Proceedings of Engineering & Technology PET -Vol 98, pp.14-20 Advances in Control Systems and Information Technology

On the Discrete Time Stepwise Safe Switching Soft Sensor Design for Water Distribution Networks

Nikolaos D. Kouvakas#1, Fotis N. Koumboulis#2, Maria P. Tzamtzi#3, Dimitrios G. Fragkoulis#4, Klimis K. Katsiavrias*5, Konstantinos Katsiavrias*6, John Dimitropoulos*7

#*Robotics, Automatic Control and Cyber-Physical Systems Laboratory, Department of Digital Industry Technologies, National and Kapodistrian University of Athens, Euripus Campus, Euboea, Greece*

1,2,3,4{nkouvak, fkoumboulis, mtzamtzi, dfragkoulis}@dind.uoa.gr,

**AKATT S.A.*

251, Agiou Dimitriou str, 173 42, Agios Dimitrios, Athens, Greece

5,6,7{c_katsiavrias, k_katsiavrias, j_dimitropoulos}@akatt.gr

***Abstract*— A discrete-time, model-based soft sensor is developed to estimate free chlorine concentration in selected sections of a water distribution network (WDN). The study considers a benchmark network with a star-like geometry, which is initially described by a set of nonlinear ordinary differential equations. The hydraulic subsystem is approximated in static form, such that steady-state flows is consistent with boundary heads and demands that drive the chlorine transport–reaction dynamics. Around a set of preselected operating points, linear approximants are obtained and the respective exact discrete-time state-space representations are derived under zero-order-hold interpolations. Based on this model, a bank of full-order Luenberger-type discrete-time observers is constructed, constituting the base of the the proposed soft sensor. The observer gains are determined through a multicriterion design procedure that combines discrete-time pole-placement requirements with a norm-based minimization objective. Simulation results on the benchmark network indicate high-fidelity chlorine estimation, with deviations limited to brief intervals during operating transitions.**

***Keywords*— WDNs, Discrete-time soft sensor, Observer bank, Mult-objective tuning, Metaheuristic Optimization**

I. Introduction

Maintaining an adequate residual chlorine concentration throughout a WDN is a primary operational objective, as this residual provides a protective barrier against microbial regrowth and contamination events after treatment (indicatively see [1]-[5] and the references therein). In practice, however, chlorine concentration is strongly location-dependent and time-varying. It is influenced by changing flow patterns, residence time, and reaction mechanisms in the water and at pipe walls. Real time monitoring of chlorine within pipes is rarely feasible due to sensor cost. As a result, utilities typically rely on sparse measurements at a small number of stations, leaving many hydraulically critical pipe sections effectively unobserved.

The problem of sparcity of real time measurements motivates the use of soft sensors that infer quality metrics, such as chlorine concentration at unmeasured locations from routinely available telemetry, such as boundary heads, flow-related signals, and reservoir quality measurements (see [6]-[9] and the references therein). One direction relies on data-driven approaches, such as neural networks and other machine-learning models, to infer variables like flow rates and key water-quality indicators. These methods are typically motivated by the need to improve real-time monitoring and operational decision-making while reducing the number of physical sensors, and several studies report improved accuracy and uncertainty handling compared to baseline predictors. A second direction focuses on observer-based soft sensors, where estimation is derived from state-space modeling and systematic estimator design. Such approaches have been used extensively for hydraulic monitoring and diagnostic tasks, including leakage detection and parameter uncertainty. Implementations frequently combine model-based estimation with recursive filtering concepts, including Extended Kalman Filter-type structures, to fuse limited pressure / flow measurements with network dynamics and deliver consistent real-time state and parameter estimates.

Since supervisory measurements are naturally acquired in discrete time, the estimation problem is posed and solved directly in a discrete-time setting. A discrete-time formulation enables a link between sampling, estimator dynamics, and real-time implementation constraints, while allowing the designer to directly shape convergence and sensitivity in the time scale at which data are actually available. Toward this goal, in the

ISSN: 1737-9334

 10.051103/PET.260597703

Proceedings of Engineering & Technology PET -Vol 98, pp.14-20

present paper, a discrete-time, model-based soft sensor, for estimating free chlorine concentration in selected sections of a WDN, is designed. A benchmark network with a star-like geometry is modeled initially by a set of nonlineal ODEs. Following, the hydraulic subsystem of the network is treated in a static manner, so that steady-state flows consistent with boundary heads and demands, drive the chlorine transport–reaction dynamics. Around preselected operating points, a linear approximant is derived and an exact discrete-time state-space model is obtained, under zero-order-hold assumptions. Using these results, a bank of full-order Luenberger-type discrete-time observers is designed, ultimately forming the basis of a soft sensor. The observer gains are selected through a multicriterion procedure that combines discrete-time pole placement constraints with a norm-based minimization goal. Simultion results on the benchmark network demonstrate high-fidelity chlorine reconstruction, with discrepancies confined to brief transition intervals.

II. A Benchmark WDN Dynamics

A. Nonlinear Model of the Benchmark

In this section, the dynamic model of a benchmark WDN will be developed, following the modelling workflow in [8]. The network has a star-like geometry centred at a single junction (see [8]). Three pipes connect the junction to three reservoirs, while an additional branch departs from the same junction toward an aggregated consumption zone. Here, each conduit is discretized using one spatial section per pipe for the flow dynamics and two spatial sections per pipe for the chlorine dynamics. The flow / chlorine concentration model of the WDN becomes:

$$E\dot{x} = \Gamma(x) + Z(x)u \tag{1}$$

where $x = [x_1 \ \cdots \ x_{10}]^T = [Q_1 \ Q_2 \ Q_3 \ H_n \ c_{1,1} \ c_{1,2} \ c_{2,1} \ c_{2,2} \ c_{3,1} \ c_{3,2}]^T$, $u = [u_1 \ \cdots \ u_5]^T = [H_1^* \ H_2^* \ H_3^* \ Q_d \ c_0]^T$, $\Gamma(x) = [\gamma_j(x)] \in \mathbb{R}^{10\times1}$, $Z(x) = [z_{i,j}(x)] \in \mathbb{R}^{10\times5}$ and their non-zero elements are

$$\gamma_1(x) = -A_1 g x_4 / L_1 - x_1^2 f(x_1, D_1, \varepsilon_1)/(2A_1D_1),\ \gamma_2(x) = A_2 g x_4 / L_2 - x_2^2 f(x_2, D_2, \varepsilon_2)/(2A_2D_2),$$
$$\gamma_3(x) = A_3 g x_4 / L_3 - x_3^2 f(x_3, D_3, \varepsilon_3)/(2A_3D_3),\ \gamma_4(x) = b^2(x_1 - x_2 - x_3)/(A_1 g L_1),$$
$$\gamma_5(x) = -\left[k_1 + 4n_{cl}x_1/(D_1^2L_1\pi)\right]x_5,\ \gamma_6(x) = 4n_{cl}x_1(x_5 - x_6)/(D_1^2L_1\pi) - k_1x_6,\ \gamma_7(x) = 4n_{cl}x_2(x_6 - x_7)/(D_2^2L_2\pi) - k_1x_7,\ \gamma_8(x) = 4n_{cl}x_2(x_7 - x_8)/(D_2^2L_2\pi) - k_1x_8,\ \gamma_9(x) = 4n_{cl}x_3(x_6 - x_9)/D_3^2L_3\pi - k_1x_9,$$
$$\gamma_{10}(x) = 4n_{cl}x_3(x_9 - x_{10})/(D_3^2L_3\pi) - k_1x_{10},\ z_{1,1}(x) = A_1g/L_1,\ z_{2,2}(x) = -A_2g/L_2,\ z_{3,3}(x) = -A_3g/L_3,$$
$$z_{4,4}(x) = -b^2/(A_1gL_1),\ z_{5,5}(x) = 12x_1/(D_1^2L_1\pi),$$
$$f(Q, D, \varepsilon) = \lambda_1\left[\varepsilon/D + D\pi\lambda_2\mu/(4Q\rho)\right]^{1/4}.$$

Note that H_i^* ($i \in \{1,2,3\}$) is boundary (actuatable) pressure heads imposed by the reservoir i, Q_i is the volumetric flow rate in conduit i, Q_d is the aggregated consumption flow rate, $c_{i,j}$ is the chlorine concentration at section j of conduit i, c_0 is the influent chlorine concentration to the network from reservoir 1, L_i and D_i are the length and inner diameter of conduit i, $A_i = \pi D_i^2/4$, ε_i is the absolute roughness of conduit i, g is the magnitude of the gravitational acceleration vector, b is the pressure wave speed, ρ and μ are the density and dynamic viscosity of the water, n_{cl} is the number of space discretization sections ($n_{cl} = 2$), k_1 is a first order chlorine reaction rate, while λ_1 and λ_2 are positive pipe friction model parameters.

In general, E is equal to the 10×10 identity matrix I_{10}. The hydraulic subsystem can be considered to be static, resulting in a quasi-static flow – chlorine model. This is a valid simplification for the present soft-sensing objective, since chlorine concentration in selected sections typically evolves on time scales that are dominated by transport and reaction processes and by slowly varying operational conditions. For the quasi-static case, this matrix takes on the block diagonal form $E = \mathrm{diag}\{0_{4\times4}, I_6\}$.

ISSN: 1737-9334

B. Continuous and Discrete Time Linear Approximant of the Quasi-Static Nonlinear Model of the WDN

Let $\overline{u}_j$ ($j=1,\ldots,5$) be the nominal values of the inputs of the system and $\overline{x}_i$ ($i=1,\ldots,10$) be the corresponding nominal values of the state variables. Furthermore, let $\overline{u}$ and $\overline{x}$ be the vectors of nominal values of the inputs and the state variables. The operating vector of the system's dynamics is denoted as the pair $\overline{o}=(\overline{u},\overline{x})$, where its elements satisfy the equality $\Theta(\overline{x},\overline{u})=\Gamma(\overline{x})+Z(\overline{x})\overline{u}=0_{10\times1}$. Assuming constant flow directions and applying series of computations it can be verified that

$$\overline{u}_1=\overline{x}_4+L_1\overline{x}_1^2 f(\overline{x}_1,D_1,\varepsilon_1)/\left(2A_1^2D_1g\right),\ \overline{u}_2=\overline{x}_4-L_2\overline{x}_2^2 f(\overline{x}_2,D_2,\varepsilon_2)/\left(2A_2^2D_2g\right),\ \overline{u}_3=\overline{x}_4-L_3\overline{x}_3^2 f(\overline{x}_3,D_3,\varepsilon_3)/\left(2A_3^2D_3g\right),\ \overline{u}_4=\overline{x}_1-\overline{x}_2-\overline{x}_3,\ \overline{x}_5=4n_{cl}\overline{u}_5\overline{x}_1/\left(D_1^2k_1L_1\pi+4n_{cl}\overline{x}_1\right),\ \overline{x}_6=4n_{cl}\overline{x}_1\overline{x}_5/\left(D_1^2k_1L_1\pi+4n_{cl}\overline{x}_1\right),\ \overline{x}_7=4n_{cl}\overline{x}_2\overline{x}_6/\left(D_2^2k_1L_2\pi+4n_{cl}\overline{x}_2\right),\ \overline{x}_8=4n_{cl}\overline{x}_2\overline{x}_7/\left(D_2^2k_1L_2\pi+4n_{cl}\overline{x}_2\right),\ \overline{x}_9=4n_{cl}\overline{x}_3\overline{x}_6/\left(D_3^2k_1L_3\pi+4n_{cl}\overline{x}_3\right),\ \overline{x}_{10}=4n_{cl}\overline{x}_3\overline{x}_9/\left(D_3^2k_1L_3\pi+4n_{cl}\overline{x}_3\right).$$

Note that the nonlinear algebraic system of equations is proven to be solvable with respect to $\overline{x}$ (see [8]). Applying series of computations, it can be verified that the linear approximant of the nonlinear model (1) is of the form

$$E\delta\dot{x}=J_s(\overline{x},\overline{u})\delta x+Z(\overline{x})\delta u \tag{2}$$

where δx is the response of the linear approximant (2) for $\delta u=\Delta u=u-\overline{u}$, that approximates $\Delta x=x-\overline{x}$ around the operating point $\overline{o}=(\overline{u},\overline{x})$, and where $J_s(\overline{x},\overline{u})=\partial\Theta(x,u)/\partial x\big|_{u=\overline{u},x=\overline{x}}$. Since the nonlinear system for the determination of the nominal values of the state variables is solvable with respect to $\overline{x}$, there exist a nonlinear vector function, mapping the nominal values of the inputs to the nominal values of the states, i.e., $\overline{x}=\sigma(\overline{u})$. Hence, the linear approximant system matrices can be expressed as $A(\overline{u})=J_s(\sigma(\overline{u}),\overline{u})$, $B(\overline{u})=Z(\sigma(\overline{u}))$ and the linear approximant becomes

$$E\delta\dot{x}=A(\overline{u})\delta x+B(\overline{u})\delta u \tag{3}$$

Define $\delta z=\begin{bmatrix}\delta z_1 & \cdots & \delta z_4\end{bmatrix}^T=\begin{bmatrix}\delta x_1 & \cdots & \delta x_4\end{bmatrix}^T$ and $\delta q=\begin{bmatrix}\delta q_1 & \cdots & \delta q_6\end{bmatrix}^T=\begin{bmatrix}\delta x_5 & \cdots & \delta x_{10}\end{bmatrix}^T$. Applying series of computations upon (3), it can be observed that the linear approximant can be rewritten as

$$A_{1,1}(\overline{u})\delta z+B_1(\overline{u})\delta u=0_{4\times1},\ \delta\dot{q}=A_{2,1}(\overline{u})\delta z+A_{2,2}(\overline{u})\delta q+B_2(\overline{u})\delta u \tag{4}$$

Applying series of computations upon (4), it can be verified that the flow variables can be eliminated from the chlorine concentration dynamics to yield

$$\delta\dot{q}=A_q(\overline{u})\delta q+B_q(\overline{u})\delta u \tag{5}$$

where $A_q(\overline{u})=A_{2,2}(\overline{u})$ and $B_q(\overline{u})=B_2(\overline{u})-A_{2,1}(\overline{u})A_{1,1}(\overline{u})^{-1}B_1(\overline{u})$. Under zero-order-hold assumptions, i.e. if the input vector $\delta u(t)$ is piecewise constant over each sampling interval, the exact discrete time model at sampling instants $t=kT_s$, where $k\in\mathbb{N}_0$ and $T_s\in\mathbb{R}^+$, takes on the form

$$\delta q(k+1)=A_d(\overline{u})\delta q(k)+B_d(\overline{u})\delta u(k) \tag{6}$$

where $A_d(\overline{u})=\exp\left(A_q(\overline{u})T_s\right)$ and $B_d(\overline{u})=\int_0^{T_s}\exp\left(A_q(\overline{u})\tau\right)B_q(\overline{u})d\tau$. The measurable variables are only the chlorine concentrations at the exit of conduits 2 and 3, i.e. δq_4 and δq_6. Thus, the measurable output vector is $\delta y(k)=C_d\delta q(k)$, where non-zero elements of $C_d=\left[(c_d)_{i,j}\right]\in\mathbb{R}^{2\times6}$ are $(c_d)_{1,4}=(c_d)_{2,6}=1$.

III. Discrete Time Soft Sensor Design Toward Chlorine Concentration Estimation

In the present section, a soft sensor, toward chlorine concentration estimation, will be designed. The soft sensor will be based on discrete time full order Luenberger type of observers, designed on preselected sets of operating points of the system, of the form

ISSN: 1737-9334

$$\Im : \delta\hat{q}(k+1) = \left(A_d(\bar{u}) - L(\bar{u})C_d\right)\delta\hat{q}(k) + B_d(\bar{u})\delta u(k) + L(\bar{u})\delta y(k) \ ; \ \delta\hat{q}(0-) = \delta\hat{q}_0 \quad (7)$$

where $L(\bar{u}) \in \mathbb{R}^{6\times 2}$ is an appropriate observer matrix, to be selected by the designer. The elements of L are to be selected by the designer such that the eigenvalues of $A_d(\bar{u}) - L(\bar{u})C_d$ are appropriately adjusted. It can readily be verified that the discrete time linear approximant (6) is observable, hence arbitrary eigenvalue assignment of $A_d(\bar{u}) - L(\bar{u})C_d$ can be achieved by appropriately selecting the elements of L. Let

$$p_0(z) = \det\left(zI_6 - A_d(\bar{u}) + L(\bar{u})C_d\right), \ p_s(z) = \det\left(zI_6 - A_d(\bar{u})\right) \quad (8)$$

Let $\pi_{o,i}$ ($i = 1,\ldots,6$) are the roots of $p_0(z)$ and $\pi_{s,i}$ ($i = 1,\ldots,6$) are the roots of $p_s(z)$. Without loss of generality, assume that $|\pi_{o,i}| \le |\pi_{o,i+1}|$ and $|\pi_{s,i}| \le |\pi_{s,i+1}|$ ($i = 1,\ldots,5$). In what follows, the observer polynomial roots are restricted to satisfy the following constraints: a) the roots of $p_o(z)$ are real, nonnegative and stable, $\pi_{o,i} \in [0,1)$, b) the roots of $p_o(z)$ are ordered and have a minimum distance between them being equal to $\gamma \in \mathbb{R}^+$ i.e. it holds that $\pi_{o,i+1} > \gamma + \pi_{o,i}$, c) regional stability is achieved i.e. it holds that $\pi_{o,i} < |\pi_{s,i}|$ and d) the H_2 norm of the resolvent matrix of the error dynamics is minimized. Regarding the last design goal, minimization of the H_2 norm of the resolvent matrix of the error dynamics corresponds to minimization of the RMS energy gain from the uncertainty in the initial estimation error to the error transient. Applying series of computations upon the first polynomial in (8), in can be verified that the design requirement of pole placement can be achieved by selecting the elements of the first column of L. Once the poles are fixed, the elements of the second column of L can be used to satisfy the norm-based design requirement. The above design requirements lead to a multicriteria optimization problem which is generally nonlinear and nonconvex. For this reason, a numerical optimization approach is required. In the present paper, a metaheuristic approach being analogous to that in [10]-[13].

It is important to mention, once more, that the above presented design procedure, produces observer parameters corresponding to a given operating point. The next step is to determine the operating area of the derived observer, namely the area where the observer behaves satisfactorily. To do so, the five-dimensional spheroid approach, introduced in [8], is proposed. Defining the spheroid $\sum_{j=1}^{5}\left[\left(\tilde{u}_j - \bar{u}_j\right)/\bar{u}_j\right]^2 = R^2$, where $\tilde{u}_j$ denotes the steady state value of u_j ($j = 1,\ldots,5$) during a step wise transition, the operating area is determined as the maximum radius of the spheroid such that all input transitions from the operating point to a new steady state value inside the spheroid, result in steady state estimation error smaller than $\varepsilon_{ss,\max}$, where $\varepsilon_{ss,\max}$ is a positive parameter selected by the observer designer. In the 5-dimensional space, if the intersection of two target operating areas, corresponding to different operating points, is not the null set, then transitions from any point within a given target operating area to any point in the intersection can be carried out accurately using the observer derived for that target operating area. Moreover, when moving from a point in the first target operating area outside the intersection to a point in the second target operating area that is also outside the intersection, an accurate transition can be realized in two stages. First, the system is driven into the intersection while the observer associated with the first area remains active. Then, once the intersection is reached, the observer is switched to the one corresponding to the second area. A switching mechanism being analogous to that in [8] may be employed.

IV. Simulation Results

To demonstrate the performance of the proposed switching observer scheme, let $L_1 = 62[\text{m}]$, $L_2 = 124[\text{m}]$, $L_3 = 80[\text{m}]$, $D_1 = 0.1[\text{m}]$, $D_2 = 0.1[\text{m}]$, $D_3 = 0.06[\text{m}]$, $\varepsilon_j = 0.35[\text{mm}]$ ($j = 1,2,3$), $g = 9.81[\text{m/s}^2]$, $\rho = 999.1[\text{Kg/m}^3]$, $\mu = 0.0011[\text{Pa s}]$, $k_1 = 0.1[\text{h}^{-1}]$, $b = 1200[\text{m/s}]$, $\lambda_1 = 0.11[-]$ and $\lambda_2 = 68[-]$, $\varepsilon_{ss,\max} = 5[\%]$. Regarding the sampling period, it will be assumed that it is $T_s = 1[\text{min}]$ which is a valid

ISSN: 1737-9334

Proceedings of Engineering & Technology PET -Vol 98, pp.14-20 **Advances in Control Systems and Information Technology**

assumption for typical primary water distribution networks. For demonstration purposes, assume that it is desirable to move from an initial point, let $u_1 = 20.0673[\text{m}]$, $u_2 = 16.9749[\text{m}]$, $u_3 = 14.0886[\text{m}]$, $u_4 = 360[\text{l/min}]$ and $u_5 = 2.0092[\text{mg/lt}]$, to a final point, let $u_1 = 30.3638[\text{m}]$, $u_2 = 14.842[\text{m}]$, $u_3 = 16.1169[\text{m}]$, $u_4 = 0.0049[\text{m}^3/\text{s}]$, $u_5 = 1.4955[\text{mg/lt}]$. To achieve this transition, consider the sets of operating points:

i. $\overline{u}_1 = 20[\text{m}], \overline{u}_2 = 17[\text{m}], \overline{u}_3 = 14[\text{m}], \overline{u}_4 = 360[\text{l/min}], \overline{u}_5 = 2[\text{mg/l}]$.

ii. $\overline{u}_1 = 24,7368[\text{m}], \overline{u}_2 = 16.0526[\text{m}], \overline{u}_3 = 14.9474[\text{m}], \overline{u}_4 = 331.5789[\text{l/min}], \overline{u}_5\ 1.7632[\text{mg/l}]$.

iii. $\overline{u}_1 = 30[\text{m}], \overline{u}_2 = 15[\text{m}], \overline{u}_3 = 16[\text{m}], \overline{u}_4 = 300[\text{l/min}], \overline{u}_5 = 1.5[\text{mg/l}]$.

Following the observer design procedure presented in the previous section using $\gamma = 10^{-5}$, for the above operating points the observer poles, the second column of L and the target operating radii are evaluated to be:

i. $R = 0.0368$, $\pi_1 = 0$, $\pi_2 = 0.0008$, $\pi_3 = 0.001$, $\pi_4 = 0.0908$, $\pi_5 = 0.0974$, $\pi_6 = 0.5459$,
$l_2 = [0.002 \quad -0.0009 \quad 0.0235 \quad 0.008 \quad 0.0135 \quad 0.1817]^T$

ii. $R = 0.238$, $\pi_1 = 0.0001$, $\pi_2 = 0.0011$, $\pi_3 = 0.0015$, $\pi_4 = 0.0176$, $\pi_5 = 0.0999$, $\pi_6 = 0.2386$
$l_2 = [0.0022 \quad -0.0008 \quad 0.0077 \quad 0.0086 \quad 0.0073 \quad 0.1359]^T$

iii. $R = 0.2831$, $\pi_1 = 0.0001$, $\pi_2 = 0.00121$, $\pi_3 = 0.00127$, $\pi_4 = 0.0077$, $\pi_5 = 0.0757$, $\pi_6 = 0.1398$
$l_2 = [0.0023 \quad -0.0009 \quad 0.0027 \quad 0.0046 \quad 0.0037 \quad 0.00971]^T$

Applying series of computations it can be verified that a) the initial point belongs in area 1 and not in areas 2 and 3, b) the final point belongs in area 3 and not in areas 1 and 2, c) the intersections of areas 1 and 2 as well as the areas 2 and 3 are not null, and d) the intersection of areas 1 and 3 are null. According to the procedure described in the previous section, transition from the initial to the final point will take place through two intermediate points, one belonging in the intersection of areas 1 and 2, and one belonging in the intersection of areas 2 and 3. Let these points be a) $u_1 = 20.4734[\text{m}]$, $u_2 = 16.9749[\text{m}]$, $u_3 = 14.0886[\text{m}]$, $u_4 = 354[\text{l/min}]$, $u_5 = 1.9513[\text{mg/lt}]$, and b) $u_1 = 26.2998[\text{m}]$, $u_2 = 15.4709[\text{m}]$, $u_3 = 15.4866[\text{m}]$, $u_4 = 318[\text{l/min}]$, $u_5 = 1.6482[\text{mg/lt}]$. Without loss of generality, it will be assumed that initially estimation and real values coincide. Finally, to better capture the dynamic characteristics associated with variations in the flow-related inputs, they will be modelled to change not as an ideal step, but via a smooth step-type transition with an approximate rise time of 1.6 seconds. Comparing estimations vs real values of the non-measurable chlorine concentrations, derived from the fully dynamic (not quasi-static) nonlinear model of the system (1), it can be verified that the observer presents very high fidelity in reconstructing the nonlinear chlorine concentration. For both sections of conduit 1 as well as the first section of conduit 3, the estimated points are essentially superimposed on the nonlinear reference over the full simulation horizon. The only noticeable deviations occur during the major downward transition around $t = 9[\text{min}]$ to $t = 11[\text{min}]$ for the first section of conduit 2 where the estimate becomes slightly higher than the nonlinear value for about three samples, consistent with a brief lag / conservative overestimation during rapid concentration decrease. The maximum estimation errors for non-measurable chlorine concentrations are 0.0273%, 0.1666%, 1.0406% and 0.2593%. It is important to mention that attempting a single step transition from the initial to the final point using the observer of the first set of operating point, the resulting estimation error is significantly higher, being 1.1455%, 4.7482%, 15.2898% and 8.0988%. The transition from the initial to the final point takes place in 17 samples / minutes.

V. CONCLUSIONS

In this work, a discrete-time, model-based soft sensor was developed for estimating free chlorine concentration in selected sections of a water distribution network. A benchmark network with a star-like geometry was represented by a set of nonlinear ordinary differential equations, and the hydraulic subsystem was subsequently approximated in a static form so that steady-state flows, consistent with boundary heads and demand withdrawals, acting as driving signals for the chlorine transport–reaction dynamics. Around

ISSN: 1737-9334

preselected operating points, a linear approximant was derived, and an exact discrete-time state-space representation was obtained under zero-order-hold assumptions. Based on this formulation, a bank of full-order Luenberger-type discrete-time observers was designed to provide the final soft-sensing scheme. The observer gain was determined through a multicriteria design procedure that combined discrete-time pole-placement constraints with a norm-based minimization objective, enabling convergence shaping while reducing amplification of estimation transients. Simulation results on the benchmark network indicated high-fidelity chlorine reconstruction, with any mismatches largely limited to brief intervals during operating transitions. Future work will focus on improving robustness and extending applicability under practical sources of uncertainty. In particular, the impact of demand variability, parameter uncertainty, and model mismatch will be addressed through robust and/or stochastic tuning of the observer gains. The observer-bank and switching strategy will also be refined to provide more systematic performance guarantees under noisy measurements.

Acknowledgment

This work was conducted within “SUB1.1 Clusters of Research Excellence (CREs)” Action of the NRRP “Greece 2.0”, co-funded by Greece and EU – NextGenerationEU, under the project YΠ3TA-0560428-DeSw.

References

[1] World Health Organization, *Guidelines for Drinking-water Quality: Fourth Edition Incorporating the First Addendum*. Geneva, Switzerland: World Health Organization, 2017.

[2] World Health Organization, *Principles and Practices of Drinking-Water Chlorination: A Guide to Strengthening Chlorination Practices in Small- to Medium-Sized Water Supplies*. New Delhi, India: World Health Organization, Regional Office for South-East Asia, 2017.

[3] Centers for Disease Control and Prevention, *About Water Disinfection with Chlorine and Chloramine*. Feb. 14, 2024.

[4] R. A. Li, J. A. McDonald, A. Sathasivan, and S. J. Khan, “Disinfectant residual stability leading to disinfectant decay and by-product formation in drinking water distribution systems: A systematic review,” *Water Research*, vol. 153, pp. 335–348, Apr. 2019.

[5] M. E. Bledsoe, A. Goel, M. Adelgren, T. M. LaPara, and R. M. Hozalski, “Effect of chloramine disinfection of community water system on Legionnaires’ disease outbreak, Minnesota, USA, 2024,” *Emerging Infectious Diseases*, vol. 32, no. 1, pp. 74–81, Jan. 2026.

[6] M. Djerioui, M. Bouamar, M. Ladjal, and A. Zerguine, “Chlorine soft sensor based on extreme learning machine for water quality monitoring,” Arabian J. Sci. Eng., vol. 44, no. 3, pp. 2033–2044, 2019.

[7] M. A. Ogundero, T. A. Fashanu, F. O. Agunbiade, K. Orolu, A. A. Yinusa, U. A. Daudu, and M. O. H. Amuda, “A soft sensor-based inference engine for water quality assessment and prediction,” *Air, Soil and Water Research*, vol. 18, no. 1, pp. 1–12, 2025, doi: 10.1177/11786221251315618.

[8] N. D. Kouvakas, F. N. Koumboulis, M. P. Tzamtzi, and D. G. Fragkoulis, “Toward the design of chlorine soft sensors via stepwise safe switching observers for a primary water distribution network,” in *Artificial Intelligence Applications in Engineering and Medicine: Proceedings of the 2nd Olympiad in Engineering Science*, D. Pavlou, H. Adeli, and J. Correia, Eds., Structural Integrity, vol. 36. Cham, Switzerland: Springer, 2026.

[9] N. D. Kouvakas, F. N. Koumboulis, M. P. Tzamtzi and D. G. Fragkoulis, “Soft Sensor Design for Water Distribution Networks using Unknown Input Safe Switching Observers,” *3rd International Conference on Frontiers of AI, Ethics, and Multidisciplinary Applications (FAIEMA)*, Stavanger, Norway, 2025

[10] N. D. Kouvakas, F. N. Koumboulis, D. G. Fragkoulis, and G. F. Fragulis, “Metaheuristic procedures for the determination of a bank of switching observers toward soft sensor design with application to an alcoholic fermentation process,” *Mathematics*, vol. 11, no. 23, Art. no. 4733, 2023.

[11] F. N. Koumboulis, D. G. Fragkoulis, N. D. Kouvakas, and A. Feidopiasti, “Soft sensor design via switching observers,” *Sensors*, vol. 23, no. 4, Art. no. 2114, 2023.

[12] T. C. Drosou, N. D. Kouvakas, F. N. Koumboulis, and M. P. Tzamtzi, “A mixed analytic/metaheuristic dual stage control scheme toward I/O decoupling for a differential drive mobile robot,” in *Frontiers of Artificial Intelligence, Ethics, and Multidisciplinary Applications: 1st International Conference on Frontiers of AI, Ethics, and Multidisciplinary Applications (FAIEMA)*, Greece, 2023, M. Farmanbar, M. Tzamtzi, A. K. Verma, and A. Chakravorty, Eds. Singapore: Springer Nature Singapore, 2024, pp. 197–214.

[13] G. L. Giannaris, N. D. Kouvakas, F. N. Koumboulis, and D. Vouyioukas, “Switching wireless control for longitudinal quadrotor maneuvers,” *J. Intell. Robot. Syst.*, vol. 102, no. 2, 2021

ISSN: 1737-9334