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Magnetohydrodynamic thermosolutal mixed convection with thermal diffusion and diffusion thermo effects using Lattice Boltzmann Method

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Abstract*— This work presents a comprehensive numerical investigation of magnetohydrodynamic (MHD) double-diffusive convection in a lid-driven cavity, incorporating the Soret (thermal diffusion) and Dufour (diffusion-thermo) effects. A novel hybrid computational approach is introduced, combining the multiple relaxation time lattice Boltzmann method (MRT-LBM) for resolving the velocity field, the single relaxation time LBM (SRT-LBM) for modelling the magnetic field, and the finite difference method (FDM) for solving the energy and species transport equations. Numerical simulations demonstrate that streamlines, isotherms, and isoconcentrations are strongly affected by the presence of a magnetic field. As the Hartmann number increases, magnetic damping of convective motion becomes pronounced, weakening buoyancy- and shear-induced vortices and leading to more stratified flow structures. This transition significantly reduces convective heat and mass transfer rates, with the system approaching a regime dominated by conduction and molecular diffusion. Moreover, the evolution of average Nusselt and Sherwood numbers under varying magnetic field strengths offers valuable insight into the coupled transport mechanisms. The robustness and accuracy of the developed hybrid scheme highlight its potential applicability to a wide range of engineering and industrial processes involving MHD flows and coupled heat and mass transfer phenomena.

Keywords— **Hybrid numerical model (LBM-FDM); MRT-LBM; SRT-LBM; MHD-thermosolutal mixed convection; Soret and Dufour effects.**

I. INTRODUCTION

The Lattice Boltzmann Method (LBM) has emerged as a powerful mesoscopic approach for simulating fluid dynamics, offering an alternative to classical numerical methods by solving simplified kinetic equations consistent with the macroscopic Navier–Stokes equations [1-3]. Its strength lies in its simplicity, adaptability to complex boundaries, and suitability for parallel computing, making it ideal for simulations on high-performance platforms such as GPUs [4]. Among the LBM collision models, the single relaxation time (BGK) model [5] is widely used due to its computational efficiency, though it often suffers from numerical instability in high Reynolds number regimes [6,7]. To overcome this limitation, the Multiple Relaxation Time (MRT) model [8] was developed, allowing independent tuning of relaxation parameters and offering better stability and accuracy. Another alternative, the entropy-based LBM [9], provides unconditional stability but at the cost of computational intensity due to implicit formulations.

For heat and mass transfer modeling, several LBM-based schemes exist. The multi-speed model [10-11], while capable, suffers from numerical instabilities and high memory usage [12]. The more popular double distribution function model, often called the passive scalar model, handles fluid flow and thermal/species transport separately [13], offering better stability. However, the hybrid method—combining LBM for fluid flow and finite difference (FDM) or finite volume (FVM) methods for thermal and species transport—has gained recognition for its accuracy and flexibility, particularly when paired with MRT [14-15].

Thermosolutal convection arises from the interplay between thermal and concentration gradients in multicomponent systems. Traditionally governed by Fourier's and Fick's laws, this process becomes more complex in the presence of cross-diffusion effects, notably the Soret (thermal diffusion) and Dufour (diffusion-thermo) effects [16-18]. The Soret effect leads to species migration under temperature gradients, while the Dufour effect causes heat flux induced by concentration gradients. Though often secondary, these effects are significant in nanofluids and reactive flows and must be included to ensure accurate simulations.

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The presence of a magnetic field introduces further complexity through magnetohydrodynamic (MHD) effects, where Lorentz forces suppress convection and alter transport mechanisms. MHD-enhanced systems are especially relevant in electronics cooling, microfluidics, and space technologies. Many studies have addressed MHD impacts on thermosolutal convection [19-21], reporting that increasing the Hartmann number (Ha) reduces flow circulation and convective transport. However, these classical models often use a static Lorentz force without dynamically solving the magnetic field.

To address this limitation, the present study proposes a hybrid numerical model that explicitly resolves the magnetic field using BGK-LBM, combined with MRT-LBM for velocity and FDM for temperature and concentration. This framework provides a more realistic simulation of MHD thermosolutal convection with Soret and Dufour effects, offering improved accuracy and deeper insight into coupled heat and mass transfer phenomena [22-24].

II. Numerical method

The Lattice Boltzmann Method based on the BGK approximation (BGK-LBM) can be expressed as:

$$f_i(\mathbf{x}+\delta x, t+\delta t) = \left(1-\frac{1}{\tau_f}\right) f_i(\mathbf{x},t) + \frac{1}{\tau_f} f_i^{eq}(\mathbf{x},t)$$

The following is an expression for the process of computing the magnetic induction equation using a similar lattice Boltzmann formulation (BGK-LBM):

$$\frac{\partial h_i}{\partial t} + \xi_i \nabla h_i = -\frac{1}{\tau_h}\left(h_i - h_i^{eq}\right)$$

Although kinetic theory does not directly govern the physical evolution of the magnetic field, the magnetic induction equation can nonetheless be modelled statistically using the Lattice Boltzmann Equation (LBE). This is possible due to the structural similarity between the induction and momentum equations, both of which are conservative and hyperbolic in nature. As a result, the discretised form of the magnetic field LBE mirrors that of the hydrodynamic LBE [25]:

$$h_i(\mathbf{x}+\xi_i\delta t, t+\delta t) = h_i(\mathbf{x},t) - \frac{\delta t}{\tau_h}\left[h_i(\mathbf{x},t) - h_i^{eq}(\mathbf{x},t)\right]$$

Here, h_i represents the magnetic field probability distribution function (PDF), ξ_i is the lattice speed for the magnetic field PDF, and τ_h denotes the magnetic field relaxation time.
$h_i = [h_{ix}, h_{iy}]$ governs the evolution of the magnetic field in a two-dimensional space, expressed as:

$$h_{ix}(\mathbf{x}+\xi_\mathbf{i}\delta t, t+\delta t) - h_{ix}(\mathbf{x},t) = -\frac{\delta t}{\tau_h}\left[h_{ix}(\mathbf{x},t) - h_{ix}^{eq}(\mathbf{x},t)\right]$$

$$h_{iy}(\mathbf{x}+\xi_\mathbf{i}\delta t, t+\delta t) - h_{iy}(\mathbf{x},t) = -\frac{\delta t}{\tau_h}\left[h_{iy}(\mathbf{x},t) - h_{iy}^{eq}(\mathbf{x},t)\right]$$

The relationship between the magnetic field relaxation parameter and the magnetic diffusivity is comparable to that between kinematic viscosity and the collision frequency [25]:

$$\sigma = (\tau_h - 0.5)\, c_s^2$$

Unlike kinematic viscosity, magnetic diffusivity is not directly linked to a physical relaxation process. Instead, it is numerically modeled through an artificial parameter known as the relaxation time. In

$$f_i^{eq} = w_i\rho\left[1 + \frac{\xi_\mathbf{i}.\mathbf{u}}{c_s^2} + \frac{(\xi_\mathbf{i}.\mathbf{u})^2}{2c_s^4} - \frac{\mathbf{u}.\mathbf{u}}{2c_s^2}\right] + \frac{\lambda_i}{2c_s^4}\left[\frac{1}{2}|\ \xi_\mathbf{i}\ |^2|\ \mathbf{B}\ |^2 - (\xi_\mathbf{i}.\mathbf{B})^2\right]$$

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this context, the equilibrium distribution functions are constructed to reflect the coupling between the fluid velocity and the magnetic field, and are expressed as follows:

$$h_{ix}^{eq} = \lambda_i \left[B_x + \frac{\xi_{iy}}{c_s^2} (v.B_x - u.B_y) \right]$$
$$h_{iy}^{eq} = \lambda_i \left[B_y + \frac{\xi_{ix}}{c_s^2} (u.B_y - v.B_x) \right]$$

It is important to note that, for two-dimensional flows, the D2Q5 lattice structure (Fig. 1) is the most preferred, as demonstrated in [25]. In this case, λi is the weighting factor expressed in D2Q5 as following: λi=1/3 for i=0 and λi=1/6 for i = 1, 2, 3, 4.

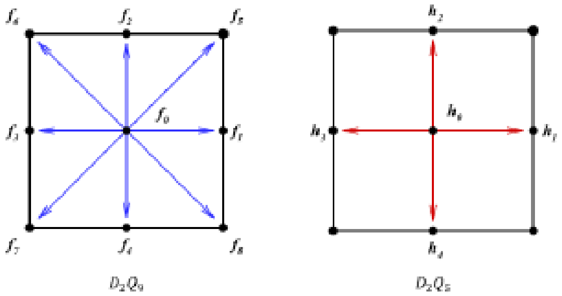

Fig. 1 D2Q9 and D2Q5 lattices

Finally, the magnetic field value is obtained by summing all the magnetic field PDFs within the lattice structure as follow:

$$B_x = \sum_{i=0}^{4} h_{ix} \qquad and \qquad B_y = \sum_{i=0}^{4} h_{iy}$$

The procedures for resolving the velocity field using the MRT-LBM and for solving the temperature and concentration fields using the finite difference method (FDM) are comprehensively detailed in [15].

III. Results and discussion

The simulation results reveal the progressive impact of the magnetic field on flow structure and transport phenomena. In the absence of a magnetic field (Ha = 0), curved isotherms and isoconcentration lines indicate strong convective heat and mass transfer, driven by the combined effects of buoyancy and the moving top wall. Steep gradients near the upper and lower walls confirm efficient thermal and solutal mixing, where the Soret and Dufour effects further enhance double-diffusive convection under shear-dominated conditions. At moderate magnetic field strength (Ha = 25), magnetic damping becomes evident. The flow develops two primary counter-rotating vortices, with the upper vortex driven by the lid motion remaining dominant. A secondary vortex appears near the lower-right corner due to frictional and pressure effects. Although convection weakens, the system still exhibits significant mixed convection behaviour.

When the Hartmann number is increased to Ha = 50, the buoyancy-induced vortex grows and nearly equals the size of the shear-induced vortex, indicating a shift toward a balance between shear and buoyancy forces. Frictional effects diminish, and both temperature and concentration fields become more uniform, highlighting a partial suppression of convection and a transition toward conduction and diffusion. At strong magnetic influence (Ha = 100), the flow structure changes dramatically. The secondary vortex vanishes, and the buoyancy-driven vortex becomes dominant, while the influence of the moving lid is significantly reduced. Isotherms and isoconcentrations become nearly linear and stratified, confirming the suppression of convective

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transport. Heat and mass transfer are now primarily governed by conduction and diffusion, signifying a regime of double-diffusive natural convection dominated by buoyancy forces.

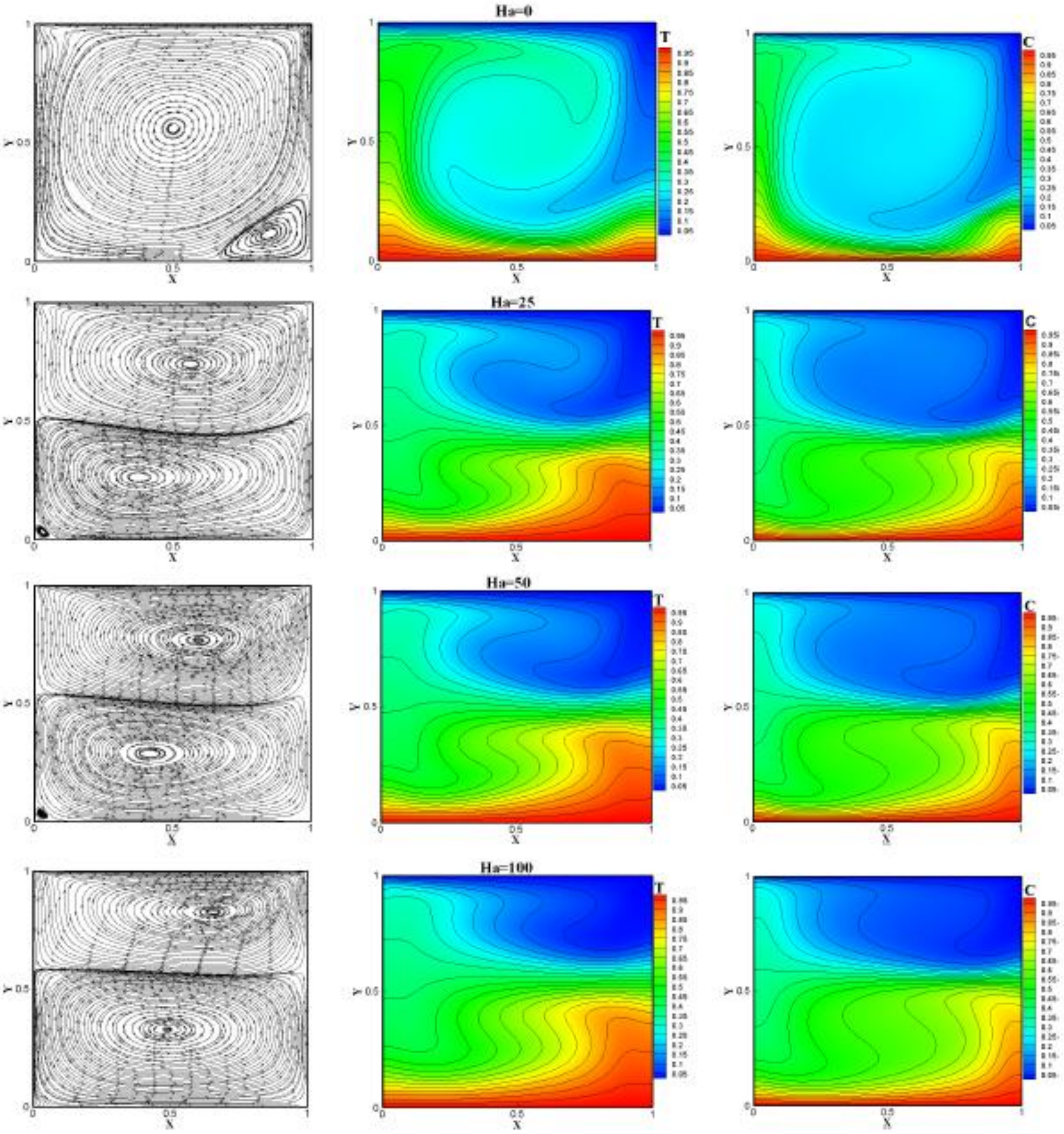

Fig. 1 Effect of Hartmann number (Ha = 0; 25; 50 and 100) on streamlines,

isotherms and isoconcentrations for Sr = Df = 0.5

To highlight the influence of the magnetic field on heat and mass transfer, Table 1 reports the variation of the average Nusselt number and Sherwood number with respect to the Hartmann number (Ha), for fixed Soret and Dufour parameters (Sr = Df = 0.5). The results align with the previously discussed trends, confirming that increasing magnetic field strength leads to a gradual reduction in the average heat and mass transfer rates. This decline is primarily attributed to the magnetic damping effect, which weakens buoyancy-induced convection and progressively suppresses the overall convective transport, thereby reducing thermal and solutal transfer efficiency.

TABLE I
EFFECT OF HARTMANN NUMBER ON NU AND SH FOR SR = DF = 0.5.

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Ha	Nusselt number	Sherwood number
0	7.236	9.726
25	4.400	5.437
50	4.176	5.180
100	3.465	4.231

IV. CONCLUSIONS

In this study, a hybrid numerical framework combining the Lattice Boltzmann Method (LBM) and the Finite Difference Method (FDM) was developed to investigate magnetohydrodynamic (MHD) double-diffusive convection within a lid-driven cavity. The fluid flow was resolved using the multiple relaxation time LBM (MRT-LBM), while the temperature and concentration fields were computed via the finite difference method. The magnetic field evolution was modelled using the single relaxation time LBM (BGK-LBM). The results demonstrate that increasing the Hartmann number (Ha) leads to a marked suppression of convective heat and mass transfer, driving the system toward conduction- and diffusion-dominated regimes. The presence of the magnetic field significantly influences the flow structure, attenuating both buoyancy- and shear-induced vortices and resulting in stratified distributions of velocity, temperature, and concentration.

Future work may focus on extending this modelling approach to explore enhanced transport mechanisms in nanofluid systems, particularly by incorporating interparticle interactions and nanoparticle dynamics to better understand their role in heat and mass transfer enhancement.

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