The choice of the LFT form for modeling with parametric uncertainties the bond graphs allowed to use a single tool for the systematic generation of indicators formal uncertain defects. These parametric uncertainties are explicitly introduced on the physical model with its graphics architecture, which displays clearly on the model of their origins.

Uncertain ARR generated are well structured, showing separately the contribution Energy uncertainties fault indicators and facilitating their evaluations in the step of decision by the calculation of adaptive thresholds for normal operation. The diagnosis performance is monitored by an analysis of the residues of sensitivity to uncertainties and defects. The defect detectability index is defined to estimate a priori detectable value of a default and to measure the impact of default on an industrial process. The parametric sensitivity index is used to determine parameters that have the most influence on the residues. From a practical standpoint, the fields of application of this method are very broad due to the energy aspect and multi physics of bond graphs and the LFT form used to model the influence of uncertainties about the system. The developed procedure is implemented on a software tool (controllab

products 20-sim version 4.0) to automate the generation of LFT models and uncertain ARR.

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