Conférence Internationale en Automatique & Traitement de Signal (ATS-2017)
Proceedings of Engineering and Technology – PET
Vol.22pp.1- 6

On the synthesis of a stabilizing state feedback control for a class of switched time-delay systems

Nawel Aoun[#*1], Marwen Kermani[*2], Anis Sakly[#3]

[#]*National Engineering School of Sousse*
Pôle Technologique de Sousse Route de Ceinture Sahloul, 4054, Tunisia, University of Sousse
[1]nawelaoun@yahoo.fr

[*]*Research Unit of Industrial Systems Study and Renewable Energy (ESIER), National Engineering School of Monastir*
Ibn El Jazzar, Skaness, 5019, Monastir, University of Monastir
[2]kermanimarwen@gmail.com
[3]anis_sakly@yahoo.fr

***Abstract*— A simple method to stabilize a class of linear switched systems that involves time-delay input is proposed. The approach, available for the case of arbitrary switching, is based on the use of the aggregation techniques as well as on the exploitation of a specific state space description of the system using the arrow form matrix. The idea consists in constructing a pseudo-overvaluing system relative to a regular vector norm and common to all the subsystems. The stability of this comparison system, expressed in terms of simple algebraic delay-independent criteria, permits to conclude to that of the original system.**

***Keywords*— Linear continuous-time switched systems; time-delayed control; arbitrary switching; state feedback control; arrow form matrix; M-matrix.**

I. Introduction

Switched systems are an important class of hybrid dynamical systems which consists of a set of continuous-time or discrete-time subsystems and a switching rule that orchestrates the switching among them [1], [3]. They have wide technological fields of applications namely in the control of mechanical systems, automotive industry, aircraft and air-traffic control, switching power converters, communication networks, etc [2], [8].

Recently, stability analysis and control synthesis of such systems have received a growing attention. One of the main concerns of researchers is the problem of stability and stabilization under arbitrary switching. In this context, guaranteeing the asymptotic stability of each subsystem individually becomes insufficient and a common Lyapunov function for all the constituent systems is required [4-7].

On the other hand, considerable interest has been attributed to time-delay systems during the last decades as time delay is an inherent feature to many physical processes. In fact, delays emerge in many engineering applications such as rolling mills, pneumatic and hydraulic systems, automotive industry and robotic systems. They generally describe propagation phenomena, material and energy transfer in interconnected systems and data transmission in communication systems.

It is well known that delays can often be a source of instability and poor control performances.

Several results on analysis and control design approaches of time-delay systems are proposed and are divided into two main categories. The first one provides a controller which can stabilize the system independently from the size of the delay whereas the second takes into consideration the size of the delay. Even though it is commonly recognized that delay independent conditions are more conservative than delay-dependent ones, they remain practical in some cases where time-delay is unknown or inestimable.

Most of the results that have been reported on stability of switched time-delay systems are based on the search of a common Lyapunov-Krasovskii functional and the proposed stability criteria are formulated in terms of Linear Matrix Inequalities (LMIs) which can sometimes meet some computation difficulties even for some low-order systems [10].

In this paper, we give an extension of the result of Mori et al. [9] to the case of switched linear time-delay systems. The approach, based on the comparison principle technique, holds for the case of arbitrary switching and represents an alternative to the problem of existence of a common Lyapunov-Krasvoskii functionals. Moreover, methods based on vector norms approach can be considered no matter the number or the order of subsystems switching among each other is important [17-21]. The description of the system by using the arrow form matrix simplifies the study.

The application of vector norms approach to switched systems to switched systems has already been introduced in [11-13] and has been extended later to time-delay switched systems in [14-16]

The remainder of this paper is organized as follows. In section II, we give the model description and some preliminaries. Section III presents the main result related to switched systems described by delayed differential equations. A numerical example is provided to show the effectiveness of the proposed method. Finally, some concluding remarks are given in section V.

Notations: The following notations will be used throughout the paper, $\Re^n$ denotes the n-dimensional Euclidean space, I_n is the identity matrix with appropriate dimensions, $\|.\|$

ISSN 2356-5608

denotes Euclidean vector norm. For any $u=(u_i)_{1\le i\le n}$, $v=(v_i)_{1\le i\le n}\in\Re^n$ we define the scalar product of vectors u and v as: $\langle u,v\rangle=\sum_{i=1}^{n}u_i v_i$. If $M=\left(m_{ij}\right)_{1\le i,j\le n}$, we denote by M^T its transpose, M^{-1} its inverse, $M^*=\left(m_{ij}{}^*\right)_{1\le i,j\le n}$ with $m_{ij}{}^*=m_{ij}$ if $i=j$ and $m_{ij}{}^*=\left|m_{ij}\right|$ if $i\ne j$ and $\left|M\right|=\left|m_{ij}\right|$, $\forall i,j$.

II. MODEL DESCRIPTION AND PRELIMINARIES

A. Model description

A linear continuous-time switched system subject to a time-delayed input and acting under an arbitrary switching rule $\sigma(t)=i$, $i\in I=\{1,..,N\}$ can be represented by the following differential equation:

$$y^{(n)}(t)+\sum_{i=1}^{N}\zeta_i(t)\sum_{j=0}^{n-1}a_{i,j}y^{(j)}(t)=u(t-\tau)\qquad(1)$$

where $y(t)\in\Re^n$ is the state vector, $u(t)\in\Re$ is the control input, τ is the time-delay, $a_{i,j}$ are constant coefficients, $\forall i=1,...,N$ and $j=0,..,n-1$, N is the number of subsystems switching between each other and $\zeta_i(t)$ is an exogenous function indicating which subsystem is being active at time t such that:

$$\zeta_i(t)=\begin{cases}1 & \text{if } \sigma(t)=i\\ 0 & \text{otherwise}\end{cases},\ i\in I\qquad(2)$$

It's obvious that $\sum_{i=1}^{N}\zeta_i(t)=1,\ \forall t\ge 0$.

The application of a state feedback control law of the form:

$$u(t)=\sum_{j=0}^{n-1}k_{i,j}y^{(j)}(t)\qquad(3)$$

with $k_{i,j}$ the components of the gain vectors $K_i=\left[k_{i,0},k_{i,1},..,k_{i,n-1}\right]^T$, yields to the following decription:

$$y^{(n)}(t)+\sum_{i=1}^{N}\zeta_i(t)\left(\sum_{j=0}^{n-1}a_{i,j}y^{(j)}(t)-\sum_{j=0}^{n-1}k_{i,j}y^{(j)}(t-\tau)\right)=0\qquad(4)$$

A change of variable of the form $x(t)=\left[y(t),y^{(1)}(t),..,y^{(n-)}(t)\right]^T$ permits the system (4) to be put under the controllable state space representation:

$$\dot{x}(t)=A_{\sigma(t)}x(t)+BK_{\sigma(t)}x(t-\tau)\qquad(5)$$

or equivalently:

$$\dot{x}(t)=A_i x(t)+BK_i x(t-\tau),\qquad\forall i\in I\qquad(6)$$

where:

$$A_i=\begin{pmatrix}0 & 1 & ... & 0\\ \vdots & \ddots & \ddots & \vdots\\ 0 & ... & 0 & 1\\ -a_{i,0} & -a_{i,1} & ... & -a_{i,n-1}\end{pmatrix},\ B=\begin{pmatrix}0\\ \vdots\\ 0\\ 1\end{pmatrix}\qquad(7)$$

B. Preliminaries

Theorem 1. [15] The matrix $A=\left(a_{ij}\right)_{1\le i,j\le n}$ is an M-matrix if the following properties are satisfied:

- $a_{ii}>0\ \ (i=1,..,n),\ \ a_{ij}\le 0\ \left(i\ne j;\ i,j=1,..,n\right)$ (8)
- all the principal minors of A are positive:
$$(A)\begin{pmatrix}1 & 2 & ... & j\\ 1 & 2 & ... & j\end{pmatrix}>0\quad\forall j=1,...,n\qquad(9)$$
- for any given positive real vector $\eta=\left(\eta_1,\eta_2,...,\eta_n\right)^T$, the algebraic equations $Ax=\eta$ have a positive solution $w=\left(w_1,w_2,...,w_3\right)^T$.

III. MAIN RESULTS

In this section, we give sufficient stabilization conditions of system (6) via the control state feedback law (3).

First, a change of base of the form $z(t)=Px(t)$ under the arrow form matrix description, allows the system to be represented by:

$$\dot{z}(t)=M_i z(t)+N_i z(t-\tau),\qquad\forall i\in I\qquad(10)$$

The arrow form matrices M_i, $i\in I$ are given by:

$$M_i=\begin{pmatrix}\alpha_1 & & & \beta_1\\ & \ddots & & \vdots\\ & & \alpha_{n-1} & \beta_{n-1}\\ \gamma_{i,1} & ... & \gamma_{i,n-1} & \gamma_{i,n}\end{pmatrix}\qquad(11)$$

where:

$$\begin{cases}\beta_j=\prod_{\substack{q=1\\ q\ne j}}^{n-1}(\alpha_j-\alpha_q)^{-1} & \forall j=1,..,n-1\\ \gamma_{i,j}=-P_i(\alpha_j) & \forall j=1,..,n-1\\ \gamma_{i,n}=-a_{i,n-1}-\sum_{j=1}^{n-1}\alpha_j & \end{cases}\qquad(12)$$

whereas the delayed-state matrices N_i, $i\in I$ are given by:

$$N_i=\begin{pmatrix}0_{n-1,n-1} & & & 0_{n-1,1}\\ \delta_{i,1} & \cdots & \delta_{i,n-1} & \delta_{i,n}\end{pmatrix}\qquad(13)$$

with:

$$\begin{cases}\delta_{i,j}=-Q_i(\alpha_j) & \forall j=1,..,n-1\\ \delta_{i,n}=-k_{i,n-1} & \end{cases}\qquad(14)$$

and P is the corresponding passage matrix such that:

$$P=\begin{pmatrix}1 & 1 & \cdots & 1 & 0\\ \alpha_1 & \alpha_2 & \cdots & \alpha_{n-1} & 0\\ \alpha_1{}^2 & \alpha_2{}^2 & \cdots & \alpha_{n-1}{}^2 & \vdots\\ \vdots & \vdots & \cdots & \vdots & 0\\ \alpha_1{}^{n-1} & \alpha_2{}^{n-1} & \cdots & \alpha_{n-1}{}^{n-1} & 1\end{pmatrix}\qquad(15)$$

Note that α_j, $j=1,..,n-1$ are distinct constant parameters that can be chosen arbitrarily and that $P_i(\lambda)$ and $Q_i(\lambda)$, $i\in I$,

ISSN 2356-5608

are the instantaneous characteristic polynomials of matrices A_i and BK_i respectively such that:

$$\begin{cases} P_i(\lambda) = \lambda^n + \sum_{p=0}^{n-1} a_{i,p}\lambda^p \\ Q_i(\lambda) = -\sum_{p=0}^{n-1} k_{i,p}\lambda^p \end{cases} \quad (16)$$

At this level, we can state the following theorem.

Theorem 2. System (6) is asymptotically stabilizable via the state feedback control law (3) under an arbitrary switching rule $\sigma(t) = i$, $i \in I$, if there exist α_j , $j = 1,...,n-1$, $\alpha_j \neq \alpha_q$ $\forall j \neq q$ such that the following inequality is satisfied:

$$-t_{c,n} + \sum_{j=1}^{n-1} t_{c,j} \left|\beta_j\right| \alpha_j^{-1} > 0 \quad (17)$$

with:

$$\begin{cases} t_{c,j} = \max\limits_{i\in I} \left|\gamma_{i,j}\right| + \left|\delta_{i,j}\right| , \quad j = 1,..,n-1 \\ t_{c,n} = \max\limits_{i\in I} \gamma_{i,n} + \left|k_{i,n-1}\right| \end{cases} \quad (18)$$

Proof. Seeing that the asymptotic stability of system (6) requires necessarily that of each individual subsystem and by applying the theorem of Mori et al., [], we can already say that each subsystem $\sum_{M_i,N_i}$ is asymptotically stable independently from the value of the time delay τ if:

$$T_i = M_i^* + \left|N_i\right|, \quad \forall i \in I \quad (19)$$

is the opposite of an M -matrix.

where:

$$M_i^* = \begin{pmatrix} \alpha_1 & & & \left|\beta_1\right| \\ & \ddots & & \vdots \\ & & \alpha_{n-1} & \left|\beta_{n-1}\right| \\ \left|\gamma_{i,1}\right| & \cdots & \left|\gamma_{i,n-1}\right| & \gamma_{i,n} \end{pmatrix} \quad (20)$$

$$\left|N_i\right| = \begin{pmatrix} 0_{n-1,n-1} & & & 0_{n-1,1} \\ \left|\delta_{i,1}\right| & \cdots & \left|\delta_{i,n-1}\right| & \left|k_{i,n-1}\right| \end{pmatrix} \quad (21)$$

and

$$T_i = \begin{pmatrix} \alpha_1 & & & \left|\beta_1\right| \\ & \ddots & & \vdots \\ & & \alpha_{n-1} & \left|\beta_{n-1}\right| \\ \left|\gamma_{i,1}\right| + \left|\delta_{i,1}\right| & \cdots & \left|\gamma_{i,n-1}\right| + \left|\delta_{i,n-1}\right| & \gamma_{i,n} + \left|k_{i,n-1}\right| \end{pmatrix} \quad (22)$$

By referring to theorem1, T_i is the opposite of an M -matrix if:

$$(-1)^j \Delta_j > 0, \qquad j = 1,...,n \quad (23)$$

It is clear that, for $j = 1,..,n-1$, the condition (23) is checked since $\alpha_j < 0$. Therefore, for $j = n$, relation (23) becomes:

$$(-1)^n \det(T_i) > 0 \quad (24)$$

which is equal to:

$$(-1)^n \left[\gamma_{i,n} + \left|k_{i,n-1}\right| - \sum_{j=1}^{n-1} \left|\beta_j\right| \; \left|\gamma_{i,j}\right| + \left|\delta_{i,j}\right| \; \alpha_j^{-1} \right] \prod_{k=1}^{n-1} \alpha_k > 0 \quad (25)$$

By dividing (25) by $(-1)^{n-1} \prod_{k=1}^{n-1} \alpha_k$, we obtain:

$$- \; \gamma_{i,n} + \left|k_{i,n-1}\right| \; + \sum_{j=1}^{n-1} \left|\beta_j\right| \; \left|\gamma_{i,j}\right| + \left|\delta_{i,j}\right| \; \alpha_j^{-1} > 0 \quad (26)$$

The extension of the result to the overall switched system passes through the following steps:

Let $w \in \Re_+^{*n}$ with components ($w_m > 0$, $\forall m = 1,..,n$) and consider the common radially unbounded Lyapunov functional for all the fuzzy subsystems $\sum_{M_i,N_i}$ given by:

$$V(x(t),t) = V_1(x(t),t) + V_2(x(t),t) \quad (27)$$

with:

$$V_1(x(t),t) = \left\langle \left|x(t)\right|, w \right\rangle \quad (28)$$

$$V_2(x(t),t) = \left\langle \left|N_c\right| \int_{t-\tau}^{t} \left|x(\theta)\right| d\theta, w \right\rangle \quad (29)$$

where: $N_c = \max\limits_{i\in I} \left|N_i\right|$.

It is clear that $V(t=0) \geq 0$.

The right Dini derivative of $V(x(t),t)$ along the trajectory of (10) yields:

$$D^+V(x(t),t)\Big|_{(10)} = D^+V_1(x(t),t)\Big|_{(10)} + D^+V_2(x(t),t)\Big|_{(10)} \quad (30)$$

where:

$$D^+V_1(x(t),t)\Big|_{(10)} = \left\langle \frac{d^+\left|x(t)\right|}{dt^+}, w \right\rangle = \left\langle \mathrm{sgn}(x(t)) \frac{d^+x(t)}{dt^+}, w \right\rangle \quad (31)$$

and

$$\mathrm{sgn}(x(t)) = \begin{pmatrix} \mathrm{sgn}\, x_1(t) & & \\ & \ddots & \\ & & \mathrm{sgn}\, x_n(t) \end{pmatrix} \quad (32)$$

Then

$$\begin{aligned} D^+V_1(x(t),t)\Big|_{(10)} &= \sum_{i=1}^{N} \xi_i(t) \left\langle \mathrm{sgn}(x(t)) \;\; M_i x(t) + N_i x(t-\tau) \;\; , w \right\rangle \\ &\leq \sum_{i=1}^{N} \xi_i(t) \left\langle \;\; M_i^* \left|x(t)\right| + \left|N_i\right| \left|x(t-\tau)\right| \;\; , w \right\rangle \\ &\leq \left\langle \;\; M_c^* \left|x(t)\right| + \left|N_c\right| \left|x(t-\tau)\right| \;\; , w \right\rangle \end{aligned} \quad (33)$$

where: $M_c = \max\limits_{i\in I} \; M_i^*$.

On the other hand, we have:

$$D^+V_1(x(t),t)\Big|_{(10)} = \left\langle \left|N_c\right| \; \left|x(t)\right| - \left|x(t-\tau)\right| \; , w \right\rangle \quad (34)$$

The sum of (34) and (33) gives:

ISSN 2356-5608

$$D^+V(x(t),t)\Big|_{(10)} = \left\langle M_c^* + |N_c| \; |x(t)|, w \right\rangle = \langle T_c |x(t)|, w \rangle \quad (35)$$

Knowing that:

$$\langle T_c |x(t)|, w \rangle = \langle T_c^T w, |x(t)| \rangle, \quad (36)$$

and if we assume that T_c is the opposite of an M -matrix, we can find a vector $\rho \in \Re_+^{*n}$ ($\rho_l > 0 \quad \forall l = 1,..,n$) satisfying the relation: $T_c^T w = -\rho$, $\forall w \in \Re_+^{*n}$.

Finally, we get:

$$D^+V(x(t),t)\Big|_{(10)} \le \langle T_c^T w, |x(t)| \rangle = \langle -\rho, |x(t)| \rangle$$
$$= -\sum_{l=1}^{n} \rho_l |x_l(t)| < 0 \quad (37)$$

Hence, the common pseudo-overvaluing matrix for all the subsystems is constructed as follows:

$$T_c = \max_{i \in I} \; M_i^* + |N_i| \; = \begin{pmatrix} \alpha_1 & & & |\beta_1| \\ & \ddots & & \\ & & \alpha_{n-1} & |\beta_{n-1}| \\ t_{c,1} & \cdots & t_{c,n-1} & t_{c,n} \end{pmatrix} \quad (38)$$

with $t_{c,j}$, $j = 1,..,n-1$ are defined in (18).

Consequently, a sufficient condition ensuring the asymptotic stability of the switched system (6) under arbitrary switching is:

$$-t_{c,n} + \sum_{j=1}^{n-1} t_{c,j} |\beta_j| \alpha_j^{-1} > 0$$

This completes the proof of theorem 2.

Remark. Suppose that for an index $i = i\max$, $T_c = T_{i\max}$ which is traduced by the fact that one of the individual comparison systems overvalues the rest of the subsystems. In this case and under some further conditions, the theorem can be simplified by the following corollary.

Corollary. System (6) is asymptotically stabilizable via the state feedback control law (3) under an arbitrary switching rule $\sigma(t) = i$, $i \in I$, if there exist α_j , $j = 1,...,n-1$, $\alpha_j \ne \alpha_q \quad \forall j \ne q$ such that the following inequality is satisfied:

- $P_{i\max}(\alpha_j)\beta_j < 0$ and $Q_{i\max}(\alpha_j)\beta_j < 0$
- $k_{i\max,n-1} < 0$
- $a_{i\max,0} - k_{i\max,0} > 0$

Proof. For $i = i_{\max}$, condition (26) can be rewritten as:

$$-\ \gamma_{i\max,n} + |k_{i\max,n-1}| \ + \sum_{j=1}^{n-1} |\beta_j \gamma_{i\max,j}| \alpha_j^{-1} + \sum_{j=1}^{n-1} |\beta_j \delta_{i\max,j}| \alpha_j^{-1} > 0 \quad (39)$$

If in addition, $P_{i\max}(\alpha_j)\beta_j < 0$, $Q_{i\max}(\alpha_j)\beta_j < 0$ and $k_{i\max,n-1} < 0$, equation (39)becomes:

$$\left(-\gamma_{i\max,n} + \sum_{j=1}^{n-1} |\beta_j \gamma_{i\max,j}| \alpha_j^{-1} \right) + \left(-k_{i\max,n-1} + \sum_{j=1}^{n-1} |\beta_j \delta_{i\max,j}| \alpha_j^{-1} \right) > 0 \quad (40)$$

Knowing that:

$$-\gamma_{i\max,n} + \sum_{j=1}^{n-1} |\beta_j \gamma_{i\max,j}| \alpha_j^{-1} = \frac{P_{i\max}(0)}{\prod_{j=1}^{n-1} (-\alpha_j)^{n-1}} \quad (41)$$

and

$$-k_{i\max,n-1} + \sum_{j=1}^{n-1} |\beta_j \delta_{i\max,j}| \alpha_j^{-1} = \frac{Q_{i\max}(0)}{\prod_{j=1}^{n-1} (-\alpha_j)^{n-1}} \quad (42)$$

it follows that:

$$P_{i\max}(0) + Q_{i\max}(0) > 0 \quad (43)$$

Finally, the stability condition is reduced to:

$$a_{i\max,0} - k_{i\max,0} > 0 \quad (44)$$

This achieves the proof of the corollary.

IV. ILLUSTRATIVE EXAMPLE

Consider a switched system composed of two second-order subsystems as described by the following differential equation:

$$y^{(2)}(t) + \sum_{i=1}^{2} \zeta_i(t) \sum_{j=0}^{1} a_{i,j} y^{(j)}(t) - \sum_{i=1}^{2} \zeta_i(t) \sum_{j=0}^{1} k_{i,j} y^{(j)}(t-\tau) = 0$$

or equivalently by the matrix form:

$$\dot{x}(t) = \sum_{i=1}^{2} \zeta_i(t) \; A_i x(t) + BK_i x(t-\tau)$$

where:

$$A_1 = \begin{pmatrix} 0 & 1 \\ -8 & -10 \end{pmatrix} \quad BK_1 = \begin{pmatrix} 0 & 0 \\ k_{1,0} & k_{1,1} \end{pmatrix}$$
$$A_2 = \begin{pmatrix} 0 & 1 \\ -3 & -7 \end{pmatrix} \quad BK_2 = \begin{pmatrix} 0 & 0 \\ k_{2,0} & k_{2,1} \end{pmatrix}$$

A change of base of matrices A_1 , A_2 , BK_1 and BK_2 into the arrow form leads to the new state space representation:

$$\dot{z}(t) = \sum_{i=1}^{2} \zeta_i(t) \; M_i z(t) + N_i z(t-\tau)$$

where:

$$\begin{cases} \gamma_{1,1} = -[\alpha^2 + 10\alpha + 8] \\ \gamma_{1,2} = -10 - \alpha \end{cases}$$
$$\begin{cases} \gamma_{2,1} = -[\alpha^2 + 7\alpha + 3] \\ \gamma_{2,2} = -7 - \alpha \end{cases}$$
$$\begin{cases} \delta_{1,1} = k_{1,1}\alpha + k_{1,0} \\ \delta_{2,1} = k_{2,1}\alpha + k_{2,0} \end{cases}$$

For an arbitrary choice of $\alpha = -1$ (hence $\beta = 1$), we have:

ISSN 2356-5608

$$M_1=\begin{pmatrix}-1 & 1\\ 1 & -9\end{pmatrix}\quad N_1=\begin{pmatrix}0 & 0\\ k_{1,0}-k_{1,1} & k_{1,1}\end{pmatrix}$$

$$M_2=\begin{pmatrix}-1 & 1\\ 3 & -6\end{pmatrix}\quad N_2=\begin{pmatrix}0 & 0\\ k_{2,0}-k_{2,1} & k_{2,1}\end{pmatrix}$$

Notice that, in this case, we can use the corollary if:

$$\begin{cases}0<k_{1,1}<k_{1,0}\\ 0<k_{2,1}<k_{2,0}\end{cases}$$

According to the corollary, to check the stability of each subsystem individually, it suffices to choose $k_{1,0}$ and $k_{2,0}$ such that:

$$\begin{cases}8-k_{1,0}>0\\ 3-k_{2,0}>0\end{cases}$$

If in addition, we have:

$$\begin{cases}k_{2,0}-k_{2,1}>k_{1,0}-k_{1,1}-2\\ k_{2,1}>k_{1,1}-3\end{cases}$$

then, we can conclude that $T_2=\begin{pmatrix}-1 & 1\\ k_{2,0}-k_{2,1}+3 & k_{2,1}-6\end{pmatrix}$ is a common candidate pseudo-overvaluing matrix for both subsystems. Thus, we can derive the following sufficient asymptotic stability conditions for the switched system:

$$\begin{cases}0<k_{1,1}<k_{1,0}\\ 0<k_{2,1}<k_{2,0}<3\\ k_{2,0}-k_{2,1}>k_{1,0}-k_{1,1}-2\\ k_{2,1}>k_{1,1}-3\end{cases}$$

For example, if we take $k_{2,1}=1$ and $k_{2,0}=2$, the obtained conditions become:

$$\begin{cases}0<k_{1,1}<4\\ 0<k_{1,0}<3+k_{1,1}\end{cases}$$

When we apply the theorem, stability domain is larger and the stability condition is written as:

$$\begin{cases}|k_{2,1}|+|k_{2,0}-k_{2,1}|<3\\ |k_{2,0}-k_{2,1}|>|k_{1,0}-k_{1,1}|-2\\ |k_{2,1}|>|k_{1,1}|-3\end{cases}$$

Figure 1 shows the stability domain obtained when applying the corollary whereas figure. 2 shows that obtained when applying the theorem.

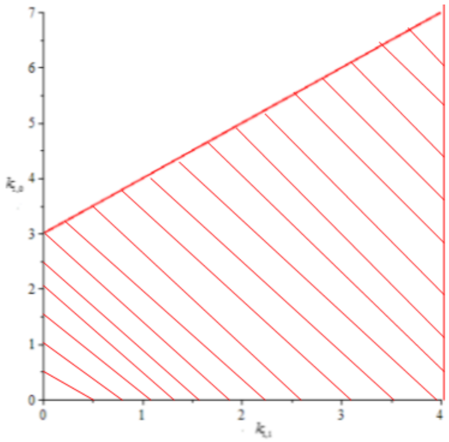

Fig. 1 Stability domain (according to the corollary)

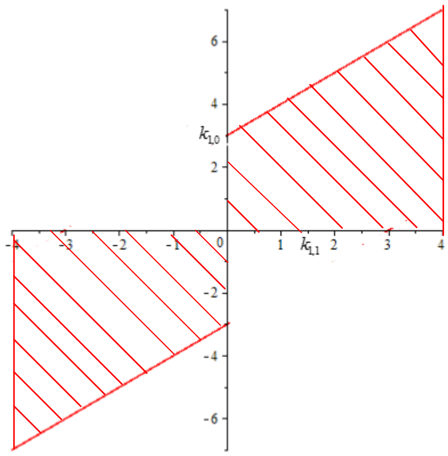

Fig. 2 Stability domain (according to the theorem)

V. Conclusion

Through this paper, we have proposed to study the stabilization by state feedback control of linear continuous-time switched systems. The approach is convenient for the case of arbitrary switching and provides simple algebraic delay-independent stability criteria. This method is particularly interesting when we deal with nonlinear systems. In fact, the choice of representing the system under the arrow form matrix is well appropriate to this case since it permits to isolate nonlinear elements in the last row or column, and thus makes the derivation of stability conditions easier.

References

[1] Z. Sun and S. S. Ge, Switched linear systems-control and design, Springer, 2005.

[2] Z. Sun, "Robust switching of discrete-time switched linear systems," Automatica, vol. 48, no. 1, pp. 239-242, 2012.

[3] D. Liberzon, Switching in systems and control, Boston, MA: Birkhaüser, 2003.

[4] D. Liberzon and A. S. Morse, "Basic problems in stability and design of switched systems," IEEE Control Systems Magazine, vol. 19, no. 5, pp. 59-70, October 1999.

[5] J. Hespanha and A. S. Morse, "Stability of switched systems with average dwell-time," Proc. of the 38th IEEE Conf. Decision and Control, pp. 2655-2660, 1999.

ISSN 2356-5608

[6] D. Liberzon, J. Hespanha and A. S. Morse, “Stability of switched linear systems: A Lie algebra condition,” Systems and Control Letters, vol. 37, no. 3, pp. 117-122, 1999.
[7] H. Lin and P. Antsaklis, “Stability and stabilizability of switched linear systems: A short survey of recent results,” IEEE Int. Symp. on Intelligent Control, vol. 1, pp. 24-29, 2005.
[8] D. Z. Cheng, “Stabilization of planar switched systems,” Systems & Control Letters, vol. 51, no. 2, pp. 79-88, 2004.
[9] T. Mori, N. Fukuma and M. Kuwahara, “Simple stability criteria for single and composite linear systems with time delays,” Int. J. of Control, vol. 34, no. 6, pp. 1175-1184, 1981.
[10] J. P. Richard, “Time-delay systems: an overview of some recent advances and open problems,” Automatica, vol. 39, pp. 1667-1694, 2003.
[11] A. Nawel, M. Kermani and A. Sakly, “On the stability and stabilization of discrete-time TS fuzzy switched systems,” Proc. 3rd Int. Conf. Automation, Control, Engineering and Computer Science (ACECS’16), vol. 13, pp. 772-778.
[12] A. Sakly and M. Kermani, “Stability and stabilization for a class of switched nonlinear systems via vector norms approach,” ISA Trans., vol. 57, pp. 144-161, July 2015.
[13] M. Kermani and A. Sakly, “Robust stability and stabilization studies for uncertain switched systems based on vector norms approach,” Int. J. of Dynamics and Control, vol. 4, no. 1, pp. 76-91, August 2014.
[14] M. Kermani and A. Sakly, “Delay-independent stability criteria under arbitrary switching of a class of switched nonlinear time-delay systems,” Advances in Difference Equations,” pp. 1-20, 2015.
[15] M. Kermani and A. Sakly, “On stability analysis of discrete-time uncertain switched nonlinear time-delay systems,” Advances in Difference Equations, 2014.
[16] M. Kermani and A. Sakly, “Stability analysis of switched nonlinear time-delay systems,” Systems Science&Control Engineering: An Open Access Journal, vol. 2, no. 1, pp. 80-89, 204.
[17] P. Borne, Nonlinear system stability: Vector norm approach system and control, Encyclopedia, t. 5, pp. 3402-3406, 1987.
[18] M. Benrejeb, D. Soudani, A. Sakly and P. Borne, “New discrete Tanaka-Sugeno-Kang fuzzy systems characterization and stability domain,” Int. J. of Computer, Communication and Control, pp. 9-19, 2006.
[19] M. Benrejeb, A. Sakly, K. Ben Othman and P. Borne, “Choice of conjunctive operator of TSK fuzzy systems and stability domain study,” Mathematics and Computers in Simulation, vol. 76, pp. 410-421, 2008.
[20] M. Benrejeb, M. Gasmi and P. Borne, “New stability conditions for TS fuzzy continuous nonlinear models,” Nonlinear Dynamics and Systems Theory, vol. 5, no. 4, pp. 369-379, 2005.
[21] M. Benrejeb, P. Borne and F. Laurent, “Sur une application de la représentation en flèche à l’analyse des processus,” Rairo Automatique/Systems Analysis and Control, vol. 16, pp. 133-146, 1982.

ISSN 2356-5608