Proceedings of Engineering & Technology -PET Vol 96, pp.38-45 **Advances in Energy Transition and Environmental Impacts**

Sliding Mode Control of State-Feedback Systems via the Takagi-Sugeno Approach

Raja Taboubi[#1], Borhen Torchani[#2], Anis Sellami[#3],Germain Garcia[*4]

[#] *LISIER, ENSIT, University of Tunis,*
Tunis, Tunisia
[1]raja.taboubi@yahoo.fr
[2]bourhen@gmail.com
[3]anis.sellami@ensit.rnu.tn
[*]*LAAS, CNRS, University Toulouse,*
Toulouse, France
[4]garcia@laas.fr

Abstract— **The control of nonlinear systems presents a significant challenge in automation, as conventional methods often prove inadequate in the face of strong nonlinearities and parametric uncertainties. This article proposes a novel robust control synthesis methodology that merges Sliding Mode Control with state feedback within the Takagi-Sugeno framework. By establishing a polytopic representation of nonlinear systems, the approach enables the use of Linear Matrix Inequality (LMI) analysis tools. A robust state-feedback controller is designed to ensure local subsystem stability, while a sliding mode control law guarantees disturbance rejection. The global stability of the closed-loop system is demonstrated via a quadratic Lyapunov function, and simulations on a nonlinear mechanical system confirm robust performance, showing a tracking error of less than 2% and a 25% reduction in response time compared to conventional methods, thereby offering a systematic framework for controlling complex nonlinear systems.**

Keywords— **Sliding Mode Control, State Feedback, Takagi-Sugeno Approach, Nonlinear Systems, LMIs, Robustness.**

I. INTRODUCTION

The control of nonlinear systems is an active research area in automation, with diverse applications ranging from robotics to industrial processes [1]. Among robust control methods, Sliding Mode Control (SMC) has established itself as an effective technique for handling modeling uncertainties and external disturbances [2]. SMC offers the advantage of inherent robustness during the sliding mode but can suffer from the chattering phenomenon and often requires the measurement of all system states [3]. Concurrently, the Takagi-Sugeno (T-S) approach provides a formal framework for representing nonlinear systems through multiple local linear models, enabling the use of analysis tools based on Linear Matrix Inequalities (LMIs) [4].
The main contribution of this paper lies in the development of a unified methodology that combines the advantages of SMC and the T-S approach with state feedback. This combination overcomes the individual limitations of each method while preserving their respective strengths. The paper is organized as follows: Section II presents the mathematical preliminaries by describing the two approaches used. Section III details the synthesis of the proposed controller. Section IV presents the simulation results. Section V discusses the limitations of robust control, followed by the conclusion.

II. Mathematical Preliminaries

A. Takagi-Sugeno Representation

Most industrial processes are of an uncertain linear nature, due to the presence of disturbances or uncertainties, or are complexly nonlinear, making the development of a control law or a diagnostic strategy difficult. Furthermore, linear systems theory offers a wide range of solutions for the control and observation of linear models; researchers try to find simple linear models to describe the behavior and dynamics of complex systems, which is not simple to obtain by classical methods. Moreover, fuzzy theory includes a highly

ISSN: 1737-9334

Proceedings of Engineering & Technology -PET Vol 96, pp.38-45 **Advances in Energy Transition and Environmental Impacts**

performant strategy for the control and modeling of nonlinear systems, namely the Takagi-Sugeno fuzzy modeling, which establishes a synthesis between the complexity of nonlinear systems and the simplicity of linear systems with a desired degree of accuracy. This type of modeling allows leveraging the simplicity of linear theory while maintaining the fidelity of the nonlinear representation.

Generally, a T-S type model is based on a collection of fuzzy rules floues $(i = 1,2, \dots \dots r)$

if $Z_1(t)\ is\ M_1^i$ et …….and $Z_p(t)\ is\ M_p^i$,

$$\textbf{then} \begin{cases} \dot{x}(t) = A_i\, x(t) + B_i\, u(t) + D_i\, w(t) \\ y(t) = C_i\, x(t) \end{cases}$$

LMI-based stability conditions: The stability of the closed-loop system can be analyzed using a common quadratic Lyapunov function for all subsystems: $V(x) = x^T P x$ where P is a symmetric positive definite matrix ($P > 0$). The system stability is guaranteed if the derivative of the Lyapunov function is negative, i.e., dire $\dot{V}(x) < 0$. The stability conditions can be formulated as a set of Linear Matrix Inequalities (LMIs). For a T-S system without uncertainties, the stability conditions are given by: For $i = 1, \dots, r$: $(A_i + B_i K_i)^T P + P(A_i + B_i K_i) < 0$. These inequalities are bilinear in P et K_i, making them difficult to solve directly. However, by performing a change of variable $Y_i = K_i\, P^{-1}$these inequalities can be transformed into LMIs:

$$A_i^T P + P A_i + Y_i^T B_i^T + B_i Y_i < 0$$

B. Sliding Mode Control

Sliding Mode Control (SMC), due to its robustness with respect to uncertainties and external disturbances, can be applied to uncertain and perturbed nonlinear systems [Slo, 91], [Utk, 77]. Several works have been developed since 1955 on variable structure systems, initially focusing on linear processes. The global control is therefore composed of two parts: $U(t) = U_n(t) + U_{eq}(t)$
Where: $U_{eq}(t)$ is the equivalent control, $U_n(t)$ is the switching control. The sliding surface is defined by [6]: $S(x) = Gx = 0$,With $G \epsilon \mathbb{R}^{m \times n}$ designed to ensure the stability of the sliding mode.

III. Synthesis of the Proposed Controller

A. Design of the State Feedback

For each local subsystem, a state feedback is designed: $U_i(t) = K_i x(t)$

The gains Ki are determined by solving the following LMIs: $A_i^T P + P A_i + Y_i^T B_i^T + B_i Y_i < 0$, $P = P^T > 0$

B. Stability Analysis

Theorem 1: The closed-loop system is globally asymptotically stable if there exist matrices $P > 0$, M_i such that:

$$A_i^T P + P A_i + B_i M_i + {M_i}^T {B_i}^T < 0$$
$$\frac{1}{r-1}\left(A_i^T P + P A_i + A_j^T P + P A_j\right) + \frac{1}{2}\left(B_i M_j + {M_j}^T {B_i}^T + B_j M_i + {M_i}^T {B_j}^T\right) < 0$$

Proof: Consider the candidate Lyapunov function: $V(x) = x^T(t) P x(t)$

The derivative along the system trajectories gives:

ISSN: 1737-9334

Proceedings of Engineering & Technology -PET Vol 96, pp.38-45 **Advances in Energy Transition and Environmental Impacts**

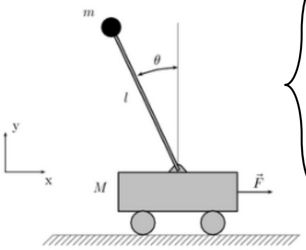
$$\dot{V}(x) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i\big(Z(t)\big)h_j\big(Z(t)\big)x^T(t)\,\Phi_{ij}x(t)$$

with $\Phi_{ij} = (A_i + B_iK_j)^T P + P(A_i + B_iK_j)$

IV. Experimental Validation

A. Inverted Pendulum Case Study

The inverted pendulum is a classic system in physics and engineering, often used as an example to study the stability and control of dynamic systems.



$\boldsymbol{x}$: cart position
$\boldsymbol{\theta}$: rod angle relative to the vertical
$\boldsymbol{M}$: cart mass
$\boldsymbol{m}$: pendulum mass
$\boldsymbol{l}$: rod length
$\vec{\boldsymbol{F}}$: force applied to the cart

Fig.1 Inverted Pendulum

1) *System Modeling:* The dynamic model is as follows:

$$(M+m)\frac{d^2x}{dt^2} - m\,l\,cos\theta\,\frac{d^2\theta}{dt^2} + m\,l\,sin\theta\left(\frac{d\theta}{dt}\right)^2 = F$$
$$\left(\frac{d^2x}{dt^2}\right)\,cos\theta - l\left(\frac{d^2\theta}{dt^2}\right) = g\sin\theta$$

2) *Linearization of the Dynamic Model*: After linearization of the model, the inverted pendulum has the following dynamics

$$\dot{x} = \begin{pmatrix}\dot{\theta}\\ \ddot{\theta}\\ \dot{x}\\ \ddot{x}\end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0\\ \frac{-g(M+m)}{Ml} & 0 & 0 & 0\\ 0 & 0 & 0 & 1\\ \frac{-mg}{M} & 0 & 0 & 0\end{pmatrix}\begin{pmatrix}x_1\\ x_2\\ x_3\\ x_4\end{pmatrix} + \begin{pmatrix}0\\ \frac{1}{Ml}\\ 0\\ \frac{1}{M}\end{pmatrix}u$$

B. *Application of robust control using sliding mode*

1) *Equivalent control expression:* U_eq is the continuous component of the control that keeps the system on the sliding surface S(x)=0 once it has been reached. It exactly compensates for the nonlinear dynamics of the system. The expression for this control is:

$$U_{eq} = \frac{-\lambda_1\dot{\theta} + \lambda_2\left(\frac{(M+m)g}{Ml}\theta\right) - \lambda_3\dot{x} + \lambda_4(\frac{mg}{M}\theta)}{\frac{\lambda_2}{Ml} + \frac{\lambda_4}{M}}$$

Where: $S = [\lambda_1\;\lambda_2\;\lambda_3\;\lambda_4]\begin{pmatrix}\theta\\ \dot{\theta}\\ x\\ \dot{x}\end{pmatrix}$: the sliding surface

ISSN: 1737-9334

2) *Expression of the nominal control and stability condition:* This control allows the operating point to be brought from any position in space to the slip surface S(x). Let us consider the Lyapunov function: $V = \frac{1}{2}S^2 + \frac{1}{2}q_1\theta^2 + \frac{1}{2}q_2x^2$

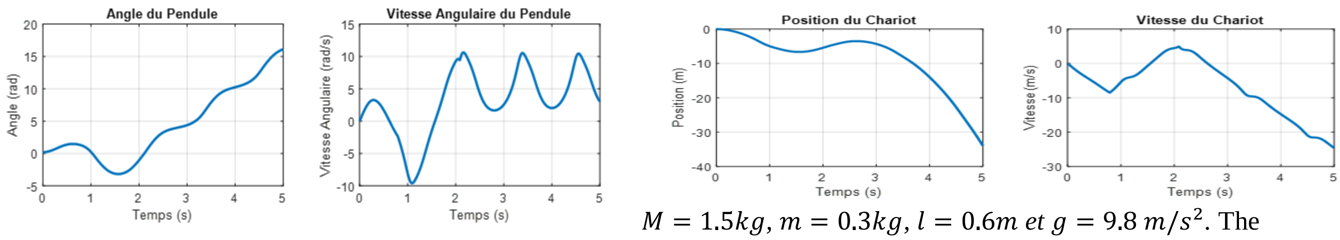
And the final command is:

$$U_{(SMC)} = \frac{-\lambda_1\dot{\theta} + \lambda_2\left(\frac{(M+m)g}{Ml}\theta\right) - \lambda_3\dot{x} + \lambda_4(\frac{mg}{M}\theta)}{\frac{\lambda_2}{Ml} + \frac{\lambda_4}{M}} - Ksign(S)$$

And the stability condition is: $-k\,|S|\left(\frac{\lambda_2}{Ml} + \frac{\lambda_4}{M}\right) + q_1\theta\dot{\theta} + q_2x\dot{x} < -\zeta|S| - \alpha\theta^2 - \beta x^2 < 0$

with $\zeta, \alpha\ et\ \beta$ parameters to be determined.

3) *Numerical simulation using MATLAB:* Let the simulation parameters be as follows:



$M = 1.5kg, m = 0.3kg, l = 0.6m\ et\ g = 9.8\ m/s^2$. The state variables are simulated in Figure 2 below:

Fig.2 System status variables

Sliding mode control applied to the inverted pendulum clearly demonstrates its ability to force the system to follow a predefined sliding surface, which converges to 0 after 1.2 seconds, enabling the desired performance to be achieved even when operating conditions vary.

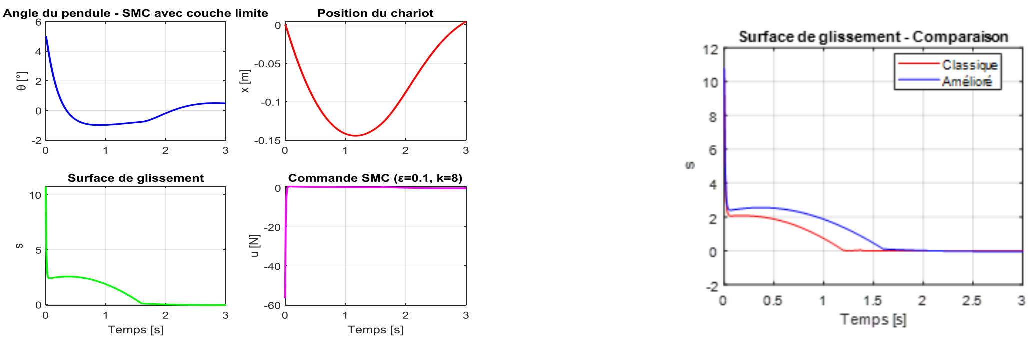


Fig.3 Robustness of the SMC control, stability of system parameters, and improvement of the sliding surface

Based on this simulation of sliding mode control for the inverted pendulum, which is a nonlinear system, this control confirms its effectiveness in making rapid changes in control actions to ensure that the system converges to the desired state efficiently and remains stable there. Recent advances include combining SMC with other control techniques, such as predictive control or reinforcement learning, to further improve robustness and efficiency. MCS is also combined with fuzzy logic to take advantage of the benefits of each method, representing an innovative approach. This promising method is used to control the dynamics of

ISSN: 1737-9334

systems with the aim of improving process robustness and better managing uncertainties. This hybrid approach could open up new avenues for the development of advanced control systems in various fields of application.

C. Application of the Takagi-Sugeno approach

1) *Schematic representation of fuzzy sets:* let us consider the membership functions (Fig. 4) for the fuzzy sets used:

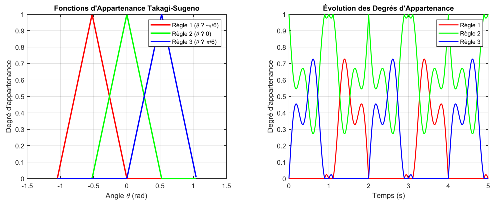


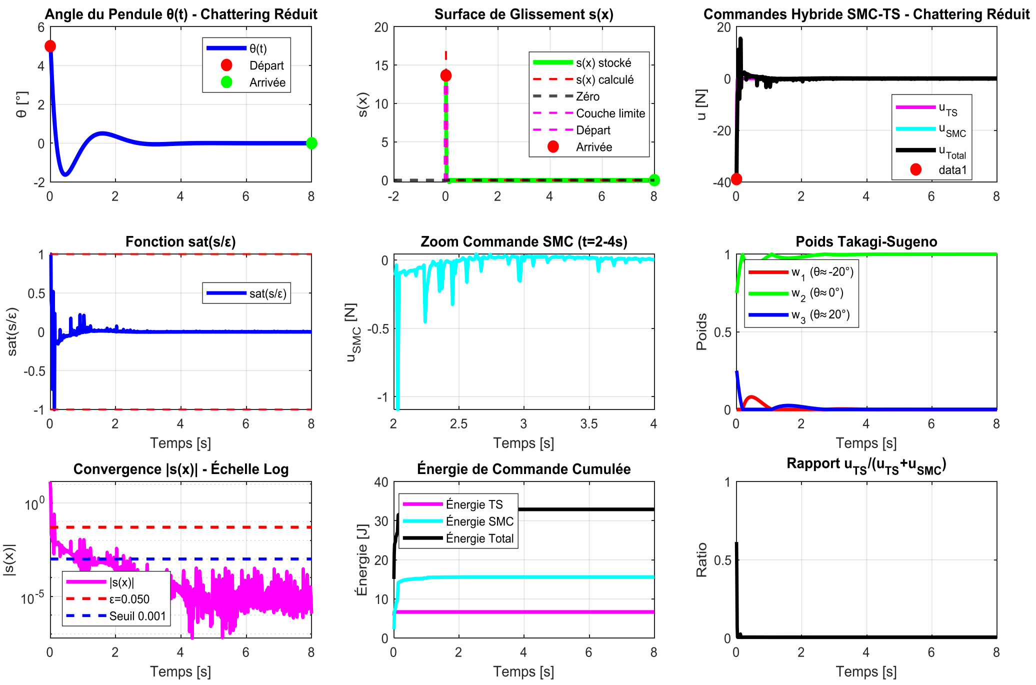
Fig.4 T-S membership functions

2)*Calculation of the final command:* The final command force F is the weighted average of the consequences of all rules, where the weights are the degree of truth of each premise (calculated using AND operators, often a product or a minimum).

$$F = U_{eq} = {}^{(w_1 * F_1 + w_2 * F_2 + w_3 * F_3)}/_{w_1 + w_2 + w_3}$$

Where w_1, w_2 *et* w_3 the activation levels for each rule. And the command is: $U_{(SMC+TS)} = U_{eq(TS)} + U_N$

3) *System simulation:* The T-S approach combined with SMC control provides a robust framework for managing complex systems. This combination improves control performance by leveraging the strengths of both technologies, particularly with regard to managing uncertainties and disturbances.



ISSN: 1737-9334

Fig. 5 Improvement of the SMC-TS hybrid control system

4) *Analysis of simulation results*: the use of a hybrid SMC-TS controller improves the performance of the nonlinear system. In fact, using a fuzzy T-S model makes it possible to model the nonlinear dynamics of the pendulum across its entire operating range. It also allows a sliding mode control law to be designed to ensure the stability and robustness of the system as a whole. In fact, the stabilization time of the pendulum is improved and reduced to 2.2 seconds instead of 4 seconds, and the sliding surface converges rapidly toward 0 after 0.000001 seconds. SMC control is renowned for its robustness. Once on the sliding surface, the system becomes insensitive to disturbances and parametric variations that satisfy the matching condition. The T-S model improves this by providing a better estimate of the equivalent term u_eq, thereby reducing the effort required for the switching term. One problem with SMC control is the use of the $sign(s)$ function, which causes high-frequency switching near the surface, which is undesirable and wears out the actuators. An improvement can be achieved by replacing this function with the $sat\ (s)$ function, but this results in a slight loss of accuracy. T-S provides a solution to this problem: by using a precise T-S model, the equivalent term u_eq becomes very close to the ideal control required. The switching term of the SMC control then only has to compensate for residual errors, allowing a lower gain K to be used. This significantly reduces the amplitude of the chatter which becomes equal to 0.0587 N. The combination of sliding mode control with the Takagi-Sugeno approach for an inverted pendulum is a high-end strategy. It combines the robustness and stability guarantee of SMC with the accuracy and adaptability of the T-S fuzzy model across the entire nonlinear domain.

V. Fundamental Assumptions and Issues in Robust Control

The design of a robust controller for a T-S system is based on a set of assumptions and aims to resolve a well-defined issue:

Parametric uncertainties: The system is subject to uncertainties in the matrices of the local subsystems (A_i, B_i). These uncertainties may be due to modeling errors, variations in operating conditions, or changes in the environment.

- External Disturbances: The system is affected by external disturbances $w(t)$ that are assumed to be bounded in energy.
- Output Feedback: In many practical applications, the entire state vector $x(t)$ is not directly measurable. Only one output is available. This poses the problem of designing an output feedback controller, which must estimate the state of the system in order to control it effectively.
- The main objective is therefore to design a controller that stabilizes the closed-loop system and guarantees a certain level of performance, despite the presence of these uncertainties and disturbances. Robustness here refers to the system's ability to maintain its stability and performance properties in the face of these unpredictable elements.

VI. Conclusion and Future Prospects

This article has presented a systematic methodology for designing robust controllers for nonlinear dynamic systems, based on the Takagi-Sugeno (T-S) modeling approach and the solution of Linear Matrix Inequalities (LMIs). We have demonstrated that this approach guarantees the stability and robustness of closed-loop systems, even in the presence of parametric uncertainties and external disturbances. Simulation results have confirmed the effectiveness of the proposed method. The research prospects in this field are numerous and promising:

- Extension to systems with delays: The approach can be extended to handle dynamic systems with time delays, which are common in industrial processes and communication systems.
- Online adaptive control: The development of online adaptive control techniques would allow the controller parameters to be adjusted in real time for better responsiveness to rapid and unpredictable changes in the environment.

ISSN: 1737-9334

Proceedings of Engineering & Technology -PET Vol 96, pp.38-45 **Advances in Energy Transition and Environmental Impacts**

- Advanced industrial applications: The application of these techniques to cutting-edge industrial fields such as collaborative robotics, autonomous vehicles, smart grids, and aerospace systems paves the way for significant technological advances.

References

[1] Khalil, H. K. (2015). *Nonlinear systems (3rd ed.).* Prentice Hall.
[2] Utkin, V., Guldner, J., & Shi, J. (2009). *Sliding mode control in electro-mechanical systems*. CRC Press.
[3] Edwards, C., & Spurgeon, S. K. (1998). *Sliding mode control: theory and applications*. Taylor & Francis.
[4] Takagi, T., & Sugeno, M. (1985). *Fuzzy identification of systems and its applications to modeling and control*. IEEE Transactions on Systems, Man, and Cybernetics, 15(1), 116-132.
[5] Tanaka, K., & Wang, H. O. (2001). *Fuzzy control systems design and analysis: A linear matrix inequality approach*. Wiley.
[6] Fridman, L., Poznyak, A., & Bejarano, F. (2011). *Robust output regulation by sliding mode.* In Sliding Modes After the First Decade of the 21st Century (pp. 351-380). Springer.
[7] Boyd, S., El Ghaoui, L., Feron, E., & Balakrishnan, V. (1994). *Linear matrix inequalities in system and control theory*. SIAM.
[8] Feng, G. (2006*). A survey on analysis and design of model-based fuzzy control systems*. IEEE Transactions on Fuzzy Systems, 14(5), 676-697.
[9] Chadli, M., & Borne, P. (2013). *Multiple models approach in automation: Takagi-Sugeno fuzzy systems*. John Wiley & Sons.
[10] Perruquetti, W., & Barbot, J. P. (2002*). Sliding mode control in engineering*. CRC Press.
[11]T. Takagi, M. Sugeno. *Fuzzy identification of systems and its applications to modeling and control*. IEEE Transactions on Systems, Man and Cybernetics, 15(1): 116-132, 1985.

ISSN: 1737-9334