Port-Hamiltonian Based Control for a Two-Stage Anaerobic Digestion Process

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Abstract— This work presents a nonlinear passive control strategy based on the Port-Hamiltonian framework for the regulation of a two-stage anaerobic digestion process, specifically designed for the treatment of tequila vinasses. The system is modeled as two interconnected continuous bioreactors representing the acidogenic and methanogenic phases. The dilution rates in each bioreactor are considered as the manipulated inputs. By formulating the Hamiltonian as the sum of squared errors around the steady state, it is shown that both the conservation matrix and the dissipation matrix depend solely on the reaction terms and yield coefficients. The proposed control approach demonstrates strong robustness in the face of set-point changes and external perturbations, as demonstrated through exhaustive numeric simulations.

Keywords --- Anaerobic digestion, Two-stages, Wastewater treatment, Passive Control, Port-Hamiltonian systems

I. INTRODUCTION

Anaerobic digestion (AD) is widely used to treat wastewater with high organic loads while producing biogas as a valuable fuel. However, its strong nonlinearity and susceptibility to substrate inhibition lead to instability [1]. Extensive research has focused on optimizing process configurations, modeling, and control to improve efficiency and stability. Several single-stage AD models have been developed to enhance monitoring and control [2]. An alternative approach involves physically separating AD into two stages based on pH selectivity. This configuration has been evaluated for various wastewater types [3], showing improved growth rates, higher organic load tolerance, shorter start-up times, high-purity biogas production, and enhanced stability [4]. Mathematical modeling of AD enables the development of control strategies to regulate organic matter degradation and ensure process stability [5]. However, few studies have explored control schemes for two-stage AD systems [6]. In contrast to adaptive or linearizing controllers that target only one or a few state variables, passive control allows stabilization of the entire system state [7]. Most research on passive control focuses on electromechanical systems, but recent studies have extended its application to chemical engineering. For example, power-shaping control has been applied to reaction systems like non-isothermal Continuous Stirred Tank Reactors (CSTRs) using Brayton-Moser and thermodynamic pseudo-Hamiltonian formulations [8]. In these approaches, thermodynamic variables such as total energy or entropy serve as Lyapunov functions. In contrast, isothermal CSTRs or systems based solely on mass balances have received less attention, possibly because stability-related physical properties are harder to identify and require more detailed dynamic formulations. Nonetheless, passive control in continuous bioreactors has recently gained increasing interest [9].

Passive control strategies are essential for mitigating instabilities in dynamical systems. In AD, they may include process monitoring, microbial management, and design modifications. Monitoring enables real-time assessment, while microbial management supports early diagnosis and optimization [10]. Design features like step-feed configurations and sludge recirculation further improve stability [11], emphasizing the value of integrating passive dynamics into control schemes, particularly for complex setups like two-stage AD. In [12], a general canonical form for feedback passivity in nonlinear systems was proposed, using bioreactor control as an example. Building on this, Port-Hamiltonian-based nonlinear controllers have been developed for chemostats regulation [13]. These methods and their application to continuous fermenters and interconnected systems have been reviewed in later work [14]. In this paper, we present a passive control strategy for regulating two-stage anaerobic digestion processes. Simulation results, utilizing a validated model for agricultural wastewater treatment, demonstrate effective and promising stabilization of this type of process, exhibiting key features such as robustness against input disturbances and step changes in the set point.

II. TWO STAGES ANAEROBIC DIGESTION SYSTEM

The two-stage AD system is shown in Fig. 1. Here, the conventional single-stage process is divided into acidogenic and methanogenic phases. In the first reactor, acidogenic bacteria convert complex organics into CO₂, H₂, and VFAs. In the second, acetogenic bacteria produce acetic acid from VFAs, which methanogens then transform into methane. A simplified model describing this configuration was developed and validated for control applications, particularly in the treatment of tequila wastewater [15], as follows:

Acidogenic reactor

$$\dot{x}_1 = \mu_1 x_1 - \alpha_1 D_1 x_1 \dot{x}_2 = \left(S_1^{in} - x_2\right) D_1 - k_1 \mu_1 x_1$$
(1)

(2)

 $\dot{x}_3 = (S_2^{in} - x_3)D_1 + k_2\mu_1x_1$ Methanogenic reactor $\dot{x}_6 = \mu_2 x_6 - \alpha_2 D_2 x_6$

$$\begin{aligned} \dot{x}_{7} &= \mu_{3}x_{7} - \alpha_{2}D_{2}x_{7} \\ \dot{x}_{8} &= (x_{4} - x_{8})D_{2} - k_{3}\mu_{2}x_{6} \\ \dot{x}_{9} &= (x_{5} - x_{9})D_{2} + k_{5}\mu_{2}x_{6} - k_{4}\mu_{3}x_{6} \end{aligned}$$





Where x_1 , x_6 (g/l) represent acidogenic biomass, and x_7 (g/l) denotes methanogenic biomass. x_2 , x_8 (g/l) are COD (g/l), while x_3 , x_9 (mmol/l) correspond to VFA concentrations in the acidogenic and methanogenic reactors. Influent concentrations are denoted by S_1^{in} (COD) and S_2^{in} (VFA). Dilution rates are represented as D_1 and D_2 (d⁻¹) respectively. Coefficients k_{1-5} are yield factors, while α_1 and α_2 represent the fraction of detached biomass [1]. Specific growth rates μ_1 , μ_2 correspond to Monod kinetics, meanwhile μ_3 follows Haldane kinetics [1]. Model parameters are listed in table 1 [16].

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$$\mu_{1} = \frac{\mu_{1max}x_{2}}{K_{S1} + x_{2}}$$
$$\mu_{2} = \frac{\mu_{2max}x_{8}}{K_{S2} + x_{8}}$$
$$\mu_{1} = \frac{\mu_{3max}x_{9}}{K_{S3} + x_{9} + (x_{9}/K_{I})^{2}}$$

Parameter Value Units 0.27 d⁻¹ μ_{1max} 0.5 d⁻¹ μ_{2max} 0.29 d^{-1} μ_{3max} $K_{\underline{S1}}$ kg COD / m² 24 *K*_{*S*2} 3.5 kg COD / m² 16 mol VFA / m³ K_{S3} 27 $(mol VFA / m^3)^1$ K_I 0.16 Dimensionless α_1 0.38 Dimensionless α_2 k_1 42.14 kg COD / kg x1 k_4 268 mol VFA / kg x₇ k_5 116.5 mol VFA / kg x₆ k_2/k_1 3.5 mol VFA / kg COD k_{5}/k_{3} 0.9 mol VFA / kg COD

TABLE I Model Parameters

(3)

One of the considerations of the dilution tank is that no chemical reaction takes place at that time, so the dynamics of the tank system is

$$\dot{x}_4 = (x_2 - x_4)D_d \dot{x}_5 = (x_3 - x_5)D_d$$
(4)

The dilution rates are represented as $D_d = F_1/V_d$. (d⁻¹), which is related to the flow of the acidogenic reactor F_1 y the volume of the tank V_d .

III. PASSIVE CONTROL APPROACH

Passive systems are characterized by the fact that the net increase in stored energy, in a certain time interval, is less than or equal to the energy supplied in the same period. Consider the system

$$\Sigma = \begin{cases} \dot{x} = f(x, u) \\ y = h(x, u) \end{cases}$$
(5)

Where $x \in X \subset \mathbb{R}^n$ is the state vector, $u \in U \subset \mathbb{R}$ is the control input, and the function $y \in Y \subset \mathbb{R}$ represents the systems outputs [17].

A system is said to be passive if there exists a storage function $V: \mathbb{R}^n \to \mathbb{R}^+$, such that for every $x_0 = X_0 \in \mathbb{R}^n$, for all $t \ge 0$, and for input function *u*, the following relation holds [18]:

$$V(x(t)) \le V(x(t_0)) + \int_0^t u(s)^T y(s) ds$$
(6)

Inequality (6), known as the dissipation inequality, states that the "stored energy" $V(x(t_1))$ at any future time t_1 is no greater than the initial energy $V(x(t_0))$ plus the total externally supplied energy $\int_{t_0}^{t_1} s(u(t), y(t)) dt$. Therefore, no internal energy can be generated, only dissipation can occur. Passivity requires that the derivative of the storage function be bounded by the input power to the system. Then we calculate the derivative of (6)

$$\dot{V}(x) \le u(t)^T y(t) \tag{7}$$

A. Acidogenic and Methanogenic reactor based on the Gibbs free energy

The inflows to the acidogenic reactor and the chemical potential of each species defines the natural output as:

$$u = \begin{bmatrix} 0\\ S_1^{in} D_1\\ S_2^{in} D_1 \end{bmatrix}, \qquad y = \begin{bmatrix} \mu_1^c\\ \mu_2^c\\ \mu_3^c \end{bmatrix}$$
(8)

Thus, the power supplied to the acidogenic reactor is

$$u^T y = D_1 S_1^{in} \mu_2^c + D_1 S_2^{in} \mu_3^c \tag{9}$$

To demonstrate the passivity of the system, a storage function based on the Gibbs free energy, defined as $G(x) = \sum_{i=1}^{3} x_i \mu_i^c$, where x_i are concentration and μ_i^c are chemical potentials, is used as storage function, i.e.:

$$\dot{V}(x) = \sum_{i=1}^{3} \mu_i^c \dot{x}_i$$
(10)

From (1), substituting in $\dot{V}(x)$

$$\dot{V}(x) = \mu_1^c [x_1 - \alpha_1 D_1 x_1] + \mu_2^c [(S_1^{in} - x_2) D_1 - k_1 \mu_1 x_1] + \mu_3^c [(S_2^{in} - x_2) D_1 + k_2 \mu_1 x_1]$$
(11)

that can be reduced to:

$$\dot{V}(x) = D_1 S_1^{in} \mu_2^c + D_1 S_2^{in} \mu_3^c + (dissipative \ terms)$$
(15)

In which the terms $D_1 S_1^{in} \mu_2^c$ and $D_1 S_2^{in} \mu_3^c$ are precisely the supplied power. Then

$$\dot{V}(x) = u^T y + (dissipative terms) \le u^T y$$
 (16)

we can conclude that the passivity condition (7) is satisfied. In the case of the methanogenic reactor, a very similar procedure is used to demonstrate the passivity condition.

B. Conditioning tank

The inflows to the tank and the chemical potential of each species defining the natural output, are given by

$$u = \begin{bmatrix} D_1 x_2 \\ D_1 x_3 \end{bmatrix}, \qquad y = \begin{bmatrix} \mu_4^c \\ \mu_5^c \end{bmatrix}$$
(17)

In this case, to demonstrate passivity, we have that storage function is

$$\dot{V}(x) = \sum_{i=4}^{5} \mu_i^c \dot{x}_i = \mu_4^c [D_d x_2 - D_d x_4] + \mu_5^c [D_d x_3 - D_d x_5]$$
(18)

The terms $-D_d x_4 \mu_4^c$ and $-D_d x_5 \mu_5^c$ represent the free energy lost through and, being negative do not increase V(x). $D_d x_2 \mu_4^c$ and $D_d x_3 \mu_5^c$ correspond to the supplied power, fulfilling the passivity condition (7).

$$\dot{V}(x) = u^T y - D_d x_4 \mu_4^c - D_d x_5 \mu_5^c \le u^T y$$
⁽¹⁹⁾

IV. PORT-HAMILTONIAN SYSTEMS AND THE IDA-PBC CONTROL METHODOLOGY

A. Port-Hamiltonian

A Port-Hamiltonian (PH) model is a system reformulation expressed as [17]:

$$\Sigma = \begin{cases} \dot{x} = [J(x) - R(x)]\nabla H + g(x)u\\ y = g(x)^T \nabla H \end{cases}$$
(20)

where, $g(x)^T \nabla H$ represents how sensitive the internal energy is to changes in the states, modulated by the port interconnection matrix, $J = -J^T$ is the interconnection matrix representing energy conservation, $R = R^T \ge 0$ contains the dissipative terms and *H* represents the stored energy. It can be demonstrated that PH systems are passive and satisfy the energy balance [19], i.e.:

$$\underbrace{\dot{H}}_{stored} \leq \underbrace{-\nabla H^T R \nabla H}_{dissipated} + \underbrace{u^T y}_{supplied} \leq u^T y$$
(21)

B. IDA-PBC design

In the Interconnection and Damping Assignment-Passivity Based Control (IDA-PBC) framework, the Hamiltonian function—representing the system total energy—is central to control design. The first step involves identifying the equilibrium point by setting the system time derivatives to zero [19]. Solving the resulting equations yields the equilibrium pair $x_d - u^*$, corresponding to the desired steady state. This pair is then set as the stabilization goal [20]. A quadratic Hamiltonian in the error state, often chosen for its simplicity and favorable stability properties:

$$H_d = \frac{1}{2} \sum_{i=1}^{n} (x_i - x_{i,d})^2$$
(22)

This choice guarantees an isolated minimum at the equilibrium point $x = x_d$, facilitating effective stabilization [21]. The system is reformulated in a Port-Hamiltonian form by identifying or constructing suitable structural matrices Q(x) and G(x), expressing the original system dynamics as:

$$\dot{x} = Q(x)\nabla H(x) + G(x)u \tag{23}$$

Thus, according with (23), the system of (1)-(4) takes the Port-Hamiltonian form

Generally, two possibilities are considered for the desired Hamiltonian: Directly setting $H_d(x) = H(x)$, or adding a corrective potential, resulting in $H_d(x) = H(x) + \Phi(x)$. The desired interconnection structure $Q_d(x) = Q(x) - \Delta R$, where ΔR is a symmetric, positive semi-definite matrix introducing additional damping into the closed-loop system [21]. The aim is to achieve a closed-loop dynamic equation

$$\dot{x} = Q_d(x) \nabla H_d(x) \tag{25}$$

The core step involves solving the matching equation

$$f(x) + G(x)b(x) = Q_d(x)\nabla H_d(x)$$
(26)

where f(x) is $Q(x)\nabla H(x)$, G(x) is g(x) and b(x) is an explicit static control law. Thus it may suffice to inject damping without fully solving the PDE [21]. In the present work, the structure is preserved by setting $Q_d(x) = Q(x)$, and the desired Hamiltonian is $H_d(x) = \frac{1}{2}(x - x_d)^T(x - x_d)$.

To enhance stability and robustness, an additional damping term is commonly integrated into the control law $u_{damp}(x) = -K_p[G(x)^T \nabla H(x)]$, with $K_p > 0$. This term ensures additional dissipation in the closed-loop system, making the time derivative of the Hamiltonian non-positive [22]. Hence, the total control input becomes

$$u = b(x) - K_p G(x)^T \nabla H(x)$$
⁽²⁷⁾

As we want to control the dilution rates of the two reactors, then we propose the function

$$u = \begin{bmatrix} D_1^* \\ D_2^* \end{bmatrix} - \begin{bmatrix} K_{11} & 0 \\ 0 & K_{22} \end{bmatrix} G(x)^T \nabla H(x)$$
(28)

That provides the closed system with additional damping injection control, where $K_{11} = 1/x_2^2$, $K_{22} = 1/x_9^2$, and D_1^* , D_2^* being the required the dilution rates.

V. RESULTS AND DISCUSSION

Process inputs with fluctuations around nominal values were introduced as perturbations to simulate realistic scenarios such as seasonal or human-activity-induced variability (Fig. 2).



Fig. 2 Process inputs: COD (S_1^{in}) and VFA (S_2^{in}) .

Fig. 3 shows the evolution of the dilution rates D_1 and D_2 , which served as control inputs. Temporary saturation of D_1 occurs around days 20, 40, 60, 80, 120, and 150, coinciding with input disturbances. Although saturation may cause biomass washout and potential process failure, this was not critical here; the system recovered promptly, and D_1 returned to its normal range. D_2 did not saturate but exhibited minor overshoots at the same times, followed by swift recovery.



Fig. 3 Control inputs: Dilution rates

Fig. 4 depicts COD dynamics in the acidogenic reactor, with a set point of 4.0 g/L (dashed line). The control strategy maintained COD near the reference value, although transient deviations occurred after perturbations. Each disturbance was followed by a damped oscillatory response, with quick return to set-point conditions. Despite the system's ability to recover, the amplitude of deviations suggests the need for more robust or predictive strategies to enhance set-point tracking and reduce risks such as organic overloading or biomass loss.



Fig. 4 Dynamical behaviour of COD in the acidogenic bioreactor (continuous line) with to the set-point (dashed line).

VFA concentration in the methanogenic reactor is shown in Fig. 5, which was regulated around a 40 mmol/L set point. The system began slightly above this value and settled smoothly, with a single undershoot near day 40 (minimum: 39.94 mmol/L). The deviation was quickly corrected, and the trajectory stabilized with minimal oscillations. The absence of overshoots or prolonged transients indicates effective control, maintaining VFA within a narrow range. This reflects high robustness of the methanogenic stage, contributing to overall process efficiency and preventing the accumulation of inhibitory metabolites.



Fig. 5 Dynamical behaviour of VFA in the methanogenic bioreactor (continuous line) with to the set-point (dashed line).

VI. CONCLUSIONS

A passive control strategy was successfully implemented in a two-stage anaerobic digestion (AD) process subject to both gradual fluctuations and abrupt step changes in the process inputs. The controller demonstrated strong robustness and reliable performance under these challenging conditions, maintaining key process variables within acceptable operational ranges and ensuring system stability. These findings highlight the effectiveness of the passive control framework for managing the full-state dynamics of mass balance models, offering a promising and scalable alternative for advanced control in complex biological systems such as AD bioreactors. The approach's inherent structural simplicity and capacity to handle nonlinear dynamics without relying on detailed model inversion further reinforce its applicability to a broad class of bioprocesses.

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