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# Robust H∞ Control for Exponential Stabilization of T-S Fuzzy Systems with Time-Varying Delay

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***Abstract*** **— This paper deals with the robust H∞ fuzzy control problem for networked control systems, where the stochastic behavior of networks, makes it difficult to ensure that the data is correctly and fully transmitted to the actuators and controllers. An exponential stabilization method for Takagi-Sugeno fuzzy systems with uncertainty, external disturbances and time-varying delay is proposed. This last is a continuous function belonging to a given bounded interval. The delay-dependent Lyapunov-Krasovskii functional approach is used for the exponential stabilization of time-delay T-S fuzzy systems and for the existence of a robust H∞ controller is established in terms of LMI. To illustrate the effectiveness of the proposed approach some examples are given.**

***Keywords*****-- Networked control system (NCS); Takagi–Sugeno (T–S) fuzzy system; time varying-delay; guaranteed cost control; Lyapunov-Krasovskii; linear matrix inequality (LMI); H∞ control; exponential stabilization.**

## I. INTRODUCTION

Network control systems offer many advantages, including ease of installation and maintenance, reduction in wiring and costs of a system, and many others, which improves reliability, efficiency and productivity in many distributed industrial control systems. This explains the very large industrial applications of the NCS, ranging from automation to large-scale surveillance [1] - [5], [10].

The exchange of data between the different parts of the network, the actuators, the sensors, and the controllers causes disturbances introduced in communication, delays and packet losses. These two last can really decrease the performances and stability of the system to control [5], [10].

Many researchers concentrated on the analysis of the stability and design of networked control systems in order to solve the weakening of the performance of these systems. [1], [2],[3]. Some results about the stability of nonlinear NCS were presented in [5], [6].

On another side, fuzzy models have been recognized as being effective and adapted to a certain degree of nonlinear dynamic complex systems representation, principally for the systems with insufficient and uncertain information, according to the fuzzy model Takagi-Sugeno(TS) [7], [8], [9].

Since there is an important gap between theory and reality and it is difficult to make hypotheses of systems models near to reality and while we will work with the networked control systems, this brings us to work with uncertain T–S fuzzy systems, and so as to minimize disturbances and increase system performance, an H∞ fuzzy controller will be proposed for uncertain T–S fuzzy systems with time variable delay [18]-[21]. And like that we can guarantee a certain robustness for the stability of networked control systems. The design of the control is made on the base of fuzzy model via the method of parallel distributed compensation (PDC). Moreover, there is another way to improve performance of the system is to guaranteed the speed of convergence to the state by the exponential stability.

Many researchers have studied problem of robust exponential stability for time-delay [11]-[13], [22]-[25], but few of them have applied the exponential stabilization with H∞ fuzzy controller for uncertain T–S fuzzy systems with time variable-delay.

In this paper, we consider the problem of controlling systems in a network. For that, we develop a new exponential stabilization for uncertain Takagi–Sugeno (T – S) fuzzy systems with external disturbance and time-varying delay. This last is a continuous function belonging to a given bounded interval.

## II. PROBLEM FORMULATION

Consider a nonlinear time delay system can be represented by the T–S fuzzy model as follows:

Plant Rule i:

IF $\theta_1(t)$ is $F_{i1}$and $\theta_2(t)$is $F_{i2}$ ... and $\theta_p(t)$ is $F_{ip}$ THEN

$$\begin{aligned}\dot{x}(t) &= (A_i+\Delta \mathrm{A_i})x(t)+(A_{di}+\Delta \mathrm{A}_{di})x(t-d(t)) \\ &\quad +(B_i+\Delta B_i)u(t)+D_i\omega(t)\\ z(t) &= C_i x(t) \\ x(t) &= \varphi(t) \quad t\in[-\max\{d_2\},0],\end{aligned} \qquad \textbf{(1)}$$

where $i = 1, 2, \ldots, r$ is the index number of fuzzy rules, $x(t)\in\mathbb{R}^n, u(t)\in\mathbb{R}^n, z(t)\in\mathbb{R}^l$ *and* $\omega(t)\in\mathbb{R}^p$ denotes the state vector, control input, measurement output vector and disturbance input vector respectively; the matrices $A_i, A_{di}, B_i, C_i,\ D_i$ are of appropriate dimensions; $\Delta\mathrm{A_i}, \Delta\mathrm{A_{di}}, \Delta\mathrm{B_i}$ denote the uncertainties in the system. $\theta_1(t)$, $\theta_2(t), \ldots, \theta_p(t)$ are the premise variables, the initial condition $\varphi(t)$ is a differentiable function or constant vector, $F_{ig}$ is a fuzzy set ($g = 1, 2,\ldots, p$). $d(t)$ is the time varying delay function and satisfies $0 < d_1 \le d(t) \le d_2$.

The inferred system is described by

$$\begin{aligned}\dot{x}(t) &= \sum_{i=1}^{r} h_i(\theta(t))[(A_i+\Delta \mathrm{A_i})x(t)+ \\ &\quad (A_{di}+\Delta\mathrm{A}_{di})\,x(t-d(t)) + (B_i+\Delta B_i)u(t)+D_i\omega(t)]\\ z(t) &= \sum_{i=1}^{r} h_i(\theta(t))[C_i x(t)]\end{aligned} \qquad \textbf{(2)}$$

Where

$$h_i(\theta(t)) = \mu_i(\theta(t)) \Big/ \sum_{i=1}^{r}\mu_i(\theta(t))$$


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$$\mu_i(\theta(t)) = \prod_{j=1}^{p} F_{ij}(\theta_j(t))$$

And $F_{ij}(\theta_j(t))$ is the grade of membership of $\theta_j(t)$ in the fuzzy set $F_{ij}$. In this paper, we assume that:

$$\mu_i(\theta(t)) \geq 0$$

And

$$\sum_{i=1}^{r} \mu_i(\theta(t))$$

For all t. Therefore,

$$h_i(\theta(t)) \geq 0, \quad \text{for i= 1, 2, …, r}$$

And

$$\sum_{i=1}^{r} h_i(\theta(t)) = 1.$$

Based on the parallel distributed compensation, the following controller rules are employed to construct the fuzzy controller.

Controller Rule i:

IF $\theta_1(t)$ is $F_{i1}$ and $\theta_2(t)$ is $F_{i2}$ … and $\theta_p(t)$ is $F_{ip}$ THEN

$$u(t) = K_i x(t), \qquad i = 1, 2, \ldots, r \qquad \textbf{(3)}$$

For simplicity, the following notations will be used:

$$\bar{A}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j \bar{A}_{ij},$$

$$\bar{A}_{ij} = (A_i + \Delta \mathrm{A_i}) + (B_i + \Delta \mathrm{B_i})K_j$$

By using these notations, the closed-loop system of (2) and (3) is shown as follows:

$$\dot{x}(t) = \bar{A}(t)x(t) + (A_{di} + \Delta \mathrm{A_{di}})x(t - d(t)) + D_i\omega(t) \qquad \textbf{(4)}$$

*Definition:* Given α > 0. The zero solution of system (1) is α exponential stable if there exist a positive number β > 0 such that every solution x(t, φ) satisfies the following condition:

$$\|x(t,\emptyset)\| \leq \beta e^{-\alpha t}\|\emptyset\|, \qquad \forall t \geq 0.$$

We introduce the following technical well-known propositions, which will be used in the proof of our results.

*Proposition 1[15]:* For any symmetric positive definite matrix M > 0, scalar γ > 0 and vector function ω : [0, γ] → $\mathbb{R}^n$ such that the integrations concerned are well defined, the following inequality holds

$$\left(\int_0^{\gamma} \omega(s)\,\mathrm{ds}\right)^T M \left(\int_0^{\gamma} \omega(s)\,\mathrm{ds}\right) \leq \left(\int_0^{\gamma} \omega^T(s) M \omega(s)\,\mathrm{ds}\right)$$

*Proposition 2(Cauchy inequality):* For any symmetric positive definite matrix $M \in \mathbb{R}^{n\times n}$ and $x, y \in \mathbb{R}^n$, we have

$$\pm 2x^T y \leq x^T M x + y^T M^{-1} y.$$

*Proposition 3(Schur Complement lemma):* Given constant symmetric matrices X, Y, Z with appropriate dimensions satisfying $X = X^T$, $Y = Y^T > 0$. Then $X + Z^T Y^{-1} Z < 0$ if and only if

$$\begin{pmatrix} X & Z^T \\ Z & -Y \end{pmatrix} < 0 \text{ or } \begin{pmatrix} -Y & Z \\ Z^T & X \end{pmatrix} < 0.$$

*Proposition 4: The overall closed-loop system is under zero initial condition.*

*Lemma:* For any real matrices $\mathbb{D}, \mathbb{F}, \mathbb{E}$ with appropriate dimensions, the following inequality holds

$$\mathbb{DFE} + \mathbb{E}^T\mathbb{F}^T\mathbb{D}^T \leq \varepsilon^{-1}\mathbb{DD}^T + \varepsilon\mathbb{E}^T\mathbb{E} \;;\varepsilon > 0, \|\mathbb{F}\| \leq 1$$

In order to attenuate the external disturbance of the fuzzy system, we introduce the $H_\infty$ performance index with $\gamma > 0$ prescribed attenuation level.

$$\int_{t_0}^{\infty} z^T(t)z(t)dt \leq \gamma^2 \int_{t_0}^{\infty} \omega^T(t)\omega(t)dt$$

## III. MAIN RESULT

Let us set

$$\lambda = \lambda_{min}(\bar{P}),$$

$$\Lambda = \lambda_{max}(\bar{P}) + d_1\lambda_{max}(\bar{Q}) + \frac{1}{2}d_2^3\,\lambda_{max} + \frac{1}{2}(d_2 + d_1) \times (d_2 - d_1)^2\lambda_{max}(\bar{S})$$

The solution $x(t,\phi)$ of the system satisfies

$$\|x(t,\phi)\| \leq \sqrt{\frac{\Lambda}{\lambda}}\, e^{\alpha t}\|\phi\|, \qquad t \in \mathbb{R}^+.$$

*Theorem:* If there exist symmetric matrices Q > 0, P > 0, T > 0, R > 0, S > 0, and Y > 0, the system (1) is robustly exponential stable with $H_\infty$ performance index and for 0 < α < 1.

The controller parameters can be chosen as $K_i = Y_i T^{-1}$, the following LMI holds:

$$\Omega = \begin{bmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{12} & \Omega_{22} \end{bmatrix} < 0. \qquad \textbf{(5)}$$

Where :

$$\Omega_{\mathrm{ij}} = \begin{bmatrix} \Psi_{11} & \Psi_{12} & \Psi_{13} & \Psi_{14} & \Psi_{15} \\ \Psi_{12} & \Psi_{22} & \Psi_{23} & \Psi_{24} & \Psi_{25} \\ \Psi_{13} & \Psi_{23} & \Psi_{33} & -T & CT \\ \Psi_{14} & \Psi_{24} & \Psi_{34} & \Psi_{44} & -T + CT \\ \Psi_{15} & \Psi_{25} & \Psi_{35} & \Psi_{45} & \Psi_{55} \end{bmatrix}; \quad \begin{matrix} i = 1, 2, \ldots, r. \\ j = 1, 2, \ldots, r. \end{matrix}$$

$$\Psi_{11} = A_iT + \Delta A_iT + TA_i' + T\Delta A_i' + B_iY + \Delta \mathrm{B_i}Y + Y'B_i' + Y'^{\Delta \mathrm{B_i}'} + Q + 2\alpha P - e^{-2\alpha d_2}R + C_i'\,C_i$$

$$\Psi_{12} = A_{di}T + \Delta \mathrm{A_{di}}T + TA_i' + T\Delta A_i' + Y'B_i' + Y'\Delta \mathrm{B_i}' + e^{-2\alpha d_2}R$$

$$\Psi_{13} = TA_i' + T\Delta A_i' + Y'B_i' + Y'\Delta \mathrm{B_i}'$$

$$\Psi_{14} = P - T + TA_i' + T\Delta A_i{}' + Y'B_i' + Y'\Delta \mathrm{B_i}'$$

$$\Psi_{15} = DT + TA_i' + T\Delta A_i{}' + Y'B_i' + Y'\Delta \mathrm{B_i}'$$

$$\Psi_{22} = A_{di}T + \Delta \mathrm{A_{di}}T + TA_{di}' + T\Delta \mathrm{A_{di}}' - e^{-2\alpha d_2}R - e^{-2\alpha d_2}S$$

$$\Psi_{23} = TA_{di}' + T\Delta \mathrm{A_{di}}' + e^{-2\alpha d_2}S\;;\; \Psi_{24} = -T + TA_{di}' + T\Delta \mathrm{A_{di}}'$$

$$\Psi_{25} = DT + TA_{di}' + T\Delta \mathrm{A_{di}}'\;;\; \Psi_{33} = -e^{-2\alpha d_2}S - e^{-2\alpha d_1}Q$$

$$\Psi_{44} = d_2^2R + (d_2 - d_1)^2S - 2T\;;\; \Psi_{55} = \begin{bmatrix} DT + TD' & T' \\ T' & \delta^2 \end{bmatrix}$$

*Proof:*

We consider the following Lyapunov- Krasovskii functional

$$V(t) = V_1(t) + V_2(t) + V_3(t) + V_4(t)$$

$$V_1(t) = x^T(t)\bar{P}x(t)$$

$$V_2(t) = \int_{t-d_1}^{t} e^{2\alpha(s-t)}x^T(s)\bar{Q}x(s)ds \qquad \textbf{(6)}$$

$$V_3(t) = d_2\int_{-d_2}^{0}\int_{t+s}^{t} e^{2\alpha(\tau-t)}\dot{x}^T(s)\bar{R}\dot{x}(\tau)d\tau ds$$

$$V_4(t) = (d_2 - d_1)\int_{-d_2}^{d_1}\int_{t+s}^{t} e^{2\alpha(\tau-t)}\dot{x}^T(s)\bar{S}\dot{x}(\tau)d\tau ds$$

It is easy to check that

$$\lambda\,\|x(t)\|^2 \leq V(t, x_t) \leq \Lambda\,\|x_t\|^2, \quad t \in \mathbb{R}^+. \qquad \textbf{(7)}$$

The derivative of $V_1(t), \ldots, V_4(t)$ is given by

$$\dot{V}_1(t) = 2x^T(t)\bar{P}\dot{x}(t) + 2\alpha x^T(t)\bar{P}x(t) - 2\alpha V_1$$

$$\dot{V}_2(t) = x^T(t)\bar{Q}x(t) - e^{-2\alpha d_1}x(t-d_1)\bar{Q}x(t-d_1) - 2\alpha V_2$$

$$\dot{V}_3(t) = d_2^2\dot{x}^T(t)\bar{R}\dot{x}(t) - d_2\int_{t-d_2}^{t} e^{-2\alpha d_2}\,\dot{x}^T(s)\bar{R}\dot{x}(s)d - 2\alpha V_3$$

$$\dot{V}_4(t) = (d_2 - d_1)^2\dot{x}^T(t)\bar{S}\dot{x}(t) - (d_2 - d_1)e^{-2\alpha d_2}\int_{t-d_2}^{t}\dot{x}^T(s)\bar{S}\dot{x}(s)ds - 2\alpha V_4$$

We apply proposition1 and the Leibniz-Newton formula:

$$-d_2\int_{t-d_2}^{t}\dot{x}^T(s)\bar{R}\dot{x}(s)ds \leq -d(t)\int_{t-d(t)}^{t}\dot{x}^T(s)\bar{R}\dot{x}(s)ds$$

$$\leq -[\textstyle\int_{t-d(t)}^{t}\dot{x}(s)ds]^T\bar{R}[\int_{t-d(t)}^{t}\dot{x}(s)ds]$$

$$= -[x(t) - x(t-d(t))]^T\bar{R}[x(t) - x(t-d(t))]$$

$$= -x^T(t)\bar{R}x(t) + 2x^T(t)\bar{R}x(t-d(t)) - x^T(t-d(t))\bar{R}x(t-d(t))$$

$$-(d_2 - d_1)\int_{t-d_2}^{t-d_1}\dot{x}^T(s)\bar{S}\dot{x}(s)ds$$


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$$\leq -(d(t)-d_1)\int_{t-d(t)}^{t-d_1} \dot{x}^T(s)\bar{S}\dot{x}(s)ds$$
$$\leq -[\int_{t-d(t)}^{t-d_1} \dot{x}(s)ds]^T\bar{S}[\int_{t-d(t)}^{t-d_1} \dot{x}(s)ds]$$
$$= -[x(t-d_1)-x(t-d(t))]^T\bar{S}[x(t-d_1)-x(t-d(t))]$$
$$= -x^T(t-d_1)\bar{S}x(t-d_1)+2x^T(t-d_1)\bar{S}x(t-d(t))- x^T(t-d(t))\bar{S}x(t-d(t))$$

Therefore

$$\dot{V}(t,x(t))+2\alpha V(t,x_t)\leq x^T(t)[\bar{Q}-e^{-2\alpha d_2}\bar{R}+2\alpha\bar{P}]x(t) + \dot{x}^T(t)[d_2^2\bar{R}+(d_2-d_1)^2\bar{S}]\dot{x}(t)+x^T(t-d_1)[-e^{-2\alpha d_1}\bar{Q}- e^{-2\alpha d_2}\bar{S}]x(t-d_1)+ x^T(t-d(t))[-e^{-2\alpha d_2}\bar{R}-e^{-2\alpha d_2}\bar{S}]x(t-d(t)) + 2x^T(t)\bar{P}\dot{x}(t)+2e^{-2\alpha d_2}x^T(t)\bar{R}x(t-d(t)) + 2e^{-2\alpha d_2}x^T(t-d_1)\bar{s}x(t-d(t)) \quad \textbf{(8)}$$

By using the following identity relation

$$-\dot{x}(t)+\bar{A}(t)x(t)+(A_{di}+\Delta A_{di})x(t-d(t))+D_i\omega(t)=0$$

we obtain:

$$2[x^T(t)X+x^T(t-d(t))X+x^T(t-d_1)X+\dot{x}^T(t)X+\omega^T(t)X]\times[-\dot{x}(t)+\bar{A}(t)x(t)+(A_{di}+\Delta A_{di})x(t-d(t))+D_i\omega(t)]=0 \quad \textbf{(9)}$$

$$J_k=\int_{t_k+\tau_k}^{t_{k+1}+\tau_{k+1}} z^T(t)z(t)-\gamma^2\omega^T(t)\omega(t)dt$$

$$J_k=\int_{t_k+\tau_k}^{t_{k+1}+\tau_{k+1}} (z^T(t)z(t)-\gamma^2\omega^T(t)\omega(t)+\dot{V}(t))dt - V(t)\Big|_{t_k+\tau_k}^{t_{k+1}+\tau_{k+1}} \quad (\textbf{10})$$

And

$$J=\lim_{N\to\infty}\sum_{k=0}^{N} J_k$$

$$J=\int_{t_0}^{\infty} z^T(t)z(t)-\gamma^2\omega^T(t)\omega(t)+\dot{V}(t)dt-V(t)\Big|_{t_0}^{\infty} \quad (\textbf{11})$$

We replace (8) in (10) and we obtain:

$$J_k\leq\int_{t_k+\tau_k}^{t_{k+1}+\tau_{k+1}} [Cx(t)]^T[Cx(t)]-\gamma^2\omega^T(t)\omega(t)+\dot{V}(t)dt + x^T(t)[\bar{Q}-e^{-2\alpha d_2}\bar{R}+2\alpha\bar{P}]x(t) + \dot{x}^T(t)[d_2^2\bar{R}+(d_2-d_1)^2\bar{S}]\dot{x}(t) + x^T(t-d_1)[-e^{-2\alpha d_1}\bar{Q}-e^{-2\alpha d_2}\bar{S}]x(t-d_1) + x^T(t-d(t))[-e^{-2\alpha d_2}\bar{R}-e^{-2\alpha d_2}\bar{S}]x(t-d(t)) + 2x^T(t)\bar{P}\dot{x}(t)+2e^{-2\alpha d_2}x^T(t)\bar{R}x(t-d(t)) + 2e^{-2\alpha d_2}x^T(t-d_1)\bar{s}x(t-d(t))-V(t)\Big|_{t_0}^{\infty} \quad \textbf{(12)}$$

Adding the zero item of (9) into (8), we have

$$\dot{V}(t,x_t)+2\alpha V(t,x_t)\leq\varepsilon^T(t)\Omega\varepsilon(t)$$

$\varepsilon=[x(t)\ x(t-d(t))\ x(t-d_1)\ \dot{x}(t)\ \omega(t)]^T$

$\Psi_{11}=X\bar{A}+\bar{A}'X+\bar{Q}+2\alpha\bar{P}-e^{-2\alpha d_2}\bar{R}+C^TC$

$\Psi_{12}=X(A_{di}+\Delta A_{di})+\bar{A}'X+e^{-2\alpha d_2}\bar{R}$

$\Psi_{13}=\bar{A}'X$ ; $\Psi_{14}=\bar{P}-X+\bar{A}'X$ ; $\Psi_{15}=XD_i+\bar{A}'X$

$\Psi_{22}=X(A_{di}+\Delta A_{di})+(A_{di}+\Delta A_{di})'X-e^{-2\alpha d_2}\bar{R}-e^{-2\alpha d_2}\bar{S}$

$\Psi_{23}=(A_{di}+\Delta A_{di})'X+e^{-2\alpha d_2}\bar{S}$

$\Psi_{24}=-X+(A_{di}+\Delta A_{di})'X$ ; $\Psi_{25}=XD_i+(A_{di}+\Delta A_{di})'X$

$\Psi_{33}=-e^{-2\alpha d_2}\bar{S}-e^{-2\alpha d_1}\bar{Q}$ ; $\Psi_{34}=-X$ ; $\Psi_{35}=XD_i$

$\Psi_{44}=d_2^2\bar{R}+(d_2-d_1)^2\bar{S}-2X$

$\Psi_{45}=XD_i-X$ ; $\Psi_{55}=XD_i+D_i'X-\gamma^2$

Pre-and Post-multiplying both sides of $\Omega$ with

$$\theta=diag\,\{T,T,T,T,T\},$$

And make the change of variables such that:
$K=YT^{-1}$, $X=T^{-1}$, $\bar{R}=T^{-1}R\,T^{-1}$, $\bar{S}=T^{-1}S\,T^{-1}$, $\bar{Q}=T^{-1}Q\,T^{-1}$.
Then we obtain (5).

we have $\Omega=\theta\Gamma\theta$
From (12) the following inequality can be shown as:

$$J\leq\lim_{N\to\infty}\sum_{k=0}^{N}\int_{t_k+\tau_k}^{t_{k+1}+\tau_{k+1}}\Omega dt-V(t)\Big|_{t_k+\tau_k}^{t_{k+1}+\tau_{k+1}} \quad (\textbf{13})$$

Combining (11) and (13), the following result is obtained,

$$\int_{t_0}^{\infty} z^T(t)z(t)-\gamma^2\omega^T(t)\omega(t)+\dot{V}(t)dt \leq\lim_{N\to\infty}\sum_{k=0}^{N}\int_{t_k+\tau_k}^{t_{k+1}+\tau_{k+1}}\Omega dt$$

It is clear it that
$z^T(t)z(t)-\gamma^2\omega^T(t)\omega(t)+V(\infty)-V(t_0)\leq 0$ if $\Omega<0$ for any nonzero $\varepsilon$ ($t$).
According to the zero initial condition, we know that the $H\infty$ performance index is satisfied.
Note that $\Gamma<0$ if and only if $\Omega<0$.
Therefore, from condition (5), we obtain

$$\dot{V}(t,x_t)+2\alpha V(t,x_t)\leq 0 \quad \textbf{(14)}$$

Integrating both sides of (14) from *0* to *t*, we obtain

$$V(t,x_t)\leq V(0,x_0)e^{-2\alpha t},\quad \forall t\in\mathbb{R}^+$$

Furthermore, taking condition (7) into account, we have
$\lambda\|x(t,\phi)\|^2\leq V(t,x_t)\leq V(0,x_0)e^{-2\alpha t}\leq\Lambda e^{-2\alpha t}\|\phi\|^2$,
Then the solution $x(t,\phi)$ of the system satisfy

$$\|x(t,\phi)\|\leq\sqrt{\frac{\Lambda}{\lambda}}e^{-\alpha t}\|\phi\|,\ \ \forall t\geq 0.$$

Which implies the closed-loop system is $\alpha-$ robustly exponentialy stable.
The conditions of the theorem have been obtained and the proof is complete.

## IV. Simulation

Consider the following nonlinear system proposed in [10]. It can be represented by the following fuzzy model:

Rule 1 $If\ x_2(t) is\ N_{11}, then$

$$\dot{x}(t)=(A_1+\Delta A_1)x(t)+(A_{d1}+\Delta A_{d1})x(t-d(t)) +(B_1+\Delta B_1)u(t)+D_1\omega(t)$$
$$z(t)=C_1x(t)$$

Rule 2 $If\ x_2(t) is\ N_{12}, then$

$$\dot{x}(t)=(A_2+\Delta A_2)x(t)+(A_{d2}+\Delta A_{d2})x(t-d(t)) +(B_2+\Delta B_2)u(t)+D_2\omega(t)$$
$$z(t)=C_2x(t)$$

where

$$N_{11}(x_2(t))=1-\frac{x_2^2(t)}{2.2},\quad N_{12}(x_2(t))=1-N_{11}(x_2(t))$$

$A_1=\begin{bmatrix}-0.1125 & -0.02\\ 1 & 0\end{bmatrix}$, $A_2=\begin{bmatrix}-0.1125 & -1.527\\ 1 & 0\end{bmatrix}$, $B_1=B_2=\begin{bmatrix}1\\0\end{bmatrix}$, $C_1=C_2=\begin{bmatrix}0.01\\0\end{bmatrix}$, $D_1=D_2=[0\quad 1]$,

$A_{d1}=\begin{bmatrix}-0.0125 & -0.005\\ 0 & 0\end{bmatrix}$, $A_{d2}=\begin{bmatrix}-0.0125 & -0.23\\ 0 & 0\end{bmatrix}$

$\Delta A_1=\Delta A_2=\begin{bmatrix}-0.1125\\0\end{bmatrix}F(t)\,[1\quad 0]$, $\Delta B_1=\Delta B_2=0$

$\Delta A_{d1}=\Delta Ad_2=0$, $F(t)=\sin(t)$, $\omega(t)=0.1\sin(t)e^{-0.1t}$

The interval nondifferentiable time-varying delay [13] is :

$$d(t)=\begin{cases}0.1+0.25sin^2t & if\quad t\in\mathfrak{T}=\mathrm{U}_{k\geq 0}[2k\pi,(2k+1)\pi]\\ 0 & if\qquad t\in R^+\backslash\mathfrak{T}\end{cases}$$

We have $0.29\leq d(t)\leq 0.35$

The membership functions are:

$$h_1(t)=\left(1-\frac{1}{1+\exp\{-3(\frac{x_2}{0.5}-\frac{\pi}{2})\}}\right)\times\frac{1}{1+\exp\{-3(\frac{x_2}{0.5}+\frac{\pi}{2})\}}$$

$h_2(t)=1-h_1(t)$.


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The initial value of the system is $\varphi(t) = (0.5 - 1)^T$ for $t \in [\bar{d}_2, 0]$.

The following parameters are obtained by solving the LMI

When α=0.08 : $K_1 = K_2 = [-2.5284\ -0.3121]$

When α=0.1: $K_1 = K_2 = [-2.5301 - 0.3521]$

Thus, the system is 0.1-exponential stabilization and the value $\sqrt{\frac{\Lambda}{\lambda}} = 1.4044$ so the solution of the closed-loop system satisfies $\|x(t,\phi)\| \leq 1.4044e^{-0.1t}\|\phi\|, \quad \forall t \geq 0.$

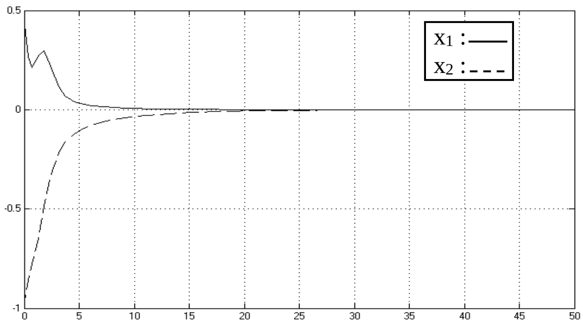


Fig.1. State responses of unforced system for $\alpha$=0.08.

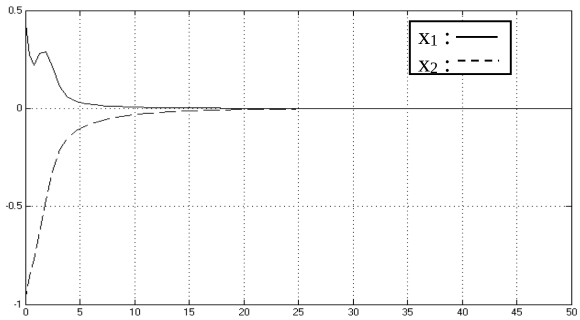


Fig.2. State responses of unforced system for $\alpha$=0.1.

Fig.1 and Fig.2. shows the state time-response of unforced system. It show that the system is stable when the initial condition is $\varphi(t) = (0.5 - 1)^T$ .

We note that when α = 0.1, the response time is faster when α = 0.08 on the other hand the excess is greater than in the former case against the second case. Our result is a little better than that obtain in [10].

## V. Conclusion

In this paper, we have considered the problem of the systems controlled in network. The new exponential stabilization for a class of nonlinear systems for uncertain Takagi–Sugeno (T–S) fuzzy systems with external disturbance and time-varying delay problem has been studied. The time delay is a continuous function belonging to a given interval, which means that the lower and upper bounds for the time-varying delay are available. The stability of time-delay T-S fuzzy system with a robust H∞ controller established by delay-dependent Lyapunov-Krasovskii functional approach and sufficient conditions for the exponential stabilization of the systems are first established in terms of LMI. An example is included to illustrate the effectiveness of the approaches proposed in this paper.

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