

# High Gain Observer for Sensorless Photo-voltaic Systems

Mohamed Stitou\*, Abderrahim El Fadili†, Fouad Giri‡, F.Z Chaoui§, Drhorhi Ismail¶  
 \*§University of Mohammed V, Ecole Normale Supérieure d'Enseignement Technique (ENSET)

Rabat-Morocco

Email: stitou.mh@gmail.com

†¶University of Hassan II, Faculté des sciences et techniques de Mohammedia (FSTM)

Mohammedia-Morocco

E-mail: elfadili.abderrahim@yahoo.fr

‡University of Caen Basse Normandie

14032 Caen France

E-mail: fouad.giri@unicaen.fr

**Abstract**—The problem of state estimation in PV systems is considered. The system consists of PV panels, a long DC cable which supplies a DC motor via a DC/DC converter. Generally, PV systems are established near to the control unit of the converter. The maximum output power tracking (MPPT) methods and control laws are based on the PVG voltage and current measurements. However, PV arrays must be placed in a site that guarantees good solar radiation. In most cases, such a site is at great distance from the control unit. Thus, the PVG voltage and current measurements become difficult. To overcome this issue, a high gain observer design is developed, in this paper, for PV systems. The observer provides estimates of PVG voltage and current using only current and voltage measurements in the other side of the DC cable (converter side). The observer convergence is formally analyzed and illustrated by simulation.

## I. INTRODUCTION

Several works in literature are dealing with PV systems control using various MPPT techniques. However, authors were not concerned in the cable effects on PV systems control. Mostly, it's implicitly considered that the DC/DC converter is connected closely to the PV generator and the cable is not long enough to alter PVG voltage and current values. In reality, there are situations where the distance between the PV arrays site and the converters is great, notably in water pumping which is one of the most popular applications in the use of PV-energy. This leads us to think about the influence of using long cables on the PV system control behaviour. In fact, DC cable equivalent electric circuit, whose parameters values are increasing proportionally as the cable length, may affect considerably the system control performances. Hence, MPPT techniques which are based on PV voltage and current measurements couldn't ensure the MPPT objective as the cable equivalent circuit is not involved in the control laws designing. The paper is composed of 5 sections. Section 2 is reserved to the system modeling while section 3 is reserved to the system observability analysis. Section 4 is devoted to the observer design and section 5 is dedicated to the observer test and simulation results.

## II. SYSTEM MODELING

Fig.1 shows the general diagram of the system. It consists of a photo-voltaic generator (PVG) connected to a boost DC/DC converter by a long two wires cable. The boost converter supplies a DC motor.

The PV generator consists of a number of PV modules associated in series and in parallel in order to provide needed levels of voltage and current. The DC cable is represented by its equivalent electrical circuit. The latter consists of a serial resistance  $R$ , a serial inductance  $L$  and two parallel capacitances  $C$ .

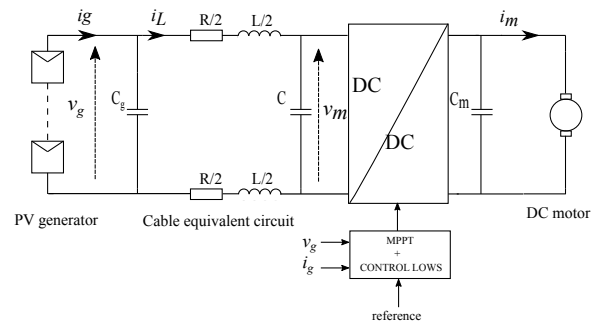


Fig. 1. System diagram.

### A. PV Generator Model

The direct conversion of the solar energy into electrical power is obtained by solar cells. The equivalent circuit of PV module and the  $(I_p, V_p)$  characteristics of the photovoltaic generator are described in [1]–[4].

The PV array module considered in this paper is the SM55. It has 36 series connected mono-crystalline cells. The module electrical characteristics are assembled in table I.

### B. DC cable Model

The DC cable considered in this work is an unshielded two wires cable. It consists of two parallel PVC covered

wires. Fig.2 shows the geometric form and the dimensions of the cable, while fig.3 presents its equivalent electrical model. Values of the cable model parameters are given in table II.

TABLE I  
ELECTRICAL SPECIFICATIONS FOR THE SOLAR MODULE

|                            |               |                    |
|----------------------------|---------------|--------------------|
| Maximum power              | $P_m$ (W)     | 55                 |
| Short circuit current      | $I_{SCR}$ (A) | 3.45               |
| Open circuit voltage       | $V_{oc}$ (V)  | 21.7               |
| Voltage at max power point | $V_m$ (V)     | 17.4               |
| Current at max power point | $I_m$ (A)     | 3.15               |
| $K_1$ (A/K)                |               | $4 \times 10^{-4}$ |

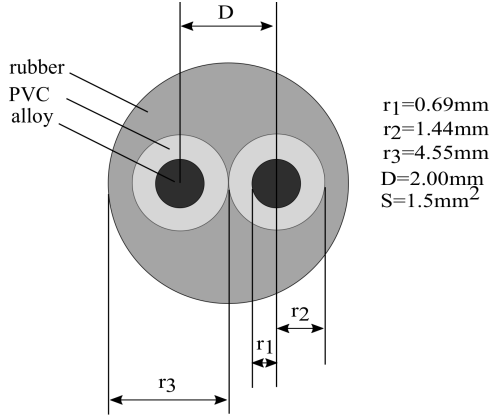


Fig. 2. DC cable section

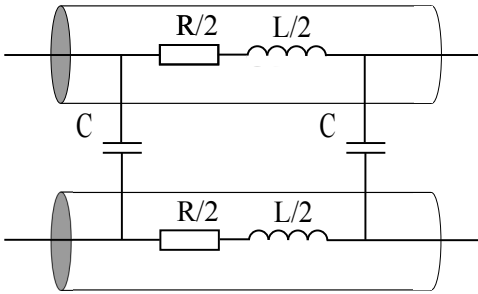


Fig. 3. Cable equivalent circuit

TABLE II  
VALUES OF CABLE MODEL PARAMETERS

|             |             |      |
|-------------|-------------|------|
| Resistance  | $R$ (mΩ/m)  | 65   |
| Inductance  | $L$ (μH/m)  | 0.28 |
| Capacitance | $C$ (pF/m)  | 60   |
| Conductance | $V_m$ (S/m) | ∞    |

### C. Modeling the system (PV generator - DC line)

The system is described by the following equations :

$$\frac{di_L}{dt} = -\frac{R}{L}i_L + \frac{1}{L}v_g - \frac{1}{L}v_m \quad (1)$$

$$\frac{dv_g}{dt} = -\frac{1}{C_g}i_L + \frac{1}{C_g}i_g \quad (2)$$

$$\frac{di_g}{dt} = \mu(t) \quad (3)$$

where,

$i_L$  is the current through the DC line,  $v_g$  is the PV generator voltage,  $i_g$  is the PV generator current,  $v_m$  is the DC/DC converter input voltage.

and  $\mu(t)$  is an unknown real bounded function which depend of the PV voltage  $v_g$ , the solar radiation  $\lambda$  and other factors.

Consider the state variables :  $x_1 = i_L$  ;  $x_2 = v_g$  ;  $x_3 = i_g$ .

The boost converter input voltage  $v_m$  is taken as the control input  $u$  of the system. Then, the system state is expressed by the following equations :

$$\frac{dx_1}{dt} = -\frac{R}{L}x_1 + \frac{1}{L}x_2 - \frac{1}{L}u \quad (4)$$

$$\frac{dx_2}{dt} = -\frac{1}{C_g}x_1 + \frac{1}{C_g}x_3 \quad (5)$$

$$\frac{dx_3}{dt} = \mu(t) \quad (6)$$

In the next section the system observability is studied by considering the DC line current  $x_1$  as the output  $y$  of the system. The state vector is noted  $x = [x_1 \ x_2 \ x_3]^T$ .

### III. OBSERVABILITY ANALYSIS OF THE SYSTEM MODEL

According to the three equations (4), (5) and (6) above, the system state has the following form (7):

$$\begin{cases} \dot{x} = G(x).x + H(x, u) + D\mu(t) \\ y = Cx \end{cases} \quad (7)$$

where  $x = [x_1 \ x_2 \ x_3]^T$  is the state vector,  $u$  is the control input of the system,  $y$  is the system output, and the matrices  $G(x)$ ,  $H(x, u)$ ,  $D$  and  $C$  are given as shown below :

$$G(x) = \begin{pmatrix} 0 & \frac{1}{L} & 0 \\ 0 & 0 & \frac{1}{C_g} \\ 0 & 0 & 0 \end{pmatrix}; H(x, u) = \begin{pmatrix} -\frac{R}{L}x_1 - \frac{1}{L}u \\ -\frac{1}{C_g}x_1 \\ 0 \end{pmatrix};$$

$$D = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}; C = [1 \ 0 \ 0]$$

Now, consider the following state transformation:

$$\Omega : R^3 \rightarrow R^3; x \rightarrow \xi = \begin{pmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \end{pmatrix}$$

$$\Omega(x) = \begin{pmatrix} \Omega_1(x) \\ \Omega_2(x) \\ \Omega_3(x) \end{pmatrix} = \begin{pmatrix} x_1 \\ \frac{1}{L}x_2 \\ \frac{1}{LC_g}x_3 \end{pmatrix} = Tx$$

Applying the transformation above, system (7) is described in the new coordinates  $\xi$  by the following equation (8):

$$\begin{cases} \dot{\xi} = A\xi + \psi(u, \xi) + b(\xi)\mu(t) \\ y = C\xi = \xi_1 \end{cases} \quad (8)$$

where the matrices  $A$ ,  $C$  and  $b(\xi)$  are defined as follows :

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}; C = [1 \quad 0 \quad 0]; b(\xi) = \begin{pmatrix} 0 \\ 0 \\ \frac{1}{LC_g} \end{pmatrix}$$

and the  $\psi(\xi, u)$  function is a triangular structure given bellow:

$$\psi(u, \xi) = \begin{pmatrix} \psi_1(\xi_1, u) \\ \psi_2(\xi_1, \xi_2, u) \\ \psi_3(\xi_1, \xi_2, \xi_3, u) \end{pmatrix} = \begin{pmatrix} -\frac{R}{L}\xi_1 - \frac{1}{L}u \\ -\frac{1}{LC_g}\xi_1 \\ 0 \end{pmatrix}$$

One can easily check that  $\psi(\xi, u)$  is Lipschitz with respect to  $\xi$  uniformly in  $u$  on  $IR^3$ . Then, system (8) above is observable. Thus, system (7) may be observable if the transformation  $\Omega$  is regular for all  $x \in R^3$ . To check such a condition, one shall only check that the Jacobian of  $\Omega(x)$  is full rank. The later is given by (9) bellow:

$$J_{\Omega(x)} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{L} & 0 \\ 0 & 0 & \frac{1}{LC_g} \end{pmatrix} \quad (9)$$

Then  $J_{\Omega(x)}$  is full rank, which implies that  $\Omega$  is regular almost everywhere in  $R^3$ , consequently system (7) is observable.

#### IV. OBSERVER DESIGN

The proposed observer for system (8) is a HG observer which is given by the following equation (10): [6]

$$\dot{\hat{\xi}} = A\xi + \psi(\hat{\xi}, u) - \theta\Delta_\theta^{-1}K(C\hat{\xi} - y) \quad (10)$$

where,

$\hat{\xi} = [\hat{\xi}_1 \quad \hat{\xi}_2 \quad \hat{\xi}_3]^T \in R^3$  is the state vector,  $u$  and  $y$  are, respectively, the input and the output of system (8).

$K = S^{-1}C^T$  is the observer gain which is chosen so that the matrix  $A - KC$  is Hurwitz.

$S$  is the symmetric positive define matrix satisfying the algebraic Lyapunov equation:  $S + A^T S + SA + C^T C = 0$ .

The function  $\psi(\hat{\xi}, u)$  has a triangular structure and is given bellow :

$$\psi(\hat{\xi}, u) = \begin{pmatrix} -\frac{R}{L}\hat{\xi}_1 - \frac{1}{L}u \\ -\frac{1}{LC_g}\hat{\xi}_1 \\ 0 \end{pmatrix}$$

$\theta$  is a positive parameter.

$\Delta_\theta$  is given by the following matrix:

$$\Delta_\theta = \text{diag}(1, \frac{1}{\theta}, \frac{1}{\theta^2})$$

#### A. Observer stability analysis

**Theorem.** Consider the system (8) and assume that the function  $\mu(t)$  is bounded. Then,

$$\exists \alpha_1 > 0; \forall \theta > \max\{1, \alpha_1\}; \exists \eta > 0; \exists \sigma_\theta > 0; \exists L_\theta > 0;$$

$$\forall u \in U \subset IR^3; \forall \xi(0) \in IR^3$$

$$\|\hat{\xi}(t) - \xi(t)\| \leq \eta \theta^2 e^{-\sigma_\theta t} \|\bar{z}(0)\| + L_\theta \mu_0$$

Where  $\xi(t)$  is an unknown trajectory of the system associated with the input  $u$ ,  $\hat{\xi}(t)$  is any observer trajectory associated with  $(u, y)$ , and  $\mu_0$  is the upper bound of  $\mu(t)$ .

**Proof.**

Let  $z(t)$  be the estimation error:

$$\begin{aligned} z(t) &= \hat{\xi}(t) - \xi(t) \\ \dot{z} &= (A - \theta\Delta_\theta^{-1}S^{-1}C^T C)z + \psi(\hat{\xi}, u) - \psi(\xi, u) \\ &\quad - b(\xi)\mu(t) \end{aligned}$$

Using the fact that:  $\theta\Delta_\theta^{-1}A\Delta_\theta = A$  and  $C^T C\Delta_\theta = C^T C$ , and considering the variable change  $\bar{z} = \Delta_\theta z$ , then considering the quadratic function  $V(\bar{z}) = \bar{z}^T S \bar{z}$ , one gets the following final result :

$$\begin{aligned} \|z\| &\leq \theta^2 \frac{\sqrt{\lambda_{max}(S)}}{\sqrt{\lambda_{min}(S)}} \|\bar{z}(0)\| \exp[-(\frac{\theta - \alpha_1}{2})t] \\ &\quad + 2 \frac{\sqrt{\lambda_{max}(S)}}{\sqrt{\lambda_{min}(S)}} \frac{\mu_0}{LC_g(\theta - \alpha_1)} \end{aligned}$$

Where,  $\alpha_1 = 2 \frac{\sqrt{\lambda_{max}(S)}}{\sqrt{\lambda_{min}(S)}} \gamma$  with  $\gamma \geq 0$  is so that

$$\|\Delta_\theta(\psi(\hat{\xi}, u) - \psi(\xi, u))\| \leq \gamma \|z\|;$$

$\lambda_{min}(S)$  and  $\lambda_{max}(S)$  are respectively the smallest and the largest eigenvalues of the function  $S$  which is the **symmetric positive define** matrix satisfying the algebraic Lyapunov equation:  $S + A^T S + SA + C^T C = 0$ .

Thus, to find the parameters used by the Theorem it suffices to set:

$$\eta = \frac{\sqrt{\lambda_{max}(S)}}{\sqrt{\lambda_{min}(S)}}; \sigma_\theta = \frac{\theta - \alpha_1}{2}$$

$$\text{and } L_\theta = 2 \frac{\sqrt{\lambda_{max}(S)}}{\sqrt{\lambda_{min}(S)}} \frac{1}{LC_g(\theta - \alpha_1)}$$

#### B. Observer equation in the original coordinates

Applying the reverse of the transformation  $\Omega$  to the observer state equation (10), one obtains the dynamic equation (11) bellow for the observer in the original coordinates  $x$  :

$$\dot{\hat{x}} = G(\hat{x})\hat{x} + H(\hat{x}, u) - \theta T^{-1}\Delta_\theta^{-1}S^{-1}C^T(C\hat{x} - y) \quad (11)$$

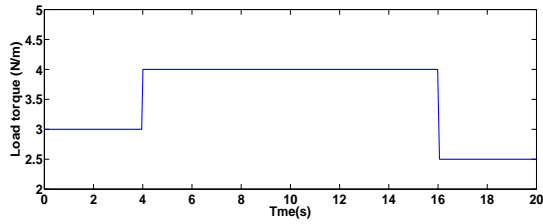


Fig. 4. Load Torque (Nm)

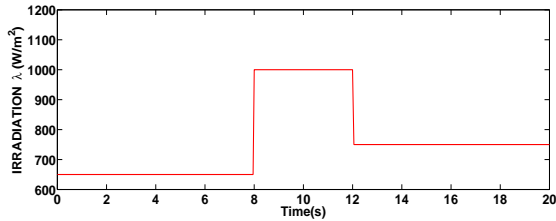


Fig. 5. Irradiation  $\lambda$  (W/m<sup>2</sup>)

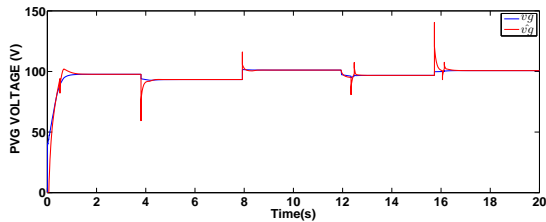


Fig. 6. Voltages  $v_g$  and  $\hat{v}_g$  for  $\theta = 50000$

## V. SIMULATION RESULTS

The simulated system diagram is realized using Powersim software (PSIM) while the observer is implemented within Matlab-Simulink using an Mfile S\_function . The system consists of a PV generator composed of a photovoltaic array, which supplies a DC motor of 0.5 KW, by an unshielded cable of 50 m.

Simulations have been achieved using the following values of the observer design parameters:  $k_1 = 8$ ,  $k_2 = 6$  and  $k_3 = 2.5$ . The observer performances have been evaluated by adjusting the gain parameter  $\theta$ .

A simulation profiles for the DC motor load torque and the solar irradiation have been defined as illustrated in Fig.4 and Fig.5. Simulations have been achieved for many values of the gain  $\theta$ . The results given here are just for two values of  $\theta$  :  $\theta = 50000$  and  $\theta = 65000$ .

We observed that the convergence of the estimated current  $\hat{i}_g$  and voltage  $\hat{v}_g$  is as perfect as the gain  $\theta$  is large. However, as Fig.8 and Fig.9 show, for heigher values of  $\theta$ , the observer becomes noise sensitive. Hence, the parameter  $\theta$  should be choosen as compromise between an acceptable convergence and noise sensitivity of the observer.

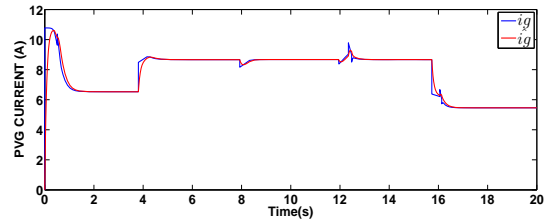


Fig. 7. Currents  $i_g$  and  $\hat{i}_g$  for  $\theta = 50000$

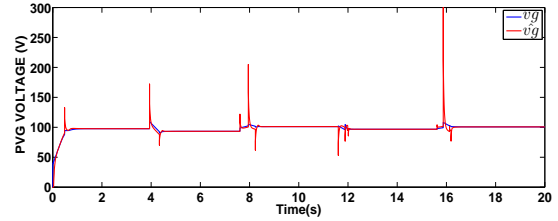


Fig. 8. Voltages  $v_g$  and  $\hat{v}_g$  for  $\theta = 65000$

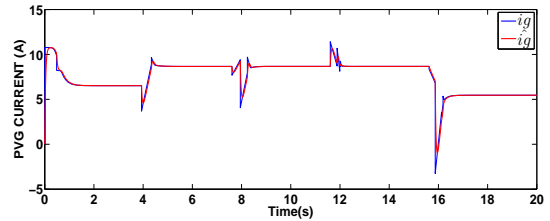


Fig. 9. Currents  $i_g$  and  $\hat{i}_g$  for  $\theta = 65000$

## VI. CONCLUSION

In the present work, a high gain observer has been synthesized and tested using a photo-voltaic system without MMPT control. It has been shown by simulation that the convergence of the PVG estimated voltage and current is guaranteed by tuning adequately the estimator gain parameter  $\theta$ .

## REFERENCES

- [1] Gow J. A., C. D. Manning. (1999) *Development of a photovoltaic array model for use in powerelectronics simulation studies*. IEEE Proceedings on Electric Power Applications, Vol. 146, No. 2, pp:193–200.
- [2] Luque A, S. Hegedus. (2003) *Handbook of Photovoltaic Science and Engineering*. John Wiley & Sons, Ltd.
- [3] Tan Y. T, D.S. Kirschen, N. Jenkins. (2004) *A model of PV generation suitable for stability analysis*. IEEE Transactions On Energy Conversion, Vol. 19, No. 4, pp: 748–755.
- [4] El Fadili A., Giri F. and El Magri A. (2014) *Reference Voltage Optimizer for Maximum Power Point Tracking in Triphase Grid-Connected Photovoltaic Systems*. International Journal of Electrical Power & Energy Systems, Vol.60, pp: 293–301.
- [5] WEENS Yannick (2006) *Modélisation des câbles d'énergie soumis aux contraintes générées par les convertisseurs d'électronique de puissance*. Université de Science et Technologie de Lille
- [6] M. Farza, M. M'Saad, L. Rossignol (2004) *Observer design for a class of MIMO nonlinear systems*. Automatica V.40, pp: 135–143
- [7] El Fadili A., Giri F. and El Magri A. (2013) *Backstepping Control for Maximum Power Tracking in Single- Phase Grid-Connected Photovoltaic Systems*. 11th IFAC International Workshop on Adaptation and Learning in Control and Signal Processing.