

# Fault Diagnosis of Buck Converter Driven DC Motor Based on a Hybrid Unknown Input Observer

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**Abstract**— In this paper a model based approach is presented for the diagnosis of parametric and discrete faults of a buck converter driven DC motor. A hybrid unknown input observer is designed based on a hybrid model of the buck converter DC motor combination. The observer parameters are synthesized using Linear Matrix Inequalities techniques. Simulation results are given to demonstrate the efficiency of the proposed method.

**Keywords**— Dynamical hybrid model, buck converter DC motor combination, Unknown Input Observer, Fault detection, Linear Matrix Inequalities

## I. INTRODUCTION

With the increasing demand for reliability and efficiency of power converters and electrical machines, fault diagnosis is becoming a very active area of research.

Many researchers investigated DC motors fault diagnosis using different approaches. In [1] parity equations are used for fault detection of multiple sensors. In [2] a combination of the parity equation and parameter estimation approach is used in the diagnosis of parametric and additive faults in a DC motor. In [3] authors designed a Luenberger observer based method for sensor fault detection. In [4] authors developed a model based method combining parity equations for residual generation with fuzzy logic for decision making.

Like most complex systems, power System exhibit a hybrid behavior which involve both continuous and discrete event dynamics [5]. In fact, electrical currents and voltages evolve according to the laws of electromagnetism and also change in a discontinuous manner according to the state of the switches. The diagnosis problem of such systems, classified as switched systems, was investigated by many researches. In fact, faults and external disturbances are coupled which increases the rate of false alarms and makes the diagnosis process more challenging. This problem was tackled in the literature for instance, by using unknown input observers [6], fault detection filter [7]-[9] and optimal fault detection filter [10].

In this paper, we propose an observer based method for the fault detection in a buck converter driven DC motor. The diagnosis method is based on a hybrid unknown input observer that provides an estimation of the measurable output and decouple the effect of the disturbances. The difference

between the measured and the estimated outputs is used to generate residuals that are sensitive to faults.

The paper is organized as follow; section II presents the hybrid unknown input observer design method. In section III the hybrid model of the studied converter-motor system and the proposed diagnosis method is presented. The simulation results and the conclusion are given in sections IV and, V respectively.

## II. HYBRID UNKNOWN INPUT OBSERVER DESIGN

Consider the linear switching system given by:

$$\begin{cases} \dot{x} = A_{\lambda(t)}x + B_{\lambda(t)}u + D_{\lambda(t)}v \\ y = H_{\lambda(t)}x \end{cases} \quad (1)$$

where  $x \in \mathbb{R}^n$  is the state vector,  $u \in \mathbb{R}^m$  denotes the known input vector,  $v \in \mathbb{R}^m$  represents the unknown input vector and  $y \in \mathbb{R}^p$  is the output vector.  $\lambda(t) : \mathbb{R}^+ \rightarrow q = \{1, \dots, N\}$  is the switching signal deciding which mode is active from a finite set of discrete states  $q$ . The matrices  $A_{\lambda(t)}$ ,  $B_{\lambda(t)}$ ,  $D_{\lambda(t)}$  and  $H_{\lambda(t)}$  are known constant matrices with appropriate dimensions.

Under certain conditions a full order unknown input observer [11] could be designed as follow :

$$\begin{cases} \dot{z} = N_{\lambda(t)}z + G_{\lambda(t)}u + L_{\lambda(t)}y \\ \hat{x} = z - E_{\lambda(t)}y \end{cases} \quad (2)$$

The estimation error is governed by the following equation:

$$\begin{aligned} \dot{e} &= \dot{x} - \dot{\hat{x}} = \dot{x} - \dot{z} + E_{\lambda(t)}\dot{y} \\ &= (M_{\lambda(t)}A_{\lambda(t)} - K_{\lambda(t)}H_{\lambda(t)})x - N_{\lambda(t)}\hat{x} \\ &\quad + (M_{\lambda(t)}B_{\lambda(t)} - G_{\lambda(t)})u \\ &\quad + M_{\lambda(t)}D_{\lambda(t)} \end{aligned} \quad (3)$$

where

$$M_{\lambda(t)} = I + E_{\lambda(t)}H_{\lambda(t)} \quad (4)$$

$$K_{\lambda(t)} = L_{\lambda(t)} + N_{\lambda(t)}E_{\lambda(t)} \quad (5)$$

Furthermore if

$$N_{\lambda(t)} = M_{\lambda(t)}A_{\lambda(t)} - K_{\lambda(t)}H_{\lambda(t)} \quad (6)$$

$$G_{\lambda(t)} = M_{\lambda(t)}B_{\lambda(t)} \quad (7)$$

$M_{\lambda(t)}D_{\lambda(t)} = 0$  (to decouple disturbance from the observer error) (8)

then equation (3) of the error dynamics becomes as follows :

$$\dot{e} = N_{\lambda(t)}e \quad (9)$$

The observer described by (2) could be designed if for every  $q$  there exist an UIO of the following form :

$$\begin{cases} \dot{z}_q = N_q z_q + G_q u + L_q y \\ \hat{x}_q = z_q - E_q y \end{cases} \quad (10)$$

where matrices  $N_q$ ,  $G_q$ ,  $L_q$  and  $E_q$  are chosen such that

$$M_q = I + E_q H_q,$$

$$K_q = L_q + N_q E_q,$$

$$N_q = M_q A_q - K_q H_q \text{ with } N_q \text{ is Hurwitz } \forall q = \{1, \dots, N\},$$

$$G_q = M_q B_q,$$

$$\text{and } M_q D_q = 0.$$

The necessary and sufficient conditions for the existence of the  $q^{\text{th}}$  UIO (10) are given by :

i.  $\text{rank}(k_q H_q) = \text{rank}(a_n)_{k=1}^n = D$ , this condition is to ensure the existence of a matrix  $E_q$  such that

$$E_q H_q D_q = -D_q \text{ for all } q = \{1, \dots, N\} \quad (11)$$

ii. all the pairs  $(M_q A_q, H_q)$  are detectable.

Furthermore the continuity of  $e(t)$  must be guaranteed by the following condition [11]

$$\text{iii. } E_1 H_1 = E_2 H_2 = \dots = E_N H_N \quad (12)$$

The general solution of (11) is given by

$$E_q = U_q + Y_q V_q \quad (13)$$

with  $U_q = -D_q (H_q D_q)^+$ ,  $V_q = I - (H_q D_q) (H_q D_q)^+$ ,  $Y_q$  is an arbitrary matrix of suitable dimensions and  $(X)^+ = (X^T X)^{-1} X^T$  is the Moore-Penrose pseudo inverse.

The error convergence could be seen as a stability problem of a hybrid dynamical system. And thus it could be demonstrated by one of the concepts of the stability analysis of hybrid systems [12]. Based on Lyapunov approach, the stability of the error dynamic could be guaranteed by the existence of a common Lyapunov function for all subsystems or the existence of multiple Lyapunov functions for each subsystem.

Consider a Lyapunov function candidate for (9) defined by :

$$V = e^T P e \quad (14)$$

$V$  is a common Lyapunov function if there exist symmetric positive definite matrices  $P$  and  $Q$  such that

$$N_q^T P + P N_q \leq -Q \quad (15)$$

By substituting the matrix  $N_q$  by its expression into (15), it yields :

$$\begin{aligned} & \left( (I + U_q H_q + Y_q V_q H_q) A_q - K_q H_q \right)^T P \\ & + P \left( (I + U_q H_q + Y_q V_q H_q) A_q - K_q H_q \right) + Q \leq 0 \end{aligned} \quad (16)$$

Inequality (16) could be reformulated as the following linear matrix inequalities (LMI's):

$$\begin{aligned} & \left( (I + U_q H_q) A_q \right)^T P + P \left( (I + U_q H_q) A_q \right) \\ & + (V_q H_q A_q)^T \bar{Y}_q^T + \bar{Y}_q (V_q H_q A_q) \\ & - \bar{K}_q H_q - H_q^T \bar{K}_q^T + Q \leq 0 \end{aligned} \quad (17)$$

$$Y_q = P^{-1} \bar{Y}_q \quad (18)$$

$$K_q = P^{-1} \bar{K}_q \quad (19)$$

### III. DIAGNOSIS OF BUCK CONVERTER DRIVEN DC MOTOR

In this section the model of the buck converter driven DC motor is described. And the proposed diagnosis method is presented.

#### A. Model of the Buck Converter DC Motor Combination

The buck converter driven DC motor shown in Fig. 1 is considered in this study.

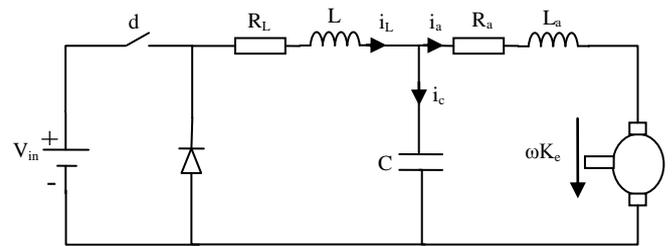


Fig. 1 Buck converter DC motor combination

The electromechanical dynamics of the buck converter driven DC motor can be modeled by the following differential equations :

$$\begin{cases} \frac{di_L}{dt} = -\frac{R_L}{L} i_L - \frac{V_c}{L} + \frac{d}{L} V_{in} \\ \frac{dV_c}{dt} = \frac{i_L}{C} - \frac{i_a}{C} \\ \frac{di_a}{dt} = \frac{V_c}{L_a} - \frac{R_a}{L_a} i_a - \frac{K_e}{L_a} \omega \\ \frac{d\omega}{dt} = \frac{K_m}{J} i_a - \frac{T_L}{J} - \frac{B_v}{J} \omega \end{cases} \quad (20)$$

where  $i_a$  is the armature current,  $i_L$  the inductor current,  $V_{in}$  the input voltage,  $V_c$  the converter output voltage and  $\omega$  the motor angular velocity.

The variable  $d$  depends on the state of the switch and is defined as follow :

$$d = \begin{cases} 0 & \text{when the switch is closed,} \\ 1 & \text{when the switch is open.} \end{cases}$$

The buck converter DC motor combination can be considered as a dynamical hybrid system without state jumps. Thus its model can be written in the form of (1) where the state vector is defined as  $x = (i_L \ V_c \ i_a \ \omega)^T$ , the torque load  $T_L$  is considered as a disturbance and the matrices  $A_q$ ,

$B_q$ ,  $D_q$  and  $H_q$ , such that  $q = \{1, 2\}$ , are given by:

$$A_1 = A_2 = \begin{pmatrix} -\frac{R_L}{L} & -\frac{1}{L} & 0 & 0 \\ \frac{1}{C} & 0 & -\frac{1}{C} & 0 \\ 0 & \frac{1}{L_a} & -\frac{R_a}{L_a} & -\frac{K_e}{L_a} \\ 0 & 0 & \frac{K_m}{J} & -\frac{B_v}{J} \end{pmatrix},$$

$$B_1 = (0 \ 0 \ 0 \ 0)^T, \quad B_2 = \left(\frac{1}{L} \ 0 \ 0 \ 0\right)^T,$$

$$H_1 = H_2 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad D = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{J} \end{pmatrix}^T$$

The considered faults consist of parametric faults characterized by changes in the armature resistance and inductance, and discrete fault caused by open switch fault in the buck converter.

#### B. Fault Diagnosis of the Converter -Motor System

It can be easily demonstrated that the necessary and sufficient conditions for this system are satisfied. Moreover  $A_1 = A_2$ ,  $H_1 = H_2$  and  $D_1 = D_2 = D$ , thus the LMI's can be simplified to :

$$\begin{aligned} & ((I+UH)A)^T P + P((I+UH)A) \\ & + (VHA)^T \bar{Y}^T + \bar{Y} (VHA) - \bar{K}H - H^T \bar{K}^T + Q \leq 0 \end{aligned} \quad (21)$$

$$Y = P^{-1} \bar{Y}$$

$$K = P^{-1} \bar{K}$$

The residuals are generated using the difference between the measured and the estimated output of the system.

$$r = [V_c - \hat{V}_c \ i_a - \hat{i}_a \ \omega - \hat{\omega}]^T \quad (22)$$

The residual evaluation function is chosen as the  $L_2$  norm of the residual vector  $r \in \mathbb{R}^p$  (23) and the threshold is chosen as a constant positive value (24).

$$\|r(t)\| = \left( \sum_{i=1}^{n_r} r_i^2(t) \right)^{\frac{1}{2}} \quad (23)$$

$$T = \sup_{t \geq 0} \|r(t)\| \quad (24)$$

A fault can be detected if the residual exceed the predefined threshold  $T$ .

$$\begin{cases} \|r(t)\| \leq T & f = 0 \\ \|r(t)\| > T & f \neq 0 \end{cases} \quad (25)$$

#### IV. SIMULATION RESULTS

The proposed method for the fault detection of the buck converter DC motor combination was implemented and demonstrated by numerical simulation.

The LMIs were solved using MATLAB LMI Toolbox. A buck converter driven DC motor converter and its corresponding UIO observer were simulated on MATLAB Simulink Stateflow. The system parameters are listed in Table 1.

TABLE I

Parameter	Value
$R_L$ buck coil resistance	0.2 $\Omega$
$L$ buck coil inductance	1.33 mH
$C$ filter capacitor	470 $\mu F$
$L_a$ armature inductance	8.9 mH
$R_a$ armature resistance	6 $\Omega$
$K_e$ back EMF constant	0.0517 V. s/rad
$K_m$ motor torque constant	0.0517 N.m/A
$J$ moment of inertia	7.95 $10^{-6}$ Kg.m <sup>2</sup>
$B_v$ viscous friction coefficient	0.001 N.m/rad/s

The buck converter operates with a switching frequency of 45kHz. And the input voltage is  $V_m = 24V$ .

Different cases were simulated considering different kinds of faults, both converter and motor related.

The considered faults are as follows:

- parametric faults caused by a change in the armature resistance of the DC-motor or a change in both the armature resistance and inductance due to inter-turn short circuit fault
- and discrete fault due to open switch fault in the buck converter.

**Case 1** no faults, torque load change at t=0.2s

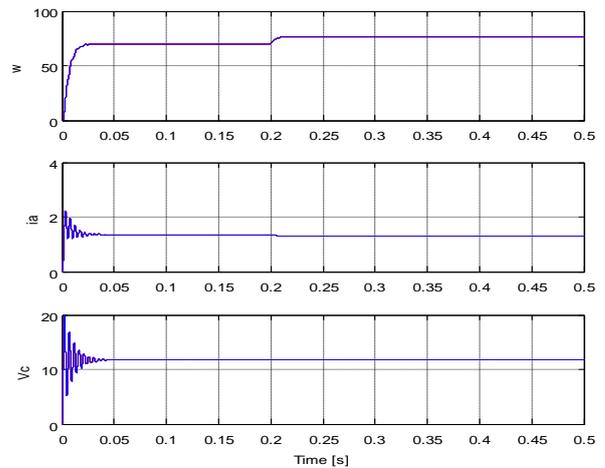


Fig. 2 Measured and estimated outputs

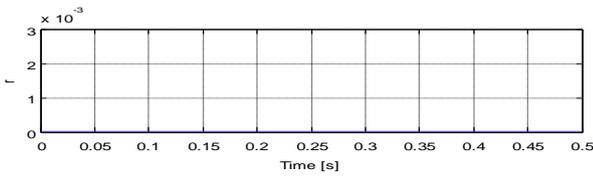


Fig. 3 Residual signal

**Case 2** open switch fault at t=0.3s

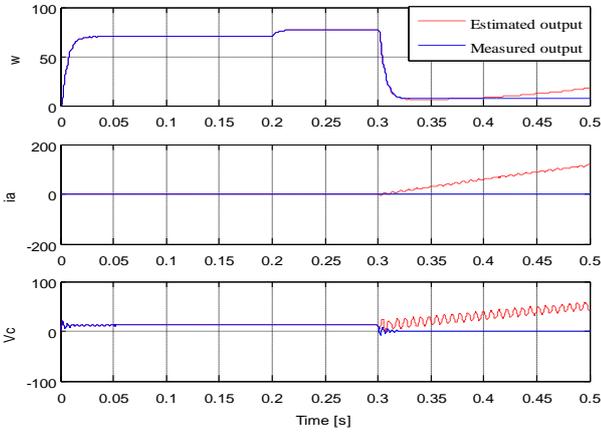


Fig. 4 Measured and estimated outputs in the presence of an open switch fault

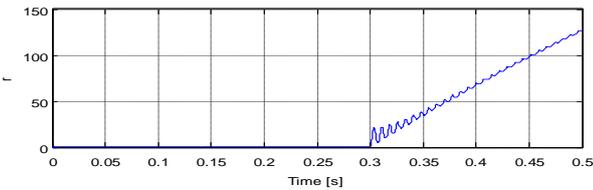


Fig. 5 Residual signal

**Case 3** Armature resistance variation at t=0.3s

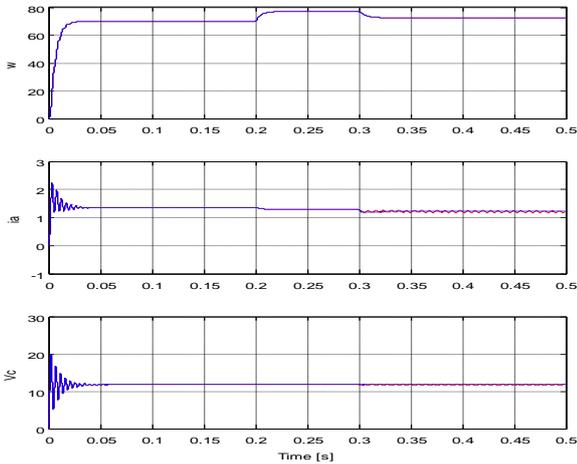


Fig. 6 Measured and estimated outputs in case of a resistance variation

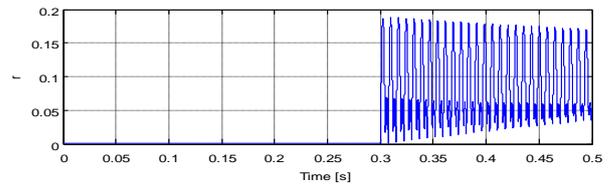


Fig. 7 Residual signal

**Case 4** Inter-turn short circuit fault at t=0.3s

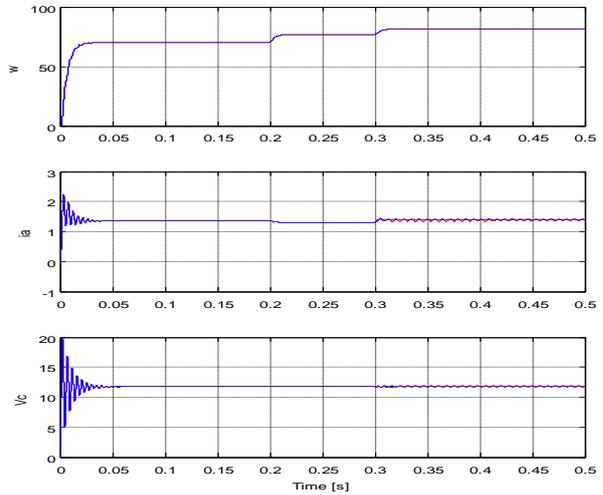


Fig. 8 Measured and estimated outputs in case of an inter-turn fault

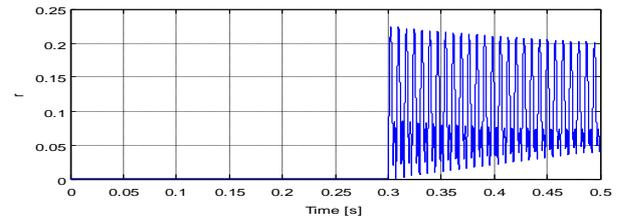


Fig. 9 Residual signal

For each case, the measured and the estimated outputs are illustrated in Fig.2, Fig.4, Fig.6 and Fig.8.

In the first case the change in the torque load impact the measurable output of the system Fig.2. However, the residual Fig.3 remains unchanged. This demonstrates the robustness of the used method against the disturbance which is here the Torque load. When a fault occurs residuals change significantly as illustrated in Fig.5, Fig.7 and Fig.9.

**V. CONCLUSIONS**

In this paper, the problem of fault diagnosis of a DC motor combined with a buck converter is investigated. A Hybrid UIO is designed to generate residuals that are sensitive to faults and indifferent to torque load disturbance.

The convergence analysis of the observer was carried out using common Lyapunov function approach. The parameters of the observer were designed based on a LMI formulation. The effectiveness of the proposed method in parametric and discrete fault detection was demonstrated by simulations.

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