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Modelling of Circular Halbach Array for an Electrodynamic Levitation of a Microgravity Platform

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Abstract— **Halbach arrays are specialized magnetic structures in which the magnetization directions of individual magnets are arranged to steer the magnetic flux into a preferred direction. The resulting magnetic field interacts dynamically with a conductive plate, generating a magnetic force. This configuration is widely employed in applications such as contactless power transfer and thrust generation. This configuration is used for levitating the microgravity (near zero gravity) platform. This study presents an analytic force model for circular Halbach array that designed for micro-gravity platform for ground-based emulations to be used in control algorithm designs. The resulting model captures the electrodynamic response of the system by accounting the design parameters and magnetic behaviours. The model is validated through comparison with finite element method (FEM) analysis results obtained using Ansys Maxwell, demonstrating both its accuracy and its suitability for control-algorithm design around the designated equilibrium condition.**

Keywords— **Magnetic levitation, Halbach array, Analytical modelling, Electrodynamic levitation, Micro-gravity platform**

I. Introduction

Halbach arrays are specialized magnetic structures in which the magnetization directions of individual magnets are arranged to steer the magnetic flux into a preferred direction [1]. The resulting magnetic field interacts dynamically with a conductive plate, generating a magnetic force. This configuration is widely employed in applications such as contactless power transfer [2] and thrust generation [3]. This configuration is used for levitating the microgravity (near zero gravity) platform.

This study develops an analytical model for the levitation force produced by circular Halbach discs designed for a micro-gravity platform. The model is validated through comparison with finite element method (FEM) analysis results obtained using Ansys Maxwell, demonstrating both its accuracy and its suitability for control-algorithm design around the designated equilibrium condition.

II. Method

In this section, the modelling of the vertical motion of the levitation disk on a copper plate has been studied. The magnetization current model was used to represent the behaviour of the permanent magnet. The vertical movement of the Halbach disk induces eddy currents [4]. Due to the surface effect observed in the eddy currents, a conductor plate with a thickness greater than the skin depth can be treated as an infinitely thick conductor plate. This allows the calculation of the electrodynamic force by modelling the permanent magnet Halbach disk moving on an infinitely thick conductor plate [5].

Wavelenght of the Halbach array "λ" and "ν" is the moving velocity on z axis. Frekans frequency of induced eddy current,

$$\omega = k\nu \tag{1}$$

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k represented as,

$$k = \frac{2\pi}{\lambda} \tag{2}$$

When permability (μ) and conductivity (σ) taken into account the skin depth is calculated as follows,

$$\delta = \sqrt{\frac{2}{\mu\sigma\omega}} \tag{3}$$

The magnetic vector potential is denoted as A_i and this potential only has a z-component (Fig. 1).

$$A_y^i(z,x) = Im\{\dot{A}_y^i(z)e^{j(kx-\omega t)}\} \tag{4}$$

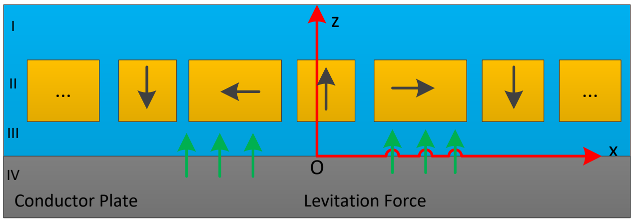

Fig. 1 Solution domain division for the whole space

For an ideal Halbach array, the magnetization currents are distributed only on the surfaces. At the same time, there are no magnetization currents inside of the Halbach array. On the upper and lower surfaces of the ideal Halbach array, the magnetization currents are defined as follows:

$$J_y(z,x) = Im\{\dot{J}_y(z)e^{j(kx-wt)}\} \tag{5}$$

$$\dot{J}_y(z+d) = \frac{B_r}{\mu_0} \tag{6}$$

and,

$$\dot{J}_y(z) = -\frac{B_r}{\mu_0} \tag{7}$$

In this case, Halbach array can be considered as the combination of two magnetization current boundaries, separated by air.

Since there are no free currents (currents bound to the conducting medium) in both Halbach array and air gap, this leads to the following result:

$$\frac{\partial^2 A_y^{II}(z,x)}{\partial z^2} + \frac{\partial^2 A_y^{II}(z,x)}{\partial x^2} = 0 \tag{8}$$

and,

$$\frac{\partial^2 A_y^{III}(z,x)}{\partial z^2} + \frac{\partial^2 A_y^{III}(z,x)}{\partial x^2} = 0 \tag{9}$$

For conductor plate, eddy currents are induced in this region, and the following expression holds,

$$\frac{\partial^2 A_y^{IV}(z,x)}{\partial z^2} + \frac{\partial^2 A_y^{IV}(z,x)}{\partial x^2} = \mu\sigma\frac{\partial A_y^{IV}(z,x)}{\partial t} \tag{10}$$

Note that the permeability of air, μ_0, is equal to the μ of the conductor plate. It is assumed that the magnetic vector potential takes the following form in each region,

$$\begin{cases} \dot{A}_y^{I}(z) = C_0e^{-kz} \\ \dot{A}_y^{II}(z) = C_1e^{kz} + C_2e^{-kz} \\ \dot{A}_y^{III}(z) = C_3e^{kz} + C_4e^{-kz} \\ \dot{A}_y^{IV}(z) = C_5e^{qz} \end{cases} \tag{11}$$

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Where q,

$$q = \sqrt{\frac{\left(k^2 + \sqrt{k^4 + \mu^2\sigma^2\omega^2}\right)}{2}} - \frac{\mu\sigma\omega}{\sqrt{2\left(k^2 + \sqrt{k^4 + \mu^2\sigma^2\omega^2}\right)}} j \quad 12$$

$C_0 \sim C_5$ are constants. Equation (11) can be verified to satisfy the boundary conditions in equations (8), (9) and (10) individually. The magnetic flux density inside the conductor plate is as follows:

$$B_x^{IV}(z) = -\frac{\partial A_y^{IV}(z)}{\partial x} = -qA_y^{IV}(z) \quad 13$$

The induced electric field intensity (electric field density) inside the conductor plate is given by:

$$E_x^{IV}(z) = -\frac{\partial A_y^{IV}(z)}{\partial t} = j\omega A_y^{IV}(z) \quad 14$$

The volumetric lifting force density inside the conductor plate can be expressed as:

$$f = -\sigma E_x^{IV} B_x^{IV} = -\sigma j\omega A^{IV} q A^{IV} \quad 15$$

The average volumetric lifting force density over one period is expressed as follows:

$$\bar{f} = -\frac{1}{2}\sigma Re\left[j\omega A_y^{IV}(z)\left(qA_y^{IV}(z)\right)^*\right] = -\frac{1}{2}\omega\sigma b|C_5|^2 e^{2az} \quad 16$$

Assuming that the width of the Halbach array in the y-direction is w, the lifting force of a Halbach array with a wavelength N can be expressed as follows:

$$F_{ideal}(z) = -N\lambda\omega \int_{-\infty}^{0} \bar{f} dz$$

$$= \frac{N\lambda\mu\omega\sigma^2 B_r^2 \left(1 - e^{-kd}\right)^2}{4k^2} \frac{v^2}{\left(\sqrt{1 + \frac{\mu^2\sigma^2 v^2}{k^2}} + 1\right)^{\frac{3}{2}} \left(\left(\sqrt{1 + \frac{\mu^2\sigma^2 v^2}{k^2}} + 1\right)^{\frac{1}{2}} + \sqrt{2}\right)} e^{-2kz} \quad 17$$

Equation (17) shows that the lifting force consists of three components. The first component depends on the structure and the materials used, and it is independent of the velocity. The second component is velocity-dependent and is controlled by the drive system. The third component indicates that there is a distinct exponential relationship between the lifting force and the levitation gap (the air gap).

If the condition $v \gg \frac{k}{(\mu\sigma)}$ is satisfied, equation (18) can be simplified as follows:

$$F_{ideal}(z) = \frac{N\lambda\omega B_r^2 \left(1 - e^{-kd}\right)^2}{4\mu} e^{-2kz} \quad 18$$

For a non-ideal Halbach array, edge (end) effects are neglected as an assumption, the distribution of magnetization currents exhibits different characteristics in different regions of the structure. Specifically, within the interior of the Halbach array, the magnetization currents are distributed in a discontinuously along the x-axis due to the segmented nature of the permanent magnet blocks. In contrast, on the upper and lower surfaces of the array, these currents become to have a continuous distribution as a result of the boundary conditions forced by the magnetic field configuration.

Accordingly, when a Fourier series expansion is applied to represent the magnetization currents, it becomes acceptable to neglect the contribution of the discontinuously distributed currents within the interior region of the Halbach array, as their influence on the overall magnetic field is comparatively limited. The majority of magnetic field intensity is generated at the surface.

Assuming that the Halbach array is composed of Mpermanent magnet blocks within a single period, and considering the symmetry of the structure, the magnetization current distributed along the upper surface over half of this period can therefore be expressed as follows:

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$$J_y(z+d,x) = \begin{cases} 0, x \epsilon \left(0, \frac{\lambda}{2M}\right] \\ \frac{B_r}{\mu_0} sin\left(\frac{2\pi}{M}\right), x \epsilon \left(\frac{\lambda}{2M}, \frac{\lambda}{2M}+\frac{\lambda}{M}\right] \\ \frac{B_r}{\mu_0} sin\left(2\frac{2\pi}{M}\right), x \epsilon \left(\frac{\lambda}{2M}+\frac{\lambda}{M}, \frac{\lambda}{2M}+2\frac{\lambda}{M}\right] \\ \vdots \\ \frac{B_r}{\mu_0} sin\left((p+1)\frac{2\pi}{M}\right), x \epsilon \left(\frac{\lambda}{2M}+p\frac{\lambda}{M}, \frac{\lambda}{2M}+(p+1)\frac{\lambda}{M}\right] \\ \vdots \\ \frac{B_r}{\mu_0} sin\left(\left(\frac{M}{2}-1\right)\frac{2\pi}{M}\right), x \epsilon \left(\frac{\lambda}{2M}+\left(\frac{M}{2}-2\right)\frac{\lambda}{M}, \frac{\lambda}{2M}+\left(\frac{M}{2}-1\right)\frac{\lambda}{M}\right] \\ 0, x \epsilon \left(\frac{\lambda}{2M}+\left(\frac{M}{2}-1\right)\frac{\lambda}{M}, \frac{\lambda}{2}\right] \end{cases} \quad 19$$

Here, p is an integer that satisfies the condition $0 \le p \le \frac{M}{2} - 2$ By applying the half-range Fourier series expansion, which is defined over half of one period, equation (19) can be rewritten as follows:

$$J_y(z+d,x) = \sum_{n=1}^{+\infty} c_n sin\left(\frac{2n\pi x}{\lambda}\right) \quad 20$$

$$c_n = \frac{4}{\lambda}\int_0^{\frac{\lambda}{2}} M_x \, sin\left(\frac{2n\pi x}{\lambda}\right) dx \quad 21$$

The dominance of the first harmonic component allows the magnetization currents to be approximately expressed as follows:

$$J_y(z+d,x) \approx c_1 sin\left(\frac{2\pi x}{\lambda}\right)$$

$$= Im\left\{\frac{B_r}{\mu_0 \pi} Msin\left(\frac{\pi}{M}\right) e^{j(kx-\omega t)}\right\} \quad 22$$

Thus, equation (6) will be modified as follows:

$$j_y(z+d) = \frac{B_r}{\mu_0 \pi} Msin\left(\frac{\pi}{M}\right) \quad 23$$

Similarly, equation (7) is rewritten as follows:

$$j_y(z) = -\frac{B_r}{\mu_0 \pi} Msin\left(\frac{\pi}{M}\right) \quad 24$$

By replacing the expression B_r with $\frac{B_r}{\pi} Msin\left(\frac{\pi}{M}\right)$ and applying it to equation (17), the lifting force for the non-ideal Halbach array is expressed as follows:

$$F(z,v) = \frac{N\mu\omega\sigma^2 B_r^2}{2\pi k^3} M^2 \sin^2\frac{\pi}{M}\left(-e^{-kd}\right)^2 \frac{v^2}{\left(\sqrt{1+\frac{\mu^2\sigma^2 v^2}{k^2}}+1\right)^{\frac{3}{2}}\left(\left(\sqrt{1+\frac{\mu^2\sigma^2 v^2}{k^2}}+1\right)^{\frac{1}{2}}+\sqrt{2}\right)} e^{-2kz} \quad 25$$

This model neglects the edge effects of a non-ideal Halbach array. This assumption can lead to deviations from the real system. Therefore, it is necessary to correct (validate) the expression for the lifting force.

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III. RESULTS

The force characteristic is generally expressed in a lumped-parameter form [6]. The magnetization–current model is obtained to represent the behavior of the permanent magnets that results force [7]. Parameter list with explanations are given in Table I.

TABLE I
SYMBOL LIST FOR FORCE MODEL

	Symbols	Explanations
1	B_r	remanence of the permanent magnet
2	M	magnet numbers per period of the Halbach array
3	μ_0	permability of the air (H/m)
4	μ	permability of the conductor plate (H/m)
5	σ	conductivity of the conductor plate (S/m)
6	z	levitation gap (m)
7	w	width of the Halbach (m)
8	d	height of the Halbach (m)
9	λ	wavelength of the Halbach array (m)
10	h	height of the conductor plate (m)
11	v	moving speed of the Halbach array (m/s)

The geometrical design of Halbach array disc is given in Fig. 2.

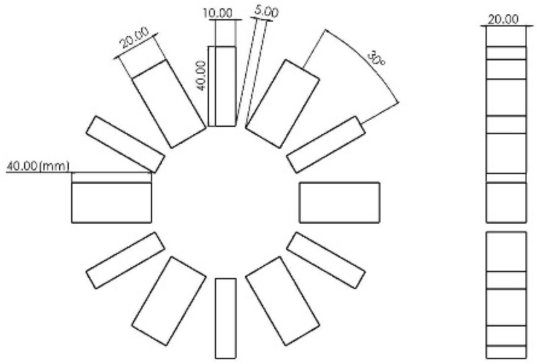

Fig. 2 Halbach array geometrical design

This model neglects the end effects of a non-ideal Halbach array, which may lead to deviations from the actual system behavior. Therefore, a correction (validation) of the levitation force expression is required. The resulting model is evaluated within the same operating range for airgap and rotational velocity by comparing it with finite element method (FEM) analysis results obtained using Ansys Maxwell in MATLAB [8]. The comparison results are presented in Fig. 3.

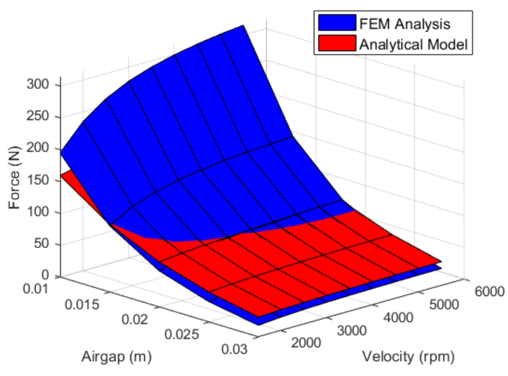

Fig. 3 FEM analysis surface results compare to analytical model

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The FEM results indicate that the levitation force increases with velocity and grows exponentially as the air gap decreases, which is consistent with the analytical model [9]. The comparison demonstrates that the analytically derived force model exhibits accurate behaviour relative to the FEM results. Although the error between the two surfaces increases for larger or smaller air gaps, the error at the designated operating point, an air gap of 0.02 m and a disk speed of 3000 rpm, is found to be 1.33%, corresponding to an accuracy of 98.67%. With this level of agreement, the model is suitable for linearization around the operating point and for subsequent development of control algorithms within the scope of the project.

IV. CONCLUSIONS

This study presents an analytic force model for circular Halbach array that is designed for micro-gravity platform for ground-based emulations to be used in control algorithm designs. The resulting model captures the electrodynamic response of the system by accounting the design parameters and magnetic behaviours. In particular, the proposed analytical framework captures the electrodynamic response of the system by incorporating key design parameters, such as geometric configuration and material properties, as well as the underlying magnetic behavior of the Halbach array. This enables a more comprehensive understanding of the interaction between the rotating magnetic field and the conductive plate surface, which is critical for precise levitation control.

The validity and applicability of the model make it suitable for integration into advanced control strategies aimed at maintaining a stable air gap under varying operating conditions. Nevertheless, the presence of parameter uncertainties and modeling approximations may lead to deviations between the predicted and actual system responses. Therefore, future work will focus on identification and modelling of parameter uncertainties and enhancing the robustness of the system by developing methods to suppress these deviations using different controller structures and add-on controller methods.

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