5ème Conférence Internationale des Energies Renouvelables (CIER– 2017)
Proceeding of Engineering and Technology –PET
Vol.27 pp.37-40

A vehicular positioning with hybrid TOA/AOA, GPS/IMU, Extended and Unscented Kalman Filter

Rabaa Ibrahmi
Sys'Com
ENIT
Tunisia
ibrahmirabaa@gmail.com

Tahar Ezzedine
Sys'Com
ENIT
Tunisia
tahar.ezzedine@enit.rnu.tn

Abstract— The mobile robots and the mobile localization are nonlinear systems. So, a linear process model is generated out of the non-linear dynamic systems in this paper. This document presents three algorithms; the first is a hybrid TOA/AOA (TS/LS) (Time Of Arrival, Angle Of Arrival)(Taylor Series-Least Square), the second is the module GPS/IMU and the finally is the Extended Kalman Filter. The module GPS/IMU and EKF can trace the objects relatively well, further reducing the positioning error.

Keywords— MS; BTS; hybrid TOA/AOA;GPS/IMU; EKF; UKF

INTRODUCTION (HEADING 1)

Now, the information of the present mobile's position is very important because it give any service of autonomous vehicle and C-ITS (cooperative Intelligent Transportation System) [1].

Use GPS is the same way to provide information of mobile's position.

This document presents in the first a hybrid TOA/AOA(TS-LS) localization algorithm which extends the Taylor Series Least Square [2] and in the second it presents a vehicular positioning with GPS/IMU.

The EKF is a method through which the state propagation equations and the sensor models can be linearized about the current state estimate.

I. HYBRID TOA/AOA LOCALIZATION ALGORITHM

In the first, designate (x_i,y_i) as the position of the ith BS and (x,y) as the position of the MS. The home BS is the first BS. t_i is the Time Of Arrival measurement at the ith BS and θ is the Angle Of Arrival measurement at the home BS. $r_i = ct_i$, where r_i is the range measurement between the ith BS and the MS, c is the speed of light.

Designate the noise free value of {r_i} as $\{r_i\}^0$, the range measurement r_i can be modeled as:

$$r_i = r_i^0 + n_i = \sqrt{(x-x_i)^2 + (y-y_i)^2} + n_i, \quad i = 1 \dots M \quad (1)$$

Where n_i (the measurement error) is modeled as a zero-mean Gaussian variable i.e, $n_i \sim N(0, \sigma_n^2)$

Utilizing (1), and presenting a new variable $R = x^2+y^2$

We have:

$$r_i^{02} = (r_i - n_i)^2 = (x - x_i)^2 + (y - y_i)^2$$

$$r_i^{02} = r_i^2 + n_i^2 - 2r_i n_i = x^2 + y^2 + x_i^2 + y_i^2 + 2xx_i + 2yy_i$$

$$r_i^2 + n_i^2 - 2n_i r_i = R + K_i - 2xx_i - 2yy_i, \quad i = 1,2 \dots M \quad (2)$$

So

$$r_1^0 \sin n_\theta = (x - x_1)sin\theta - (y - y_1)cos\theta \quad (3)$$

and

$$r_1^0 n_\theta \approx -x_1 sin\theta + y_1 cos\theta + xsin\theta - ycos\theta \quad (4)$$

Let $Z = [x \quad y \quad R]^T$, and revising (2), (4) in the matrix form, we obtain:

$$\varphi = h - G_a Z_a^0 \quad (5)$$

Where

$$h = \begin{bmatrix} (r_1^2 - K_1) & / & 2 \\ (r_M^2 - K_M) & / & 2 \\ -x_1 sin\theta & + & y_1 cos\theta \end{bmatrix} \quad (6)$$

$$G_a = \begin{bmatrix} -x_1 & -y_1 & 1/2 \\ -x_M & -y_M & 1/2 \\ -sin\theta & cos\theta & 0 \end{bmatrix} \quad (7)$$

Note from (1) that $r_i = r_1^0 + n_i$:

$$n_i r_i - n_i^2/2 = n_i(r_1^0 + n_i) - n_i^2/2 = n_i r_1^0 + n_i^2 - n_i^2/2$$

$$n_i r_i - n_i^2/2 = n_i r_1^0 + n_i^2/2$$

φ is created to be

$$\varphi = \begin{bmatrix} BE + 0.5E.E \\ r_1^0 n_\theta \end{bmatrix} \quad (8)$$

where

$$B = diag\{r_1^0, r_2^0 \dots r_M^0\}, \quad E = [n_1 \ n_2 \dots \ n_M]^T \quad (9)$$

and the covariance matrix Ψ is calculated as

$$\Psi = E(\varphi\varphi^T) = E[B(EE^T)B^T - 0.5B(EE^T)E^T + 0.5E(EE^T)B^T + 1/4E(EE^T)E^T]$$

$$\Psi = E[B(EE^T)B^T + 1/4E(EE^T)E^T]$$

$$\Psi = E[\varphi\varphi^T]$$

ISSN 2356-5608

$$\Psi = B'\begin{bmatrix} 4\sigma_n^2 I_M & 0 \\ 0 & \sigma_\theta^2 \end{bmatrix} B' + \begin{bmatrix} 2\sigma_n^4 + I_M + \sigma_n^4 1_M & 0 \\ 0 & 0 \end{bmatrix} \quad (10)$$

Accepting the independence of x, y and R the Maximum Likelihood (ML) estimator of Z_a is

$$Z_a = \arg\min\{(h - G_a Z_a)^T \Psi^{-1} (h - G_a Z_a)\}$$
$$= (G_a^T \Psi^{-1} G_a)^{-1} G_a^T \Psi^{-1} h \qquad (11)$$

II. GPS/IMU

A. GPS

GPS (The Global Positioning System) is a satellite-based navigation system that produces information correlated to location of object equipped with GPS receiver [3].
In this section, we define the vehicular positioning with GPS/IMU [4] for trajectory tracking of the MS and we improve the localization accuracy.

B. The estimation errors with GPS/IMU

Designated the obtained initial value of (x, y) as $(x\hat{}, y\hat{})$, we obtain:

$$\hat{x} = \hat{x} + \Delta x, \quad \hat{y} = \hat{y} + \Delta y \qquad (12)$$

where Δy and Δx are the estimation errors.

Utilizing (1) into Taylor Series and restraining the first order terms, we obtain:

$$r_i^0 = \hat{r}_i + \frac{\hat{x} - x_i}{\hat{r}_i}\Delta x + \frac{\hat{y} - y_i}{\hat{r}_i}\Delta y \qquad (13)$$

Where $\hat{r}_i = \sqrt{(\hat{x} - x_i)^2 + (\hat{y} - y_i)^2}$

Substituting (12) into (4), we have:

$$r_1^0 n_\theta = (\hat{x} - x_1)\sin\theta - (\hat{y} - y_1)\cos\theta + \Delta x \sin\theta - \Delta y \cos\theta \qquad (14)$$

Denote $Z'_a = [\Delta x \;\; \Delta y]^T$ and using (13), (14) in the matrix form, we obtain:

$$\varphi' = h' - G'_a Z'_a \qquad (15)$$

Where

$$\varphi' = [n_1 \;\; n_2 \ldots n_M \; (r_1 - n_1) n_\theta]^T \quad (16)$$

$$h' = [r_1 - \hat{r}_1 \ldots r_M - \hat{r}_M \;\; (\hat{x} - x_1)sin\theta - (\hat{y} - y_1)cos\theta]^T$$
$$(17)$$

$$\text{C.}\quad G'_a = \begin{bmatrix} \frac{\hat{x}-x_1}{\hat{r}_1} & \ldots & \frac{\hat{x}-x_M}{\hat{r}_M} & -sin\theta \\ \frac{(\hat{y}-y_1)}{\hat{r}_1} & \ldots & \frac{(\hat{y}-y_M)}{\hat{r}_M} & cos\theta \end{bmatrix} \qquad (18)$$

The covariance matrix $\psi' = E[\varphi' \varphi'^T]$ can be easily calculated.
The WLS estimation of (15) is then writing by:

$$Z'_a = (G'^T_a \Psi'^{-1} G'_a)^{-1} G'^T_a \Psi'^{-1} h' \quad (19)$$

and the covariance matrix of Z_a' is:

$$var(Z'_a) = \left(G'^T_a \Psi'^{-1} G'_a\right)^{-1} \qquad (20)$$

The location estimate can then be updated expanding (12)

$$\hat{x} = \hat{x} + \Delta x, \quad \hat{y} = \hat{y} + \Delta y \qquad (12)$$

III. SIMUTATONS RESULTS

We obtain four conventional algorithms (TOA (TS-LS), TOA/AOA (TS-LS), TOA/AOA positioning with single BS and TOA/AOA positioning with multiple utilizing LS)
In the first figure, we compare the performance of the TOA/AOA (TS-LS) with the other algorithms.
The cellular networks utilize a hexagonal layout that we use it.
Three BSs are deployed at:

- $(0m, 0m)$
- $(2000\sqrt{3}m, -2000m)$
- $(2000\sqrt{3}m, 2000m)$.

The MS is randomly arranged within a $2000\sqrt{3}m \times 4000m$ rectangle comprised the three BSs .
The standard deviation of the Time Of Arrival measurement error σ_n is disposed to $200m$, and the other parameters utilized in the simulation are defined in Table I, unless otherwise stated.
The performance measure of the algorithm is obtained as the Root Mean Square Error (RMSE) defined in (21), and is computed over 10,000 independent runs.

$$RMSE = \sqrt{\frac{1}{N}\sum_{i=1}^{N}[(\hat{x}_i - x_1^0)^2 + (\hat{y}_i - y_1^0)^2} \quad (21)$$

TABLE I. PARAMETERS SETTING IN THE SIMULATION

Standard Deviation σ	System Model		Observation Model		
	ax (m/s^2)	*ay* (m/s^2)	θ (°)	*v* (*m/s*)	*a* (m/s^2)
Value	0.1	0.1	2	3	1

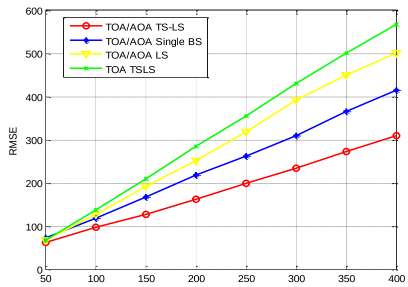

Fig. 1. RMSE based on the measurement error of TOA

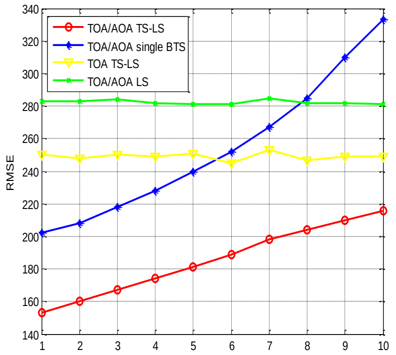

Fig. 2. RMSE as a function of the measurement error of the angle of arrival AOA

IV. SCENARIO

1) Find the estimated position utilizing the GPS/IMU module of BTS Home

$\hat{X}_K^-, \hat{Y}_K^-$ are vehicle's position that are estimated as longitude and latitude.

$$\hat{x}_k^- = \begin{bmatrix} (V_{Long}^-)_{k-1} + \Delta t.(a_{Long}^-)_k \\ (ACC_x - g.sin\theta_{k-1}^-).cos\theta_{k-1}^- \\ \theta_{k-1}^- + \Delta t.(GYRO_y)_k \end{bmatrix}$$

Where

- ✓ a_{Long} is the longitudinal acceleration
- ✓ V_{Long} is the longitudinal velocity.
- ✓ $GYRO_y$ is the y-axis angular velocity.
- ✓ $GYRO_z$ is z-axis angular velocity.
- ✓ θ is pitch angle.
- ✓ ACC_x is x-axis acceleration.
- ✓ g is acceleration of gravity.

$$z_k = \begin{bmatrix} (V_{GPS})_k \\ (V_{GPS})_k \\ \left(\tan^{-1}\frac{V_Z}{V_{XY}}\right)_k \end{bmatrix}$$

Where

from NMEA of GPS data, we have:

- ✓ Y_{GPS} is the latitude
- ✓ X_{GPS} is the longitude
- ✓ V_{GPS} is the velocity

And

- ✓ V_{XY} is velocity determined by variation of altitude and longitude.
- ✓ V_Z is velocity determined by variation of latitude

2) Compare this position to the exact position of BTS

$$\left.\begin{matrix} (x,y)_{(BTS)} \\ (x,y)_{(GPS)} \end{matrix}\right\} = (\Delta x, \Delta y)$$

3) Find the error Δx and Δy:

$$(\Delta x, \Delta y) = (x,y)_{(BTS)} - (x,y)_{(GPS)}$$

4) Calculate the estimated vehicular positioning

$$\hat{x}_k^- = \begin{bmatrix} (V_{Long}^-)_{k-1} + \Delta t.(a_{Long}^-)_k \\ (ACC_x - g.sin\theta_{k-1}^-).cos\theta_{k-1}^- \\ \theta_{k-1}^- + \Delta t.(GYRO_y)_k \end{bmatrix}$$

$$z_k = \begin{bmatrix} (V_{GPS})_k \\ (V_{GPS})_k \\ \left(\tan^{-1}\frac{V_Z}{V_{XY}}\right)_k \end{bmatrix}$$

5) Correct the position of the vehicle with the error found (Δx et Δy)

V. TRACKING ALGORITHMS

A. EKF

In this section, we illustrate the EKF and UKF[5] nonlinear filtering for trajectory tracking of the MS and to more ameliorate the localization accuracy.

EKF[6]:

The basic frame for the EKF necessitates the estimation of the state of a discrete-time nonlinear dynamic system[5],

$$\begin{cases} x_k = \dot{f}(x_k, u_k) \\ y_x = h(x_k) \end{cases}$$

where $\boldsymbol{x_k}$ is the unobserved state of the system and $\mathbf{Y_k}$ is the only observed signal. The *process* noise $\boldsymbol{u_k}$ drives the dynamic system

The idea is to approximate the nonlinear functions by linearization:

Equations of linear approximation:

Model of prediction: $\phi \approx \frac{\partial f(x,u)}{\partial x}$

Model of measures: $H_k \approx \frac{\partial h(x)}{\partial x}$

The equations are now linearized.

B. UKF

The Unscented Kalman Filter (UKF) is a recursive state estimator based on the Unscented Transform, which is a method to approach the covariance and the mean of a random variable undergoing a nonlinear change[5].

1) Numerical result

a) Simulation results for tracking algorithms

We analyze the performance of the algorithm for tracking utilizing UKF and EKF in this section.

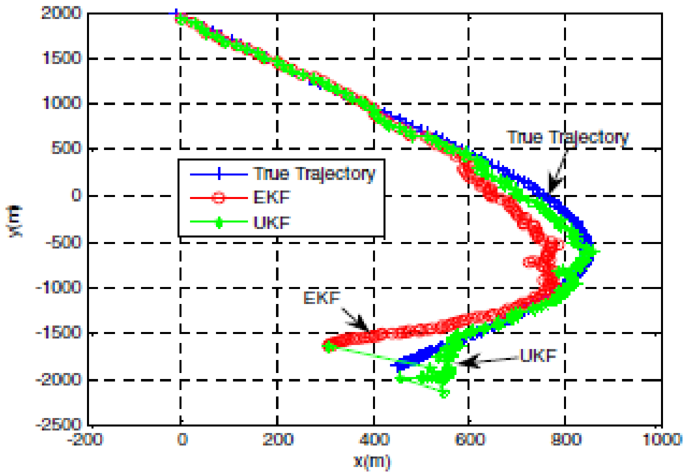

Fig.3. Sample source trajectory and tracking using the EKF and UKF

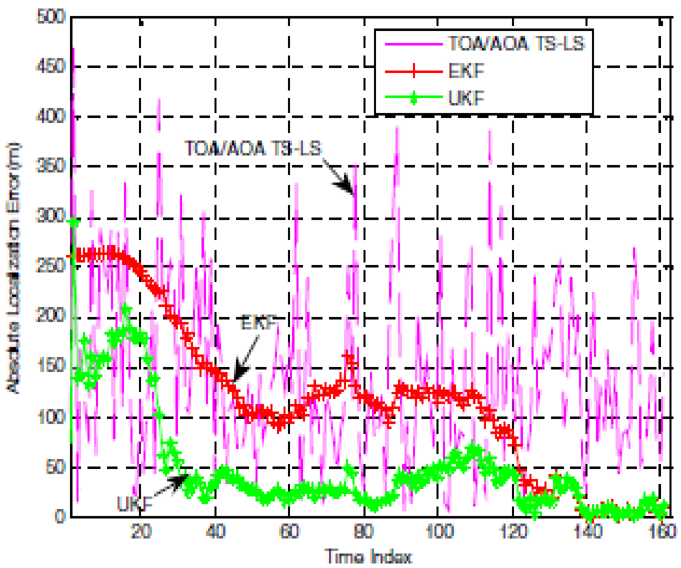

Fig.4. Absolute localization error for TOA/AOA TS-LS, EKF and UKF

VI. CONCLUSION

This document present a hybrid TOA/AOA (Time of Arrival/Angle of Arrival). The algorithm use the Taylor Series and Least Square (TS-LS) method. In addition, the module GPS/IMU can track the objects, further reducing the positioning error. Simulation results that illustrate the proposed TOA/AOA (TS-LS) can make better performance than conventional schemes in localization accuracy. UKF is generated to give closer tracking of the route than EKF that it truly takes variance of the noises and the statistical mean.

REFERENCES

[1] J. Kim, S. Lee, "A vehicular positioning with GPS/IMU using adaptive control of filter noise covariance" ICT Express 2, 2016, pp. 41-46.

[2] A. K. Wong, "An AGPS-Based Elderly Tracking System," *1st Internantional Conference on Ubiquitous and Future Networks, IEEE ICUFN, June* 2009.

[3] S.T Abbas, Z. Ahmed and A. Inam "Study on localization of moving objects Using Wireless Sensor Networks", ICTC, IEEE, 2015.

[4] C. O. Andrei and A. Kukko, "Precise carrier phase-based point positioning of boad-mounted tessestrial remote sensing platform" IEEE, 2014

[5] H. Qasem and L. Reindl, "Unscented and extended kalman estimators for nonlinear indoor tracking using distance measurements," in *Positioning, Navigation and Communication, 2007. WPNC '07. 4th Workshop on,* 2007, pp. 177-181.

[6] C. O. Andrei and A. Kukko, "Precise carrier phase-based point positioning of boad-mounted tessestrial remote sensing platform" IEEE, 2014