

Discrete Time switching surfaces optimization for an Induction Motor

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Abstract— In this paper, the discrete time sliding mode control of an induction motor is investigated. The main contribution of this work is the design of switching surfaces which defined by an addition of an integral term into the considered surfaces. A comparative study with the conventional proportional integral techniques is proposed. Finally, the validity of the proposed control is demonstrate through simulations.

Keywords—discrete time sliding mode control; induction motor; switching surfaces, conventional proportional integral.

I. INTRODUCTION

Nowadays, the recourse of the controllers is frequently used in different control law. In practical applications, the digital devices are widespread in the last decades. So, the implementation of each control law is divided in two tasks: the first one, the realization of continuous time control using the Zero-Order-Hold. And the second one, obtain a discrete time model and design the control law in discrete time. The first method may not present a high performance, because in this case the sampling time is limited in the lower value. The second method, present a drawback in the sampled model which have be difficult to compute, especially in the case of nonlinear systems. For this reason, the discrete time control present an important area of research considering the nonlinear systems, [1], [2], [3], [4], [5], and [10].

The induction motor is one of the most preferred actuator in industrial applications which presented a low maintenance cost, reliability and robustness [11]. The mathematical model of the induction motor is nonlinear. It constitute an important area of research in the advanced control. More recently control law are investigated in discrete time such as: Indirect Field Oriented Control (IFOC), Field Oriented Control (FOC), Direct Torque Control (DTC), Fuzzy Logic Control (FLC) and Sliding Mode Control (SMC).

In [8] and [9], Castillo proposed a discrete time sliding mode control which the sliding surfaces are presented by the error between the variable to be controlled and its references. A discrete time sliding mode optimization which added an integral term is discussed in the research work. This paper is organized into six sections. After introduction, the problem

formulation is developed. Then, the control strategy is presented. The fourth section is about the stability analysis. In the section five, the simulation results are interpreted and discussed to validate the best control method. The last section gives a comparative study and finally, a conclusion is drawn.

II. PROBLEM FORMULATION

The dynamical model of the induction motor in the stationary reference frame (d-q) under same assumptions is presented by:

$$\begin{cases} \frac{di_{sd}}{dt} = -\left(\frac{1}{\sigma\tau_s} + \frac{1-\sigma}{\sigma\tau_r}\right)i_{sd} + \frac{1-\sigma}{\sigma M_{sr}\tau_r}\varphi_{rd} + \frac{1-\sigma}{\sigma M_{sr}}\varphi_{rq}w + \frac{1}{\sigma L_s}V_{sd} \\ \frac{di_{sq}}{dt} = -w_{dq}i_{sd} - \left(\frac{1}{\sigma\tau_s} + \frac{1-\sigma}{\sigma\tau_r}\right)i_{sq} - \frac{1-\sigma}{\sigma M_{sr}}w\varphi_{rd} + \frac{1-\sigma}{\sigma M_{sr}\tau_r}\varphi_{rq} + \frac{1}{\sigma L_s}V_{sq} \\ \frac{d\varphi_{rd}}{dt} = \frac{M_{sr}}{\tau_r}i_{sd} - \frac{1}{\tau_r}\varphi_{rd} - w\varphi_{rq} \\ \frac{d\varphi_{rq}}{dt} = \frac{M_{sr}}{\tau_r}i_{sq} + w\varphi_{rd} - \frac{1}{\tau_r}\varphi_{rq} \\ \frac{dw}{dt} = \frac{3}{2} \frac{np^2}{j} \frac{M_{sr}}{L_r} (\varphi_{rd}i_{sq} - \varphi_{rq}i_{sd}) - \frac{np}{j} (C_r - C_f) \end{cases} \quad (1)$$

The Euler discretization is given in the system equations (1), we give a sampled period T_e . So, the sampled dynamics of the system equations (1) is expressed by [6]:

$$\begin{cases} a) \quad i_{sd}(k+1) = i_{sd}(k) - Te \left(\frac{1}{\sigma\tau_s} + \frac{1-\sigma}{\sigma\tau_r} \right) i_{sd}(k) + \frac{Te(1-\sigma)}{\sigma M_{sr}\tau_r} \varphi_{rd}(k) \\ \quad + \frac{Te(1-\sigma)}{\sigma M_{sr}} \varphi_{rq}(k) w(k) + \frac{Te}{\sigma L_s} V_{sd}(k) \\ b) \quad i_{sq}(k+1) = i_{sq}(k) - Te w_{dq} i_{sd} - Te \left(\frac{1}{\sigma\tau_s} + \frac{1-\sigma}{\sigma\tau_r} \right) i_{sq}(k) - \frac{1-\sigma}{\sigma M_{sr}} w(k) \varphi_{rd}(k) \\ \quad + \frac{Te(1-\sigma)}{\sigma M_{sr}\tau_r} \varphi_{rq}(k) + \frac{Te}{\sigma L_s} V_{sq}(k) \\ c) \quad \varphi_{rd}(k+1) = \varphi_{rd}(k) + Te \frac{M_{sr}}{\tau_r} i_{sd}(k) - \frac{Te}{\tau_r} \varphi_{rd}(k) - w(k) \varphi_{rq}(k) \\ d) \quad \varphi_{rq}(k+1) = \varphi_{rq}(k) + Te \frac{M_{sr}}{\tau_r} i_{sq}(k) + w(k) \varphi_{rd}(k) - \frac{Te}{\tau_r} \varphi_{rq}(k) \\ e) \quad w(k+1) = w(k) + Te \frac{3}{2} \frac{np^2}{j} \frac{M_{sr}}{L_r} (\varphi_{rd}(k) i_{sq}(k) - \varphi_{rq}(k) i_{sd}(k)) \\ \quad - Te \frac{np}{j} (C_r(k) - C_f(k)) \end{cases} \quad (2)$$

The control law applied to this type of systems is an indirect field oriented control by sliding mode in discrete time. The designing of the switching surfaces and sliding mode control is presented such as:

- The robust sliding mode dynamics.
- The system is maintained on the sliding manifold in a finite time regardless the uncertainties.

So, the selected switching surfaces is important regardless control design and satisfactory performances. The main contribution of this work is the design a novel form of the switching surfaces which presented on adding an integral term of the considered surfaces.

III. CONTROL LAW DESIGN

In this section, we developed a discrete time sliding mode control based on the Indirect Field Oriented Control. The control objectives is to design three regulators: two controllers relative to the direct and quadratic stator current and one controller relative to the speed.

A. Sliding Surfaces Design

The selection of the sliding surfaces defining the representative regulators are defining by the error between the selected variable and its references [7], [8] and [9]. In order to ameliorate the performances given to the studied system, an integral term is added to the selected surfaces. These later are designed by:

$$\begin{cases} s_w = c_1 \varepsilon_w + c_4 \int \varepsilon_w dt \\ s_d = c_2 \varepsilon_d + c_5 \int \varepsilon_d dt \\ s_q = c_3 \varepsilon_q + c_6 \int \varepsilon_q dt \end{cases} \quad (3)$$

The sampled dynamics of the system equation (3) is follows:

$$\begin{cases} a) s_w(k+1) - s_w(k) = c_1(\varepsilon_w(k+1) - \varepsilon_w(k)) + Te c_4 \varepsilon_w(k) \\ b) s_d(k+1) - s_d(k) = c_2(\varepsilon_d(k+1) - \varepsilon_d(k)) + Te c_5 \varepsilon_d(k) \\ c) s_q(k+1) - s_q(k) = c_3(\varepsilon_q(k+1) - \varepsilon_q(k)) + Te c_6 \varepsilon_q(k) \end{cases} \quad (4)$$

B. Control law synthesis

The control law applied to the induction motor is given by (Figure 1).

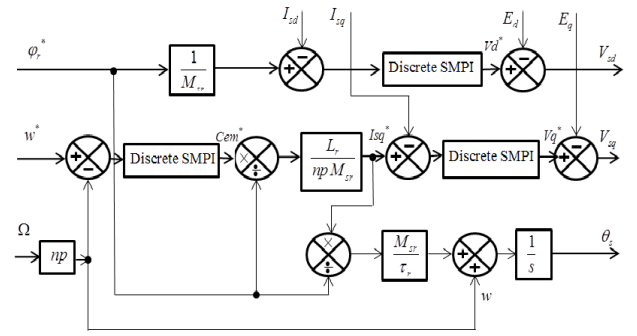


Fig. 1. Indirect Field Oriented Control (IFOC) by sliding mode in discrete time.

This method based on modeling the machine in (d-q) stationary reference frame. That is $\varphi_{rd} = 0$ and $\varphi_{rq} = 0$. The control strategy is given by two components: the first one is the equivalent control which determined in solving the equation $s(k+1) = 0$. And the second one is the nonlinear control.

$$U = U_{eq} + U_{nl} \quad (5)$$

B.1. Speed controller

Using the equation (4-a) and replacing the equation (2-e) in equation (4-a), we get:

$$\begin{aligned} s_w(k+1) - s_w(k) = & c_1 w^*(k+1) - c_1 w^*(k) - c_1 w(k) \\ & - c_1 Te \frac{3 np^2}{2 j} \frac{M_{sr}}{L_r} (\varphi_{rd}(k) i_{sq}(k)) \\ & + c_1 Te \frac{np}{j} (C_r(k) - C_f(k)) + c_1 w(k) \\ & + Te c_4 w^*(k) - Te c_4 w(k) \end{aligned} \quad (6)$$

The nonlinear component is defined by:

$$s_w(k+1) - s_w(k) = -QTe s_w(k) - K_w Te \text{sign}(s_w(k)) \quad (7)$$

So, the global control is:

$$i_{sq}(k) = \frac{2jL_r}{3c_1 Te np^2 M_{sr} \varphi_{rd}(k)} \begin{bmatrix} c_1 w^*(k+1) - c_1 w^*(k) \\ + c_1 Te \frac{np}{j} (C_r(k) - C_f(k)) \\ + Te c_4 w^*(k) - Te c_4 w(k) \\ + QTe s(k) + KTe \text{sign}(s(k)) \end{bmatrix} \quad (8)$$

B.2. Direct stator current controller

Under equation (4-b) and equation (2-a), we obtain:

IV. SIMULATION RESULTS

To validate the proposed control, the selected induction motor allows the following characteristics: 85/140V ; 3.5/6A ; $f = 50\text{Hz}$; $R_s = 3.45\Omega$; $R_r = 2.95\Omega$; $L_s = 0.1442\text{H}$; $L_r = 0.1442\text{H}$; $M_{sr} = 0.1342\text{H}$; $j = 0.01\text{Kgm}^2$; $np = 2$. The coefficients $c_1, c_2, c_3, c_4, c_5, c_6$ and the gains k_v, k_d, k_q relative to the control law, are adjusted to the values: $c_1 = 0.00018$, $c_2 = 0.0099$, $c_3 = 0.00000009$, $c_4 = 0.000000002$, $c_5 = 0.0000003$, $c_6 = 0.000000009$, $k_v = 115$, $k_d = 3000$ and $k_q = 115, k_d = 3000, k_q = 3000$. These coefficients and gains have been tuned until the obtain of the validate results.

$$s_d(k+1) - s_d(k) = c_2 I_{sd}^*(k+1) - c_2 I_{sd}^*(k) - c_2 \left[\begin{array}{l} I_{sd}(k) - Te \left(\frac{1}{\sigma \tau_s} + \frac{1-\sigma}{\sigma \tau_r} \right) I_{sd}(k) \\ + \frac{Te(1-\sigma)}{\sigma M_{sr} \tau_r} \varphi_r(k) + \frac{Te}{\sigma L_s} V_{sd}(k) \end{array} \right] + c_2 I_{sd}(k) + Te c_5 I_{sd}^*(k) - Te c_5 I_{sd}(k) \quad (9)$$

The nonlinear control is:

$$s_d(k+1) - s_d(k) = -QTe s_d(k) - K_d Te \text{sign}(s_d(k)) \quad (10)$$

The control law relative to the direct stator current controller is given by:

$$V_{sd}(k) = \frac{\sigma L_s}{c_2 Te} \left[\begin{array}{l} c_2 I_{sd}^*(k+1) - c_2 I_{sd}^*(k) \\ + c_2 Te \left(\frac{1}{\sigma \tau_s} + \frac{1-\sigma}{\sigma \tau_r} \right) I_{sd}(k) \\ - c_2 \frac{Te(1-\sigma)}{\sigma M_{sr} \tau_r} \varphi_r(k) \\ - c_2 \frac{Te}{\sigma L_s} + c_2 I_{sd}(k) + Te c_5 I_{sd}^*(k) \\ - Te c_5 I_{sd}(k) + QTe s_d(k) \\ + K Te \text{sign}(s_d(k)) \end{array} \right] \quad (11)$$

B.3. Quadratic stator current controller

Referring to (4-c) and (2-b), we define:

$$s_q(k+1) - s_q(k) = c_3 I_{sq}^*(k+1) - c_3 I_{sq}^*(k) - c_3 \left[\begin{array}{l} I_{sq}(k) - Te w_{dq} I_{sq}(k) \\ - Te \left(\frac{1}{\sigma \tau_s} + \frac{1-\sigma}{\sigma \tau_r} \right) I_{sq}(k) \\ - \frac{1-\sigma}{\sigma M_{sr}} w(k) \varphi_r(k) - \frac{Te}{\sigma L_s} V_{sq}(k) \end{array} \right] + c_3 I_{sq}(k) + Te c_6 I_{sq}^*(k) - Te c_6 I_{sq}(k) \quad (12)$$

The nonlinear control is:

$$s_q(k+1) - s_q(k) = -QTe s_q(k) - K_q Te \text{sign}(s_q(k)) \quad (13)$$

The control law relative to the quadratic stator current controller is defined by:

$$V_{sq}(k) = \frac{\sigma L_s}{c_3 Te} \left[\begin{array}{l} -c_3 I_{sq}^*(k+1) + c_3 I_{sq}^*(k) + c_3 I_{sq}(k) \\ - c_3 Te w_{dq} I_{sq}(k) - c_3 Te \left(\frac{1}{\sigma \tau_s} + \frac{1-\sigma}{\sigma \tau_r} \right) I_{sq}(k) \\ - c_3 \frac{1-\sigma}{\sigma M_{sr}} w(k) \varphi_r(k) - c_3 I_{sq}(k) \\ - Te c_6 I_{sq}^*(k) + Te c_6 I_{sq}(k) \\ + QTe s_q(k) + K_q Te \text{sign}(s_q(k)) \end{array} \right] \quad (14)$$

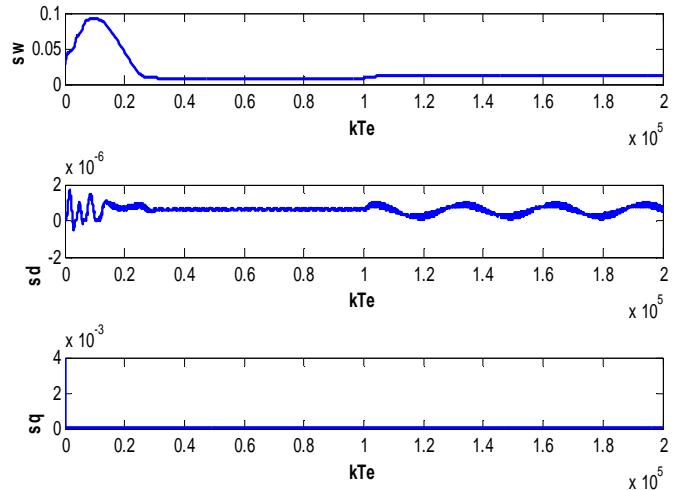


Fig. 2. Discrete time sliding surfaces evolutions

To verify the criteria of the sliding mode, Fig.2 shown the switching surfaces relative to three controllers. We remark that these surfaces reach to zero. The test condition is : nominal speed 157rad/s and nominal flux 0.8Wb.

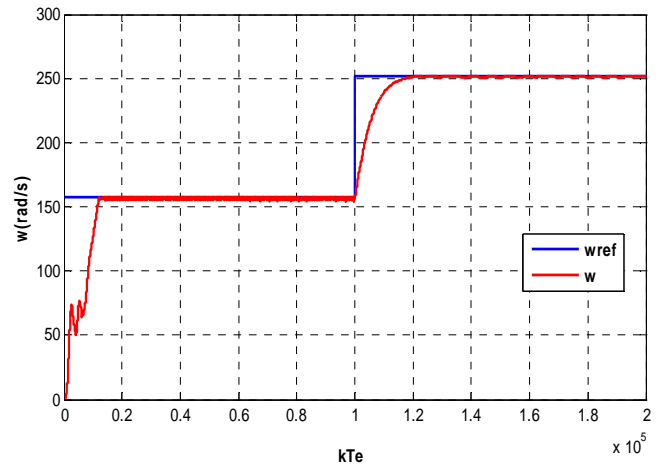


Fig. 3. Evolution of the speed

Fig.3 presents the characteristics of the speed which started without overshoot in 157rad/s and we increasing this value at 250rad/s at the time 0.18s. The settling time is 0.18s.

V. COMPARATIVE STUDY

For testing the performances given to the considered control, it is necessary to compare with other methods. In this section, we make a comparison results between the conventional PI and the PISM. We remark that the PISM given a best results than the conventional PI. So, the sliding mode techniques improve the characteristics given to the studied system, such as the settling time with PISM is 0.2s and with conventional PI is about 0.38s. Then, the characteristics with conventional PI presents an overshoot against is not the case with PISM.

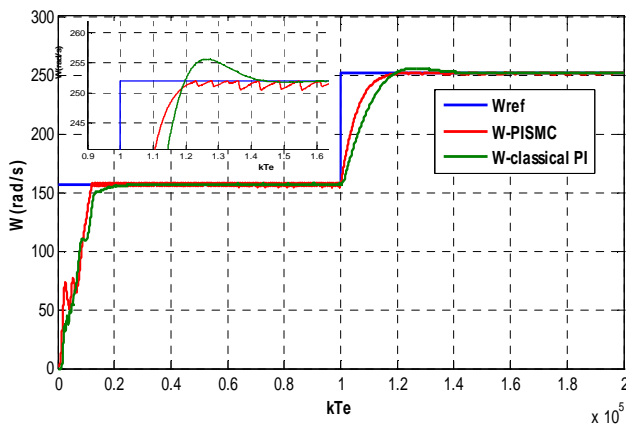


Fig. 4. Evolution of the speed with different methods

VI. CONCLUSION

In this paper, we have shown the design of a discrete time sliding mode control. This approach is based on an optimization to the sliding surfaces. The simulation results

prove the effectiveness of the proposed techniques. Finally, this method is compared with a conventional PI.

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