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Predictive Control of dc-dc Buck Power Converters: Theoretical Analysis and Experimental Results

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***Abstract*— This paper deals with the problem of controlling buck power converter using Model Predictive Control technique. The control objective is to ensure a tight regulation of the output voltage and asymptotic stability of the closed loop system. The simulations and experimental results showed that the output voltage perfectly tracks its reference.**

I. INTRODUCTION

During the past few decades, the use of power converters has become very popular for a wide range of applications, including drives, energy conversion, traction, and distributed generation [3]. They are devices that provide a supply to the electric machines by the conversion of an electrical signal. DC-DC converters are some of the most important circuits within the family of power circuits. Due to their intrinsic nonlinearity, these systems represent an interesting field for control algorithms [5]. The control of power converters has been extensively studied, and many new control schemes are presented.

Model Predictive Control (MPC) has emerged as a promising control alternative for power converters. It'is an optimal control strategy based on numerical optimization with a variety of advantages. MPC is a control strategy that obtains the control action by solving, at each sampling instant, an optimization which forecasts the future system behavior over a finite horizon [1]. MPC algorithms is very simple to design and implement, it can control large scale systems with many control variables [6].

Predictive control presents several advantages that make it suitable for the control of power converters: Concepts are intuitive and easy to understand, it can be applied to a variety of systems, constraints and nonlinearities can be easily included, multivariable case can be considered, and the resulting controller is easy to implement ([1], [2], [3], [11], [13], [14]).

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In this paper, a digital control of dc-dc buck power converter using the MPC approach is proposed and validated both by simulations and experiments.

The paper is organized as follows. In Section 2, dc- dc buck power converter is modeled. Section 3 is devoted to the predictive control of the converter. Section 4 illustrates the performances of the proposed approach by numerical simulation and experimental results. A conclusion and References list end the paper.

II. DC-DC BUCK POWER CONVERTER PRESENTATION AND MODELING

A. Dc-dc buck power converter presentation

The DC-DC Buck converter is one of the basic power electronic circuits, and it has been widely used in the fields of DC power supplies and DC motor speed regulating systems. [15]. Fig. 1 shows a typical pulse wide modulation (PWM)-based dc–dc buck power converter structure, where iL is the inductance current and vc the average output capacitor voltage, L the inductance of the circuit, C the capacitor of the circuit, R the load resistance of the circuit, E the voltage of the external source, u the binary input signal, and the duty ratio function α∈ [0, 1] the control signal of PWM. In this model, all involved passive components are subject to linear laws.

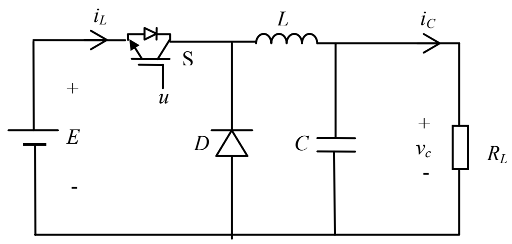

Fig. 1 DC-DC buck converter.

B. Continuous time model of buck power converter

It is shown in many places (see e.g. [4]) that the switching model of dc-dc buck power converter is

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$$\frac{di_L}{dt} = -\frac{1}{L} v_c + u\frac{E}{L} \quad (1a)$$

$$\frac{dv_c}{dt} = -\frac{v_c}{R_L C} + \frac{i_L}{C} \quad (1b)$$

For control design purpose, it is more convenient to consider the following averaged model, obtained by averaging the model (1) over one switching period [7]

$$\dot{z} = Az + B\alpha \quad (2)$$

Where $z = [z_1 \quad z_2]^T$ is the state vector, z_1 and z_2 denote the average input current (i_L) and the average output capacitor voltage (v_c), respectively. The control input for the above model is the duty ratio α. The state matrices A and B are given by

$$A = \begin{bmatrix} 0 & \frac{-1}{L} \\ \frac{1}{c} & \frac{-1}{R_L C} \end{bmatrix} \quad (3a)$$

$$B = \begin{bmatrix} \frac{E}{L} \\ 0 \end{bmatrix} \quad (3b)$$

The control objective is to enforce the output voltage z_2 to track a given constant reference voltage $V_d > 0$. The desired equilibrium point of (2) is then obtained

$$z_{2e} = V_d\ ,\ z_{1e} = \frac{V_d}{R_L}\ ,\ \alpha_e = \frac{V_d}{E} \quad (4)$$

Taking $x_1 = z_1 - z_{1e}$, $x_2 = z_2 - z_{2e}$ and $\mu = \alpha - \alpha_e$, then the following new representation is obtained

$$\dot{x} = Ax + B\mu \quad (5)$$

On the basis of (5), a discrete time model and an optimal control law will be determined.

C. Discrete time model over the prediction horizon

The Model Predictive Control is based on the optimization of a cost function (Leuer & Bocker 2015). We use a dynamic model to predict the future response of the controlled plant. In this paper, and in order to develop our proposed MPC controller, the following discrete-time state-space representation is used

$$x(k+1) = Ax(k) + C\mu(k) \quad (6)$$

where $x(k)$ is the state vector and $\mu(k)$ the input signal at the k^{th} sampling period. The matrix C is defined as follows

$$C = \begin{bmatrix} B & 0 & \cdots & 0 \\ AB & B & \cdots & 0 \\ \vdots & \vdots & \ddots & \\ A^{N-1}B & A^{N-2}B & \cdots & B \end{bmatrix} \quad (7)$$

Given a predicted input sequence, the corresponding sequence of state predictions is generated by simulating the model forward over the prediction horizon, of say N sampling intervals. The predicted state sequence generated by the linear state-space model with input sequence $\mu(k)$ can be written as follows [6]

$$x(k+i+1/k) = Ax(k+i/k) + B\mu(k+i/k), \quad i = 0,...,N \quad (8)$$

where $\mu(k+i/k)$ and $x(k+i/k)$ indicate input and state vectors at time $k+i$ that are predicted at time k . The initial condition of (8) (at the beginning of the prediction horizon) is defined as follows

$$x(k/k) = x(k) \quad (9)$$

It follows, using (8) and (9), that the predicted state sequence can be written in the following compact notation

$$x(k+i/k) = A^i x(k) + C_i v(k)\ , \quad i = 0,...,N \quad (10)$$

with C_i is the i^{th} block row of C and $v(k)$ is a vector defined as follows

$$v(k) = \begin{bmatrix} \mu(k/k) \\ \mu(k+1/k) \\ \vdots \\ \mu(k+N-1/k) \end{bmatrix} \quad (11)$$

Equation (10) can also be written in more compact form as follows

$$X(k) = Mx(k) + Cv(k) \quad (12)$$

where

$$X(k) = \begin{bmatrix} x(k+1/k) \\ x(k+2/k) \\ \vdots \\ x(k+N/k) \end{bmatrix} \quad (13)$$

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and

$$M = \begin{bmatrix} A \\ A^2 \\ \vdots \\ A^N \end{bmatrix} \tag{14}$$

III. MODEL PREDICTIVE CONTROL DESIGN

Minimizing the predictive performance cost, which is defined in terms of the predicted sequences μ , x , allows us to compute the predictive control feedback law.

In this paper the quadratic cost function is considered with the following general form

$$J(k) = \sum_{i=0}^{N} \left[x^T(k+i/k) Q x(k+i/k) + \mu^T(k+i/k) R \mu(k+i/k) \right] \tag{15}$$

Where Q, R are positive definite matrices (Q may be positive semi-definite).

Replacing $x(k+i/k)$ in (15) by its expression given in (12) and collecting terms gives

$$J(k) = v^T(k) H v(k) + 2x^T(k) F^T v(k) + x^T(k) G x(k) \tag{16}$$

where

$$H = C^T \widetilde{Q} C + \widetilde{R} \tag{17a}$$

$$F = C^T \widetilde{Q} M \tag{17b}$$

$$G = M^T \widetilde{Q} M + Q \tag{17c}$$

with

$$\widetilde{Q} = \begin{bmatrix} Q & 0 & \cdots & 0 \\ 0 & \ddots & & \vdots \\ \vdots & & Q & 0 \\ 0 & \cdots & 0 & Q \end{bmatrix} \tag{18a}$$

$$\widetilde{R} = \begin{bmatrix} R & 0 & \cdots & 0 \\ 0 & \ddots & & \vdots \\ \vdots & & R & 0 \\ 0 & \cdots & 0 & R \end{bmatrix} \tag{18b}$$

From (16), one can show that the cost function $J(k)$ depends on the control input vector $v(k)$. The optimal control law can be obtained by minimizing $J(k)$

$$v^*(k) = \arg\min_u J(k) \tag{19}$$

We can obtain the optimized solution $v^*(k)$, by considering the gradient of J with respect to v

$$\nabla_v J = 2Hv + 2Fx \tag{20}$$

Clearly, the minimum point of J will be obtained when $\nabla_v J = 0$. The optimal solution $v^*(k)$ is unique only if H is non-singular, and then is given by

$$v^*(k) = -H^{-1} F x(k) \tag{21}$$

If H is singular then the optimal $v^*(k)$ is non-unique, and a particular solution of $\nabla_v J = 0$ is computed as follows

$$v^*(k) = -H_{Left}^{-1} F x(k) \tag{22}$$

where

$$H_{Left}{}^{-1} = \left(H^T H\right)^{-1} \times H^T \tag{23}$$

is the left inverse of H.

Implementing the first element of the optimal prediction $v^*(k)$ at each sampling instant k defines a receding horizon control law. Since H and F are constant and we have a single-input this is the linear time-invariant feedback controller given by

$$\mu(k) = Kx(k) \tag{24}$$

where

$$K = \begin{cases} -[1 \quad 0 \quad 0 \quad 0] H^{-1} F & \text{if } H \text{ is non-singular} \\ -[1 \quad 0 \quad 0 \quad 0] H_{Left}^{-1} F & \text{if } H \text{ is singular} \end{cases} \tag{25}$$

Recalling that $\mu = \alpha - \alpha_e$, the input signal (which is the duty ratio α) of the averaged model (2) is then obtained

$$\alpha(k) = Kx(k) + \frac{V_d}{E} \tag{26}$$

The main result of this paper is summarized in the following proposition

Proposition 1: Consider the buck power converter model (5) with the state vector $x = z - z_e$ (where z_e is an equilibrium point of (2)), associated with the optimal control law (24),

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then the closed loop system is globally asymptotically stable. It follows that the output voltage vc tracks its reference signal Vd.

Proof: The proof is based on using the optimal cost as a Lyapunov function. Let us denote the cost J(k), and the optimal J*(k). The optimal cost at time k is obtained with the control sequence $\left[\mu^*(k/k) \quad \cdots \quad \mu^*(k+N-1/k)\right]$. The use of a * is a generic notation for variables related to optimal solutions. It follows, using the same notations of ([10], [8]), that

$$J(k+1) \le J^*(k) - x^T(k/k)Qx(k/k) - \mu^{*T}(k/k)R\mu^*(k/k) \tag{27}$$

As Q and R are positive definite matrices, it follows that

$$J(k+1) \le J^*(k) \tag{28}$$

We know that $J(k+1) \ge J^*(k+1)$ (by the definition of the optimal solution), then we have

$$J^*(k+1) \le J^*(k) \tag{29}$$

Which proves that $J^*(k)$ is a decaying sequence, hence $x(k)$ converges to the origin. This clearly shows the asymptotic stability of the closed loop system. The vanishing of $x(k)$ imply the vanishing of $x_2 = z_2 - V_d$ which in turn implies that the output voltage vc converges to its desired value Vd. This ends the proof of the proposition. □

The performances of the proposed MPC controller (26) will now be checked by numerical simulations and experimental results.

IV. SIMULATION AND EXPERIMENTAL RESULTS

A. Optimal gain calculation

The controlled power Buck converter characteristics, considered in this paper, are listed in Table 1. It is worth nothing that these parameters are used both in simulations and experiments.

TABLE I
CONTROLLED SYSTEM PARAMETERS

Parameter	**Value**
Input voltage E	12 V
Inductance L	4mH
Output Capacitor C	220μF
Output load resistor R_L	20Ω
PWM Switching frequency	25kHz
Sampling period T	0.1ms

The matrices A and B of the averaged model (2) are then obtained as follows

$$A = \begin{bmatrix} 0 & -250 \\ 4.54\times10^3 & -227.3 \end{bmatrix};\ B = \begin{bmatrix} 3\times10^3 \\ 0 \end{bmatrix} \tag{30}$$

The matrices Q and R in (15) are chosen as follows

$$Q = I_2 \quad ; \quad R = 0.01 \tag{31}$$

Considering the prediction horizon N=4 the following matrices (given by (17a-b)) are then obtained

$$F = \begin{bmatrix} -3.2984\times10^{25} & -1.6633\times10^{25} \\ -1.1313\times10^{22} & -3.0343\times10^{21} \\ 3.1289\times10^{19} & 1.5244\times10^{19} \\ 3.6979\times10^{15} & -3.7859\times10^{14} \end{bmatrix} \tag{32a}$$

$$H = \begin{bmatrix} 2.1939\times10^{26} & 4.32\times10^{22} & -2.0170\times10^{20} & 2.3244\times10^{15} \\ 4.32\times10^{22} & 2.1227\times10^{19} & -4.226\times10^{16} & -1.022\times10^{13} \\ -2.0170\times10^{20} & -4.226\times10^{16} & 1.859\times10^{14} & 0 \\ 2.3244\times10^{15} & -1.022\times10^{13} & 0 & 9\times10^{6} \end{bmatrix} \tag{32b}$$

The calculation the optimal gain K from (25) gives

$$K = \begin{bmatrix} 0.0144 & 0.0070 \end{bmatrix} \tag{33}$$

B. Simulation results

A simulation was performed using MATLAB software. The simulation diagram of the buck power converter control is shown in Fig.2.

Fig.3 illustrates the output voltage v_c in presence of constant reference signal V_d=7V. This figure clearly shows that the output voltage tracks perfectly its reference. Fig.4 and Fig.5 illustrate, respectively, the inductor current i_L and the duty ratio α .

Fig.6, Fig. 7 and Fig.8 illustrate the behavior of the controlled system in presence of a varying reference signal V_d between 6V and 10V. Fig.6 also shows that the output voltage tracks its reference.

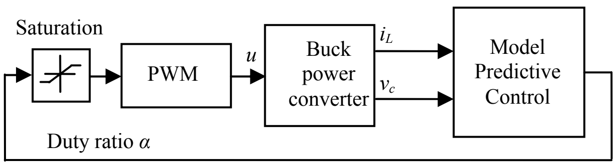

Fig. 2 Block diagram of the buck converter control

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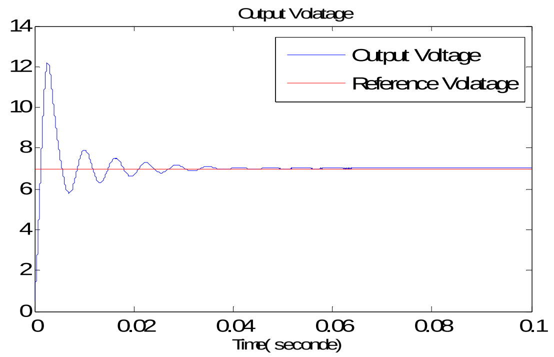

Fig. 3 The output voltage *vc* in presence of a constant reference signal *Vd*=7V.

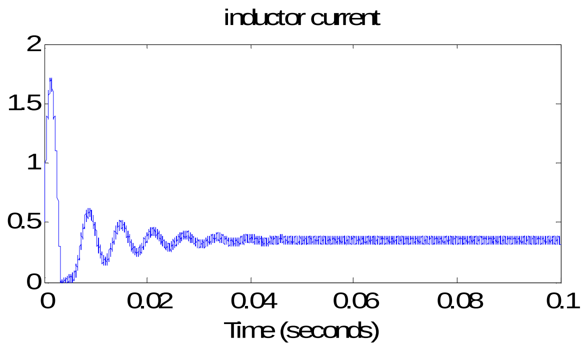

Fig.4 The inductor current *iL* in presence of a constant reference signal *Vd*=7V.

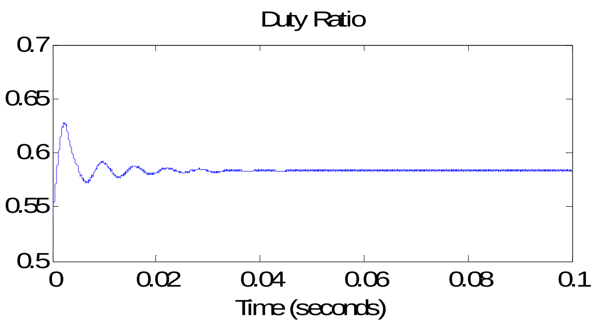

Fig. 5 The Duty ratio α in presence of a constant reference signal *Vd*=7V.

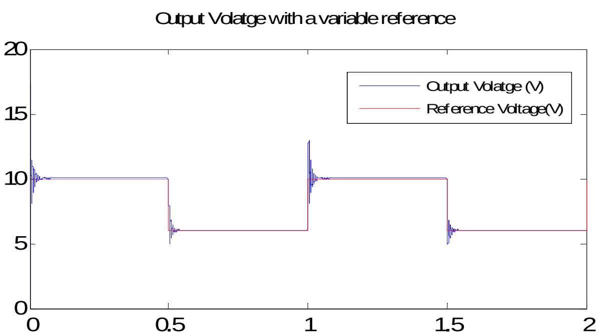

Fig.6: The output voltage *vc* in presence of a varying reference signal *Vd* between 6V and 10V.

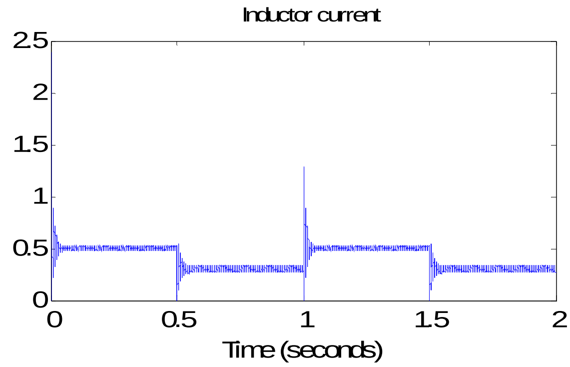

Fig.7 The inductor current *iL* in presence of a varying reference signal *Vd* between 6V and 10V.

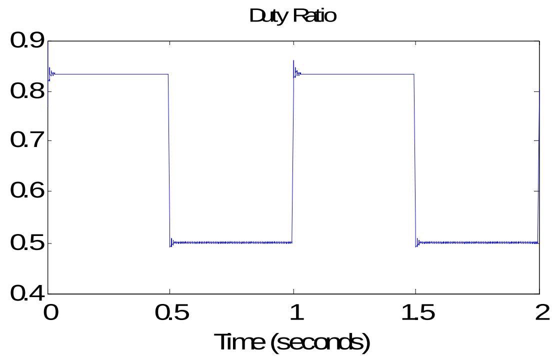

Fig.8 The Duty ratio α in presence in presence of a varying reference signal *Vd* between 6V and 10V.

C. *Experimental results*

To implement the MPC proposed controller, an experimental test bench was developed. It consists essentially of (see Fig. 8):

- a 300W Programmable DC Electronic Load from BK Precision operated under multiple modes such as constant current (CC), constant voltage (CV), constant power (CW), and constant resistance (CR).
- a power supply from BK Precision.
- Hall Effect voltage and current sensors.
- a digital oscilloscope.
- a dc-dc buck power converter.
- A dspace DS1104 with Control Desk software plugged in a Pentium 4 personal computer.

Fig.9 illustrates the output voltage v_c and the inductor current i_L in presence of constant reference signal V_d=7.5V, while Fig. 10 shows the performances in presence of a varying reference between 5V and 8V. Also, the figures show that the output voltage tracks perfectly its reference.

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V. CONCLUSIONS

This paper proposes a Model Predictive Control (MPC) of dc-dc buck power converter. It is shown using theoretical analysis, numerical simulation and experimental results that the proposed MPC approach ensures an asymptotic stability of the closed loop system and a tight regulation of the output voltage to its desired value.

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Fig. 8: View of the experimental test bench

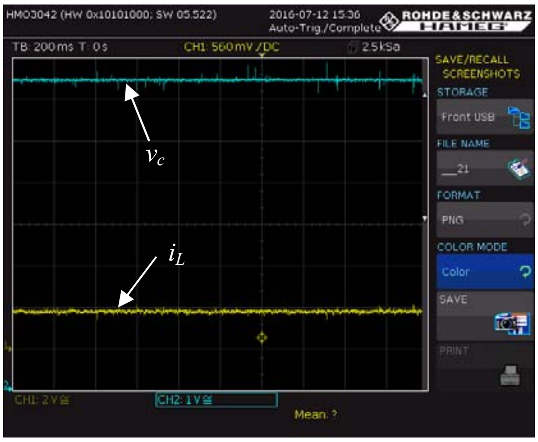

Fig. 9: The output voltage *vc* and the inductor current *iL* in presence of a constant reference signal *Vd*=7.5V

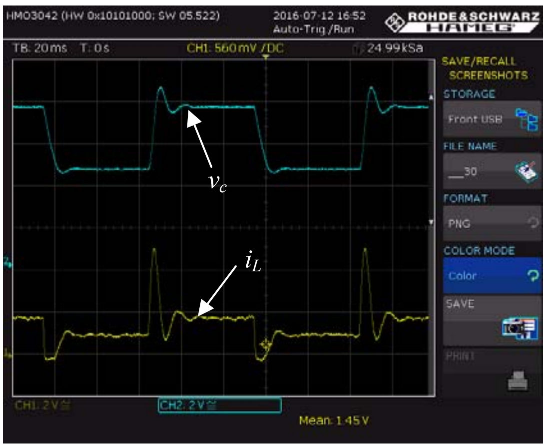

Fig. 10 The output voltage vc and the inductor current iL in presence of a varying reference signal Vd between 5V and 8V

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