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Internal Model Control of MIMO Discrete Under-Actuated Systems Via Squaring Matrix Transforms

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***Abstract*— Internal model control (IMC) is an established technique in continuous-time linear control for SISO an MIMO fully-actuated systems but has not been developed for discrete-time under-actuated systems. In this paper we present a new IMC structure to the multivariable under-actuated systems, which is based on a specific inversion principle of the model plant. Simulated examples are presented to prove the effectiveness of the proposed control method to ensure set-point tracking, stability and disturbance rejection.**

***Keywords*— Internal model control; under-actuated systems; stability; disturbance rejection; specific inversion ;**

I. Introduction

Under-actuated systems offer challenging control problems to solve operational inconveniences with great interest from theoretical point of view. Non-square system is a common industrial process in fields of the practical engineering. The number of the input variables does not equal to that of the outputs, e.g. This class of systems are abundant in real life; examples of such systems include, but are not limited to, surface vessels, spacecraft, underwater vehicles, helicopters, road vehicles, mobile robots, space robots and under-actuated manipulators[12,13] .

In dispite of their generality in industry, the analysis and control for under-actuated systems need further research. In recent years, much works focus these field [2,6,7]. In order to contribute to this research area, we propose in this paper to apply an interesting Internal Model Control approach (IMC), to a class of discrete multivariable under-actuated systems.

Control methodologies such as dynamic inversion and Moore-Penrose control require an inversion of the input influence matrix. However, if the transfer function system matrix is non-square direct inversion is not possible [16].

During the early to mid 1970, internal model control was an active research area starting with [5]. Specifically, treats the disturbance rejection problem and ensure stability.

The specificity of this IMC structure resides in the use of a special controller which is an approximate inverse of the model plant. The use of this controller ensures a high level of robustness [1,3,4,5,6].

The analysis of the stability of elements of the internal model control has been conducted in the literature by numerous fundamental researches that depend on the type of systems considered and the scope. There are many methods studying the stability of linear discrete multivariable systems. These stability criteria can be classified into two main categories namely the frequency criterion using the notion of the characteristic equations and the time criterion based on Lyapunov theory.

All of the results in [1-5] on internal model control are confined to continuous-time systems. Analogous results for discrete-time systems are not available in the literature.

The purpose of this paper is to propose a new IMC method control for the discrete-time under-actuated systems. In the controller procedure, a simple design is presented such that initial conditions are taken into account; the controller has a good performance of tracking ability, excellent robustness and good control performance [8].

The influence of the model parameters and external disturbances will be also discussed.

In the present paper, we develop an alternative approach to internal model control that is directly applicable to discrete-time under-actuated systems . Using this approach, we simultaneously solve the command following and disturbance rejection problem in discrete-time. We also present simulations examples to prove the effectiveness of the proposed control method.

II. Problem Formulation

The basic IMC structure was designed for linear SISO systems and afterwards for linear MIMO fully-actuated systems [3,11,14]. The IMC structure of multivariable

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processes is given in Fig. 1, where $C(\text{z})$ is the transfer matrix of the controller, $M(\text{z})$ is the transfer matrix of the plant model.

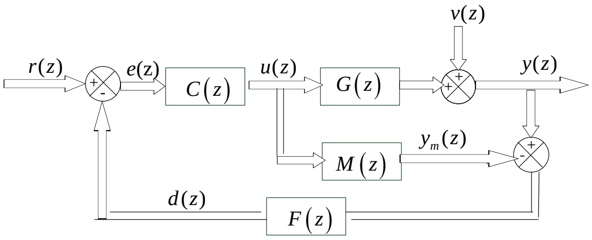

Fig. 1 IMC structure for multivariable discrete fully-actuated systems

However, the corresponding responses speed of the system output will be decreased. Hence, in order to improve the robustness while obtain a fast response speed, a first-order filter $F(z)$ is recommended to be added in the basic IMC structure as shown in Fig. 1 [1,3,10].

Where

r is the reference signal; ν a disturbance signal affecting the system; u the manipulated input signal, applied for both of the process G and its model M ; and the signal d represents the calculated difference between the process output signal y and the model output one y_m .

When the model is perfect, we have $M(z) = G(z)$, the signal d is reduced to the perturbation signal ν .

The control structure is then equivalent to an open loop scheme. From Fig. 1, the following equation can be derived without taking into account the robustness filter $F(\text{z}) = I$.

$$\begin{bmatrix} y \\ u \end{bmatrix} = \begin{bmatrix} GFC & I_m - CFM \\ C & -CF \end{bmatrix} \begin{bmatrix} r \\ \nu \end{bmatrix} \tag{1}$$

The Internal Model Control (IMC), is stable if and only if the process, the process model and the controller $C(z)$ are stable in open loop [5]. The realization of an IMC controller that equal to inverse of the model expression is essential in order to ensure perfect set-point tracking.

The synthesis of a controller C is the inverse of the chosen model if it's realizable in order to ensure perfect set-point tracking. This inversion presents the main problem of the IMC approach for linear discrete over-actuated systems. In fact, the realization of the direct model's inverse is difficult or not possible for most physical systems.

The model inversion is impossible too in the case of discrete under-actuated systems, because for them such the number of control inputs is equal to n and the number of outputs is equal to m (m is less than n), the transfer function of the process G is of dimension $(m \times n)$ making it a rectangular matrix where reversal is impossible. $G(z)$ is a process with $'n'$ inputs and $'m'$ outputs $(n \langle m)$ given as:

$$G(z) = \begin{pmatrix} G_{11}(\text{z}) & G_{12}(\text{z}) & \dots & G_{1n}(\text{z}) \\ G_{21}(\text{z}) & G_{22}(\text{z}) & \dots & G_{2n}(\text{z}) \\ \vdots & \vdots & \ddots & \vdots \\ G_{m1}(\text{z}) & G_{m2}(\text{z}) & \dots & G_{mn}(\text{z}) \end{pmatrix} \tag{2}$$

The matrix M must be chosen close the G , but as we explained previously, the inversion problem requires that the matrix M be square. In order to remedy this problem of inversion of model $M(z)$, it is necessary to use inversion techniques, we quote for example methods, virtual outputs [9-11], non-square effective relative gain (NERGA) [15], Moore-Penrose pseudo-inverse technique [16].

The objective of this paper is to solve this problem of inversion of model and then we explain our proposed solution by describing the changes we have introduced to this IMC structure so that it becomes applicable to linear discrete under-actuated systems. This control problem includes both disturbance rejection and influence of the model parameters.

III. THE CONTROL STRUCTURE DESIGN FOR DISCRETE UNDER-ACTUATED SYSTEMS

In this section, we propose firstly to modify the basic IMC structure so that it becomes applicable to under-actuated systems. Secondly we design an approximate inverse of the model plant [10].

A. The changes made to the basic IMC structure

We propose a simulation synoptic for the control of the discrete multivariable under-actuated systems by the internal model control represented by the figure 4.

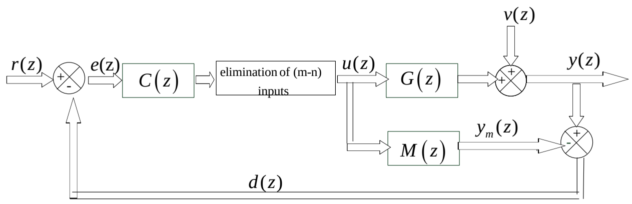

Fig. 2 The propose IMC synoptic for discrete under-actuated systems

The system $G(z)$ chosen in this paper is a MIMO under-actuated system, it’s non square so $M(z)$ is also, to solve this problem is added $(m \times (m\text{-}n))$ transfer functions to the matrix $M(z)$ in order to make it a square matrix of dimension $(m \times m)$; therefore, the transfer matrix of a linear

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discrete under-actuated systems can be square and the inversion model can be realised. The transfer matrix that will add to make the under-actuated systems square is its size $((m-n),m)$. This matrix has the following form:

$$\begin{pmatrix} y_1(\mathrm{z}) \\ y_2(\mathrm{z}) \\ \vdots \\ y_m(\mathrm{z}) \end{pmatrix} = \begin{pmatrix} M_{1,n+1}(\mathrm{z}) & M_{1,n+2}(\mathrm{z}) & \cdots & M_{1,\mathrm{m}}(\mathrm{z}) \\ M_{2,n+1}(\mathrm{z}) & M_{2,n+2}(\mathrm{z}) & \dots & M_{2,\mathrm{m}}(\mathrm{z}) \\ \vdots & \vdots & \ddots & \vdots \\ M_{m,n+1}(\mathrm{z}) & M_{m,n+2}(\mathrm{z}) & \dots & M_{m,m}(\mathrm{z}) \end{pmatrix} \begin{pmatrix} u_{n+1}(\mathrm{z}) \\ u_{n+2}(\mathrm{z}) \\ \vdots \\ u_m(\mathrm{z}) \end{pmatrix} \quad (3)$$

The $(m\times(m-n))$ functions can be chosen first-order transfer function in order to make the transfer matrix of the non-square system, up to have a square transfer matrix that can be reverse, to simplify the study and not affect the system stability [10].

On the other hand , a new function will be used to elimnate the excess control inputs acting on the process by the use of usual arithmetic operators.

B. Controller design

The IMC controller design by using the inversion method proposed in [10], is extended to linear discrete-time multivariable under-actuated systems, so we can obtain:

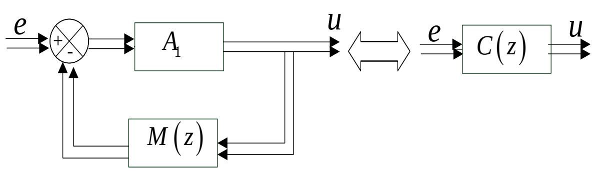

Fig. 3 Structure for model inversion

A_1 reversal of the matrix is an invertible square matrix; it must ensure the stability conditions of the controller.
The expression of IM controller can be obtained :

$$C(z) = \frac{A_1}{\mathrm{I_m} + A_1 M(\mathrm{z})} \quad (4)$$

Where I_m is the identity matrix; A_1 is a diagonal matrix; its coefficients are selected to satisfy the conditions of stability [16,1,5]. In order to better explain our study, A_1 can be expressed by:

$$A_1 = a \times \mathrm{I_m}, a \in \mathbb{R}^+ \quad (5)$$

With such a choice of A_1, and if we choose a high value of a, we obtain a small value of $\frac{1}{a}$ which allows to approximate $\frac{1}{\frac{1}{A_1} + M(z)}$ with $M(\mathrm{z})^{-1}$.

The stability of the controller C(z) depends of stability of the process control and of the model. After the equation (4), the controller is stable if all the poles of the characteristic equation are strictly lower module 1. Since M is stable, a suitable choice of the matrix A_1 ensures therefore the controller stability [10,12,14].

IV. SIMULATION RESULTS

In this part, we discuss the influence of the external disturbance and the perfection or not of the modeling.

Let's consider a (2×1) stable process with one control u1 and two outputs y1 and y2, is described by the following transfer matrix G.

$$G = \begin{bmatrix} G_1 \\ G_2 \end{bmatrix} \quad (6)$$

The process outputs is given by

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} G_1 \\ G_2 \end{bmatrix} [u_1] \quad (7)$$

Therefore and we as explained previously, the model transfer function is of dimension (2×2) in order to ensure invertibility conditions of the matrix M. The model is expressed by the following transfer matrix .

$$M = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \quad (8)$$

The model outputs is given by

$$\begin{bmatrix} y_{m1} \\ y_{m2} \end{bmatrix} = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \quad (9)$$

In order to test and validate the proposed design controller presented in Fig. 3 of multivariable under-actuated systems, two cases will be considered. In the first one, we show the influence of model parameters, and in the second one, the influence of external disturbances.
Our process transfer function $G(s)$ of study is shown as follows:

$$G(s) = \begin{bmatrix} \dfrac{2}{s^2+3s+2} \\ \dfrac{0.8}{s^2+1.5s+0.8} \end{bmatrix} \quad (10)$$

We can apply the study developed in [10], therefore the bilinear method of discretization are applied for the process G(s) and the process model M(s). The application of Jury stability criterion allows us to assess the necessary and sufficient condition of the controller stability.

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For a simpling frequency $T_e = 0.2\,s$, the closed-loop system defined by Fig. 1 is stable for a gain value $10\times I_m \le A_1 \le 70\times I_m$.

A. Case of perfect and imperfect modeling

IMC also allows us to introduce the notion of a perfect controller as an important theoretical tool.

When the model is perfect, the transfer function of model is expressed as :

$$M = \begin{bmatrix} \dfrac{2}{s^2+3s+2} & M_{12} \\ \dfrac{0.8}{s^2+1.5s+0.8} & M_{22} \end{bmatrix} \qquad (11)$$

The added functions M_{12} and M_{22} can be chosen first-order transfer function in order to make the transfer matrix of the non-square system, up to have a square transfer matrix that can be reverse, to simplify the study and not affect the system stability. In the case the transfer function of model is given by

$$M = \begin{bmatrix} \dfrac{2}{s^2+3s+2} & \dfrac{3}{s+1} \\ \dfrac{0.8}{s^2+1.5s+0.8} & \dfrac{2}{s+1} \end{bmatrix} \qquad (12)$$

For a unit step reference applied at $T = 0\,s$, the simulation results for a gain $A_1 = 64$ are the following:

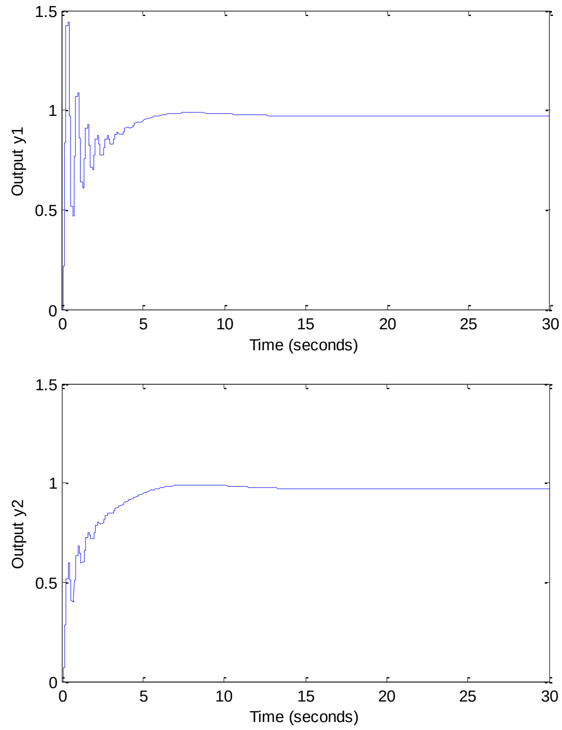

Fig. 4 Evolution of The outputs signals $y_1(t)$ and $y_2(t)$ of the sampled system with perfect modeling

When the model is not perfect, the transfer function of model is expressed as :

$$M = \begin{bmatrix} \dfrac{2}{s^2+4s+2} & \dfrac{3}{s+1} \\ \dfrac{0.8}{s^2+3s+0.8} & \dfrac{2}{s+1} \end{bmatrix} \qquad (13)$$

The IMC structure is based on an accurate linear model but modeling can't be too precise. Such a model can't provide a perfect description of the process behaviour, so we study the case of an imperfect model and test its parameters effects on the system evolution. From Fig. 5, we can see that, the system answers quickly and the reference signal is assured.

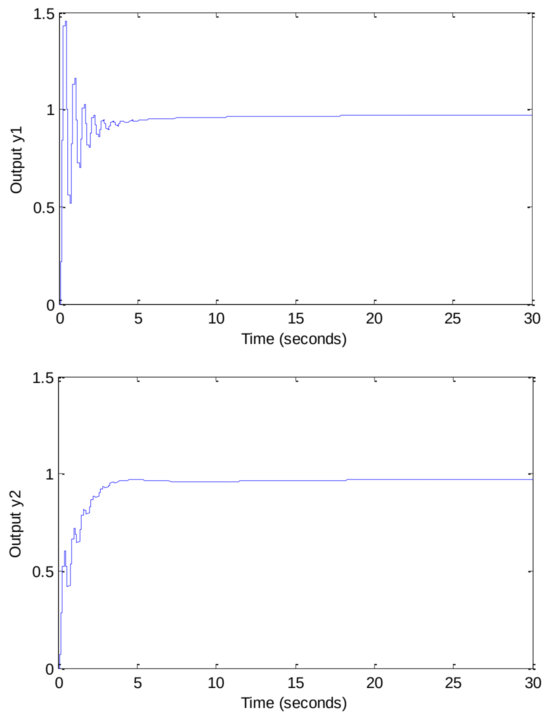

Fig. 5 Evolution of The outputs signals $y_1(t)$ and $y_2(t)$ of the sampled system with not perfect modeling

We have demonstrated the robustness of the proposed approach for internal model control of discrete multivariable process under-actuated when the system and its model may be different. This controller achieves perfect set-point satisfaction despite model/plant mismatch $M(z) \ne G(z)$.

B. Case of external disturbance

To testify the robustness of the system, a step disturbance was added at, t=15s. The disturbance signal is expressed by equation (14).

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$$v(s) = \begin{bmatrix} \frac{e^{-15s}}{s} \\ \frac{e^{-15s}}{s} \end{bmatrix} \quad (14)$$

The case of not perfect modeling is considered. The model is expressed equation (12).

The simulation results are shown in Fig.6. We can see that the proposed IMC of under-actuated systems , has a good performance on tracking given value and overcoming disturbance, but also has good stability and control quality both. Moreover, the designed controller is easy to compute.

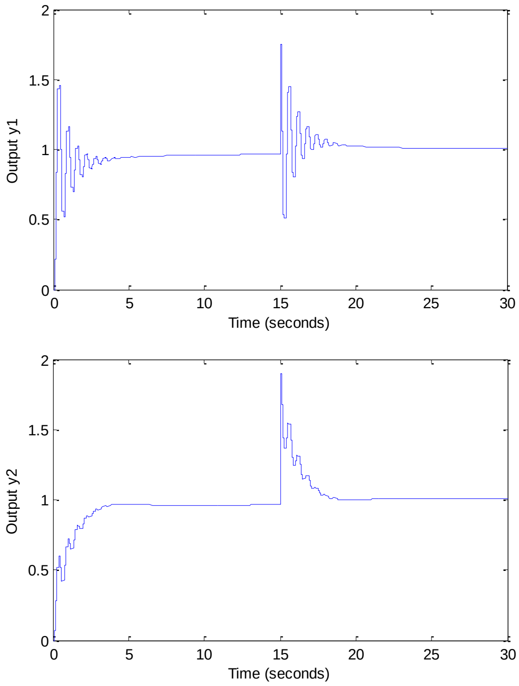

Fig. 6 Evolution of The outputs signals $y_1(t)$ and $y_2(t)$ of the sampled system with not perfect modelling and external disturbance

V. CONCLUSIONS

For the multivariable system with the number of inputs is less to that of outputs are habitually met in system industries, we proposed a new method based on internal model control for under-actuated system has been presented in this paper. The simulation results show that this method has better control performance and good robustness than other control methods for under-actuated process.

IMC structure disposes of the closed-loop stability issue altogether and thus gives the designer the opportunity to address the central issues of control system performance and robustness directly.

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