

Backstepping control for shunt active power filters

Controller Design and Average Performance Analysis

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Abstract—the problem of controlling shunt active power filters (APF) is addressed in presence of nonlinear loads. A Backstepping controller (BC) is proposed for active power filter (APF) in this paper. Firstly, the dynamic model for APF is build in which both the system parameter variations is considered. Then, the Backstepping method is applied in the design of current control system to deal with the nonlinearity of APF, and the proportional-integral correction to the voltage loop has been developed. The performances of the proposed controller are formally analysed using tools from the Lyapunov stability and linearisation methods. Finally, simulation in MATLAB validates the effectiveness of the proposed control approach.

Keywords—Parallel active power filter; harmonics; averaging model; Backstepping control; Stability of Lyapunov.

I. INTRODUCTION

With the increasing of non-linear loads such as diode or thyristor front-end rectifiers, switching power supplies power electronic devices in power system, power quality problems are deteriorating and attract more attention. Active power filters (APFs) are widely applied in power system to deal with distorted currents caused by the non-linear loads [1, 5, and 8].

The single-phase shunt active power filters are nowadays a good solution, since they can solve harmonic currents problems, and also compensate the power factor. Shunt active filters have various advantages over passive ones, since they don't need to be configured to a specific harmonic, but, all harmonics are simultaneously compensated.

The problem of controlling single phase shunt APFs has been given a great deal of interest and several control strategies have been proposed over the last decade. In this paper we will be interested to the Backstepping control. [2, 3] In control theory, Backstepping is a technique developed circa 1990 by Petar V. Kokotovic and others, for designing stabilizing controls for a special class of non-linear dynamical systems. These systems are built from subsystems that radiate out from an irreducible subsystem that can be stabilised using some other method. Because of this recursive structure, the designer can start the design process at the known stable

system and "back out" new controllers that progressively stabilize each outer subsystem. The process terminates when the final external control is achieved. Hence, this process is known as Backstepping.

The non-linear control theories for non-linear system develop quit rapidly after the proposing of Lyapunov theory, and researchers start to study on various non-linear control methods. The Backstepping method which receives widely attention once proposed is applied successfully in aerospace and robot control [4, 7, and 8]. The control strategy consists of two loops: an inner loop current to ensure the correction of the power factor compensation of reactive and deforming powers of the nonlinear load and an outer loop regulating the DC bus voltage of the active filter. It's formally shown, using tools from the Lyapunov stability and the averaging theory, that all control objectives are actually achieved in the mean. The theoretical results are illustrated by simulation.

The configuration of this work is organised as follows: The APF modelling is described in Section 2; a nonlinear control method, Backstepping control for active power filter. Based on the state space average model of APF, Backstepping method is adopted to design feedback control law. The design method of Backstepping control is described step by step in detail in Section 3; the system stability is proved by Lyapunov stability criterion. Through analysing the difference from conventional local linearisation methods in Section4; the theoretical performances are confirmed by simulation in Section 5. A conclusion and a references list end the paper.

II. TOPOLOGY AND MODELLING

The single phase shunt APF under study has the structure of Fig.1. It consists of a single-phase IGBT based half-bridge inverter and an energy storage capacitor c , placed at the DC side. From the AC side, the APF is connected, in parallel with a nonlinear load, to the main AC voltage source through filtering inductor L_f . The role of the APF is to produce reactive and harmonic components to compensate undesirable current harmonics produced by the non-linear load doing so; the filter-load association behaves as a pure resistive load

which amounts to make the fundamental component of load current in phase with the main AC voltage. The IGBT-based inverter operates in accordance to the well known Pulse Width Modulation principle [9].

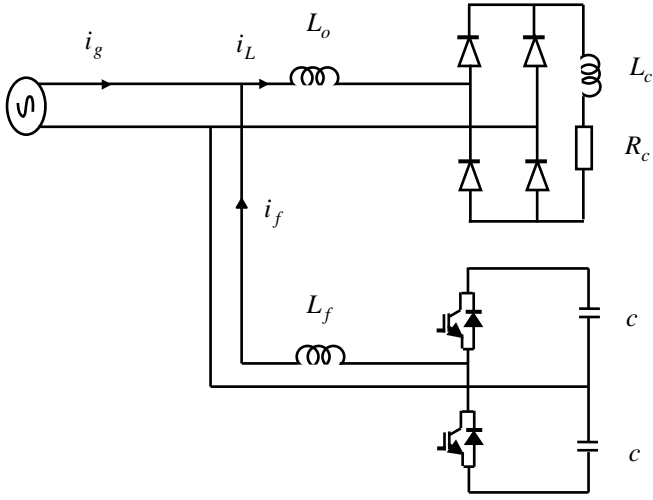


Fig. 1 Single-phase parallel active filter connected to a network with a non-linear load.

To study the behaviour of the active filter single phase with the electrical network, it must first establish a mathematical model of the whole network; shunt active power filters and non linear load.

Applying the usual Kirchoff's laws to the single phase shunt APF one easily gets:

for: $\mu = 1$

$$L_f \frac{di_f}{dt} = v_1 - v_g \quad (1.a)$$

$$C \frac{dv_1}{dt} = -i_f \quad (1.b)$$

$$C \frac{dv_2}{dt} = 0 \quad (1.c)$$

for: $\mu = -1$

$$L_f \frac{di_f}{dt} = -v_2 - v_g \quad (2.a)$$

$$C \frac{dv_1}{dt} = 0 \quad (2.b)$$

$$C \frac{dv_2}{dt} = i_f \quad (2.c)$$

where the switching function μ of the inverter is defined by:

$$\mu \begin{cases} 1si & T_1 \text{ est ON, } T_2 \text{ est OFF} \\ -1si & T_1 \text{ est OFF, } T_2 \text{ est ON} \end{cases}$$

Combining (1) and (2), one obtains the instantaneous model of whole system:

$$L_f \frac{di_f}{dt} = \frac{v_1 - v_2}{2} + \frac{[v_1 + v_2]}{2} \mu - v_g \quad (3.a)$$

$$L_f \frac{di_f}{dt} = \frac{V_d}{2} + \frac{V_s}{2} \mu - v_g \quad (3.b)$$

with:

$$\begin{cases} v_1 - v_2 = V_d \\ v_1 + v_2 = V_s \end{cases} \quad (3.c)$$

$$C \frac{d[v_1 - v_2]}{dt} = -\mu i_f$$

III. CONTROL DESIGN

A. Objective

Active filters are used to make the association including the nonlinear load and the APF look (from the power supply net) as a perfect resistor. This goal, commonly referred to PFC, will be achieved if the supply net current i_g enforced, using a suitable controller (yet to be found), to perfectly match a reference signal of the form $i_g^*(t) = \beta v_g(t)$. As long as the PFC objective is concerned, the (constant) value of β is not important. Later, the value of β will be online tuned to meet the DC voltage regulation objective. Since the supply net voltage $v_g(t)$ is sinusoidal, the previous requirement amounts to ensure that the net current is sinusoidal and in phase (or opposite phase) with the net voltage $v_g(t)$.

B. Current inner loop design

$$L_f \frac{di_f}{dt} = \frac{V_d}{2} + \frac{V_s}{2} u - v_g \quad (4.a)$$

$$C \frac{d[V_1 - V_2]}{dt} = -u i_f \quad (4.b)$$

The basic design rule of Backstepping method is to decompose the complicated non-linear system into several subsystems that no more than the system orders. In each subsystem, an appropriate virtual feedback is chosen, and then a virtual control will be introduced, the error between them is used to form Lyapunov function. The errors of each subsystem get access to asymptotic characteristics by designing control, and the non-linear controller with global asymptotic stability is obtained at last [6].

According to the mathematic model obtained above, the active power filter is a first order system, and its design of Backstepping control includes the following one step.

The shunt APF is used to compensate distorted current, so the output current i_f is the one to be controlled. Here defining i_f^* which the detected result of harmonic current as the control target is of i_f .

The design process of Backstepping controller is divided into two steps: firstly, we should construct virtual control function, and secondly construct the actual control law. We

take the controller of phase A as an example. The detailed design procedure is developed as follows [10]:

Step one: Define the first error variable as:

$$e = L_f (i_f - i_f^*) \quad (5.a)$$

Compute the derivative of e

$$\frac{de}{dt} = L_f \frac{di_f}{dt} - L_f \frac{di_f^*}{dt} \quad (5.b)$$

$$\frac{de}{dt} = \frac{v_1 - v_2}{2} + \frac{V_{dc}}{2} u - v_g - L_f \frac{di_f^*}{dt} \quad (5.c)$$

Set a Lyapunov function as:

$$v = \frac{1}{2} e^2 \quad (6.a)$$

The corresponding virtual control is chosen as:

$$\alpha = -k e \quad (6.b)$$

where $k > 0$

Finally, the control law is designed as:

$$u = \frac{2}{V_s} \left(-k e + V_g + L_f \frac{di_f}{dt} + \frac{V_d}{2} \right) \quad (6.c)$$

C. Section Headings Outer control loop design

The outer-loop is expected to generate the signal β so that the output voltage v_o is regulated to a given reference value v_o^2 . To this end, one must first establish the relation between β and the output voltage v_o (Fig. 2).

- Relation between β and v_o

The output voltage v_o varies in response to the signal β as follows

$$c \frac{dv_o}{dt} = \frac{1}{v_o} [f_1 + f_2] \quad (7.a)$$

with:

f_1 : The terms to mean not null.

f_2 : The terms to mean null.

So, we can write:

$$\frac{c}{2} \frac{d\bar{v}_s^2}{dt} = f_1 = -v_g i_L + \beta L_f i_L \dot{v}_g + v_g i_L L_f \dot{\beta} + \beta v_g^2 \quad (7.b)$$

$$\beta v_g L_f i_L - L_f v_g^2 \beta \dot{\beta} \quad (7.b)$$

$$\frac{c}{2} \frac{d\bar{v}_s^2}{dt} = f_2 = \left(1 - L_f \dot{\beta} \right) \left(\frac{\beta E^2}{2} + \frac{i_{L1}}{2} E \cos \varphi_1 \right) \quad (7.c)$$

- Squared output voltage regulation

The signal β stands as a control input in the first order system (7.b).

$$\beta = \frac{b}{b+s} (k_p e_2 + k_i e_3) \quad (8.a)$$

where s denotes the Laplace variable and (b, k_p, k_i) are any positive real constants.

$\dot{\beta}$ can be computed using the following equation:

$$\dot{\beta} = (k_p e_2 + k_i e_3 - \beta) \quad (8.b)$$

IV. CONTROL SYSTEM ANALYSIS

Theorem 1: Consider the shunt active power filter, sketched by Fig. 1, represented by its average model, in closed-loop with the cascade controller composed of:

The inner regulator consisting of the control law (6.c), and the outer regulator (7.b),

where the design parameters (s, b, k_p, k_i) are positive constant.

Then, the global closed-loop control system has the following properties:

The error $e_1 = L_f (x_1 - x_1^*)$ vanishes exponentially fast (where $x_1^* = \beta v_g - i_L$).

Introduce the augmented state vector:

$$X^*(t) = f(t, X) \quad (9.a)$$

$$\begin{cases} e_1 = s \\ e_2 = z \\ e_3 = \int z dt \\ e_4 = \beta \end{cases} \quad \text{with } z = y^* - y \quad (9.a)$$

Calculation of the equilibrium point:

$$\dot{e}_{1,2,3,4} = 0 \rightarrow X^*(t) = \begin{pmatrix} 0 \\ 0 \\ \frac{I_{L1}}{E k_i} \cos(\varphi_1) \\ \frac{I_{L1}}{E} \cos(\varphi_1) \end{pmatrix} \quad (9.c)$$

The stability of the equilibrium point:

Will now be analysed using the indirect Lyapunov method. The linearised model around the equilibrium point is written as [12].

with J is the Jacobian matrix.

$$\begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \\ \dot{e}_3 \\ \dot{e}_4 \end{bmatrix} = J = \left. \frac{\partial f}{\partial e} \right|_{e=e^*} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{E^2}{2} \\ 0 & 1 & 0 & 0 \\ 0 & b k_p & b k_i & -b \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{bmatrix} \quad (10.a)$$

with J is the Jacobian matrix.

Before applying the Routh-Hurwitz stability criterion, it should be determined the Eigenvalues of the matrix M.

$$\det(J - SI) = \begin{vmatrix} -S & 0 & 0 & 0 \\ 0 & -S & B & C \\ 0 & 1 & -S & 0 \\ 0 & bk_p & bk_i & -S - b \end{vmatrix} \quad (10.b)$$

$$\det(A - SI) = S^4 + bS^3 - \frac{E^2 b}{2} k_p S^2 - \frac{E^2 b}{2} k_i S \quad (10.c)$$

Applying the Routh-Hurwitz stability criterion, we find these conditions: $(k_p > 0, k_i > 0, k_p > \frac{k_i}{b})$

V. SIMULATION

The complete control system and simulated using the MATLAB/SIMPOWER (V.R2010a). The controlled part is a PAF system including the non-linear load based on a bridge rectifier, supplying a load which consists of a resistor R in series with an inductor L, with the numerical values of Table 1. All involved electrical components are simulated using SIMPOWER toolbox which offers a quite accurate representation of power elements. Presently, the ODE14x (extra-potation) solver is selected with fixed step time $10^6 s$.

A. Simulation parameters

The values of the parameters are in Table 1:

TABLE I
 PARAMETERS FOR SINGLE-PHASE PARALLEL ACTIVE FILTER, NETWORK AND A NONLINEAR LOAD.

Network		
$E = 220\sqrt{2}$		
Power active filter		
$c = 8.8mF$	$L_f = 50mH$	
Rectifier		
$L_o = 10mH$	$L_r = 50mH$	$R_r = 10\Omega$
Regulator parameters		
$\lambda = 100$	$f_p = 10Khz$	

B. Simulation Results

The validation of this control is made using the Matlab-Simulink software, if the diode bridge supplies a RL load type.

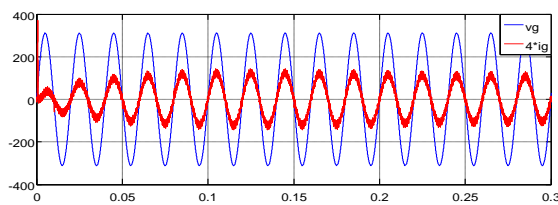


Fig. 4. Grid voltage and current

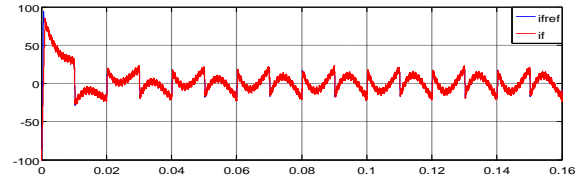


Fig. 6. current reference i_{fref} and i_f

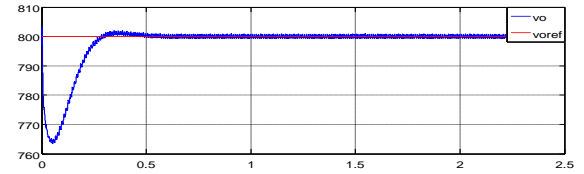


Fig. 7. Output voltage v_o

VI. CONCLUSION

A non-linear control strategy is proposed in this paper for single-phase shunt active power filters. The aim is to achieve current harmonics and reactive power compensation, in presence of non-linear and uncertain loads, as well as tight voltage regulation at the inverter output capacitor. To this end, a cascade non-linear controller is developed and analysed making use of advanced tools from the control approach of Backstepping control design, system averaging theory. It's formally established that the control objectives are actually achieved in average with a quite satisfactory accuracy. The formal results are confirmed by several simulations that also show the controller robustness against load change and uncertainty. So, the objectives of command have been achieved in terms of prosecution, the current i_f is well followed his reference, and regulation, the voltage v_o is stabilized at its reference value, therefore the single-phase shunt active power filters is an effective solution for the compensation of disturbances generated by the load non-linear.

REFERENCES

- [1] A. Bhattacharya, C. Chakraborty, "A Shunt Active Power Filter With Enhanced Performance Using ANN-Based Predictive and Adaptive Controllers," IEEE Transactions on Industrial Electronics, 58, pp. 421-428. 2011.
- [2] Petar V.Kokolovic, "The Joy of Feedback Nonlinear and Adaptive," IEEE Control Systems, VOL. 12, NO. 3. June 1992.
- [3] Rogelio Lozano and Bernard Brogliato, "Adaptive Control of Robot Manipulators with Flexible Joints," IEEE TRANSACTIONS ON AUTOMATIC CONTROL, VOL. 37, NO. 2. FEBRUARY 1992.
- [4] F. J. Lin, P. H. Shen, S. P. Hsu, "Adaptive backstepping sliding mode control for linear induction motor drive," IEE Proceedings-Electric Power Applications, 149, pp. 184-194. 2002.
- [5] J. F. Petit, G. Robles, H. Amaris, "Current Reference Control for Shunt Active Power Filters Under Nonsinusoidal Voltage Conditions," IEEE Transactions on Power Delivery, 22, pp. 2254-2261. 2007.
- [6] Z. Youping, B. Fidan, P. A. Ioannou, "Backstepping control of linear time-varying systems with known and unknown parameters," IEEE Transactions on Automatic Control, 48, pp. 1908-1925. 2003.
- [7] D. Zhao, T. Zou, S. Li, and all, "Adaptive backstepping sliding mode control for leader-follower multi-agent systems," IET Control Theory & Applications, 6, pp. 1109-1117. 2012.

- [8] J.J. Ge, Z.M. Zhao, J.J. Li, "BACKSTEPPING CONTROL FOR ACTIVE POWER FILTER WITH LCL FILTER," Renewable Power Generation Conference (RPG 2013), 2nd IET, pp. 1 – 4. [10.1049/cp.2013.1761](https://doi.org/10.1049/cp.2013.1761), 2013
- [9] José A.P, "Analyzing the Stability of the FDTD Technique by Combining the von Neumann Method with the Routh–Hurwitz Criterion," IEEE transactionson microwave theory and techniques, vol. 49, no. 2. February 2001.
- [10] Lihua Deng, Juntao Fei, Changchun Cai, "Shunt Active Power Filter Based on a Novel Sliding Mode Backstepping Control for Three-phase Three-wire System," 14th International Conference on Control, Automation and Systems (ICCAS 2014) Oct. 22-25, 2014 in KINTEX, Gyeonggi-do, Korea. 2014.
- [11] JI, Y., LU, J., QIU, J, "Stability of equilibrium points for incommensurate fractional-order nonlinear systems," Proceedings of the 35th Chinese Control Conference, July 27-29, Chengdu, China 2016.
- [12] JI, Y., LU, J., QIU, J, "Stability of equilibrium points for incommensurate fractional-order nonlinear systems," Proceedings of the 35th Chinese Control Conference, July 27-29, Chengdu, China 2016.