

Entropy analysis of a second degree fluid prescribed wall heat flux over a stretching surface in the presence of a transverse magnetic field

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Abstract— This paper presents the application of the second law analysis of thermodynamics to viscoelastic magneto hydrodynamic flow over a stretching surface where the heat process is considered namely prescribed wall heat flux. The velocity and temperature profiles are obtained by the analytical solution of highly nonlinear momentum equation and confluent hyper geometric Kummer's functions and used to compute the entropy generation number. The effects of the viscoelastic and magnetic parameter, the Prandtl number, the heat source/heat sink parameter on velocity and temperature profiles are presented. The influence of the same parameters, the Hartmann number, the dimensionless group parameter and the Reynolds number on the entropy generation are also discussed.

Keywords— Entropy analysis, Magneto-hydrodynamic, Stretching surface, Viscoelastic fluid .

I. INTRODUCTION

The studies of non-Newtonian fluids generation entropy have received much attention because the power needed in stretching a sheet and heat transfer rate in non-newtonian from those of newtonian fluid. The study of MHD flow of viscoelastic fluid over a continuously moving surface has wide range of applications in technological and manufacturing processes in industries. This concerns the production of synthetic sheets, aerodynamic extrusion of plastic sheets, cooling of metallic plates, etc.

Chang[1]and Rajagopal et al. [2] presented an analysis on flow of viscoelastic fluid over stretching sheet. Heat transfer cases of these studies have been considered by Dandapat & Gupta, [3] and Vajravelu & Rollins[4], while flow of viscoelastic fluid over a stretching surface under the influence of uniform magnetic field has been investigated by Andersson [5]. The effects of a transverse magnetic field and electric field on momentum and heat transfer characteristics in viscoelastic fluid over a stretching sheet taking into account viscous dissipation and ohmic dissipation is presented by Abel et al. [6].

The effects of non-uniform heat source, viscous dissipation and thermal radiation on the flow and heat transfer in a viscoelastic fluid over a stretching surface was considered in Prasad et al.[7]. Subhas et al. [8] analyzed the momentum and heat transfer characteristics in a hydromagnetic flow of viscoelastic liquid over a stretching sheet with non-uniform heat source. . Chen [9] studied the magneto-hydrodynamic flow and heat transfer characteristics viscoelastic fluid past a stretching surface, taking into account the effects of Joule and viscous dissipation, internal heat generation/absorption, work done due to deformation and thermal radiation. Although the forgoing research works have covered a wide range of problems involving the flow and heat transfer of viscoelastic fluid over stretching surface they have been restricted, from thermodynamic point of view, to only the first law analysis.

The contemporary trend in the field of heat transfer and thermal design is the second law of thermodynamics analysis and its related concept of entropy generation minimization. Entropy generation is closely associated with thermodynamic irreversibility, which is encountered in all heat transfer processes.

Different sources are responsible for generation of entropy such as heat transfer and viscous dissipation Bejan [10.11] The analysis of entropy generation rate in a circular duct with imposed heat flux at the wall and its extension to determine the optimum Reynolds number as function of the Prandtl number and the duty parameter were presented by Bejan [12] Mahmud & Fraser [13.14.15] applied the second law analysis to fundamental convective heat transfer problems and to non-Newtonian fluid flow through channel made of two parallel plates. The study of entropy generation in a falling liquid film along an inclined heated plate was carried out by Saouli&Aïboud-Saouli [16]. The application of the second law analysis of thermodynamics to viscoelastic magneto hydrodynamic flow over a stretching surface was carried out by Aïboud&Saouli [17.18]. Irreversibility analysis in a

couple stress film flow along an inclined heated plate with adiabatic free surface has been studied by Adesanya and Makinde [19]. Entropy generation and energy conversion rate for the peristaltic flow in a tube with magnetic field has also been investigated by Akbar [20]. Makinde [21] has investigated entropy analysis for MHD boundary layer flow and heat transfer over a flat plate with a convective surface boundary condition. Entropy analysis for an unsteady MHD flow past a stretching permeable surface in nano-fluid has been studied by Abolbashari et al. [22]. Chemical reaction effect on MHD free convective surface over a moving vertical plane through porous medium has been studied by Tripathy et al. [23]. S. Baag et al. [24] analyzed the entropy generation by applying second law of thermodynamics to magnetohydrodynamic flow, heat and mass transfer of an electrically conducting viscoelastic liquid past on a stretching surface in a porous medium.

The objective of this paper is to study the entropy generation in viscoelastic fluid over a stretching sheet with prescribed wall heat flux in the presence of uniform transverse magnetic field.

II. FORMULATION OF THE PROBLEM

In two-dimensional Cartesian coordinate system x, y we consider magneto-convection, steady, laminar, electrically conductor, boundary layer flow of a second grade fluid caused by a stretching surface in the presence of a uniform transverse magnetic field. As shown in Figure 1, the x -axis is taken in the direction of the main flow along the plate and the y -axis is normal to the plate with velocity components u, v in these directions under the usual boundary layer approximations, the governing equations are:

Continuity Equation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

Momentum Equation:

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} + K_0 \left[\frac{\partial}{\partial x} \left(u \frac{\partial^2 u}{\partial y^2} \right) - \frac{\partial u}{\partial y} \frac{\partial^2 u}{\partial x \partial y} + \frac{\partial u}{\partial y} \frac{\partial^2 u}{\partial y^2} + v \frac{\partial^3 u}{\partial y^3} \right] - \frac{\sigma B_0^2}{\rho} u \quad (2)$$

Energy Equation:

By using the usual boundary layer approximations, the equation of energy with temperature dependent heat source/sink in the flow direction is given by:

$$\rho c_p \left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k \frac{\partial^2 T}{\partial y^2} + Q(T - T_\infty) \quad (3)$$

Where Q is the rate of internal heat generation (positive) or absorption (negative).

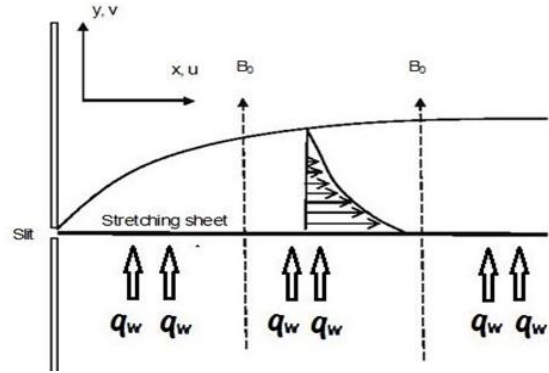


Fig.1. Physical model for the flow.

The appropriate boundary conditions for velocity field are:

$$y = 0, \quad u = u_p = \lambda x, \quad v = 0 \quad (4a)$$

$$y \rightarrow \infty, \quad u \rightarrow 0, \quad \frac{\partial u}{\partial y} \rightarrow 0 \quad (4b)$$

The thermal conditions for the energy Equation 3 are:

$$\frac{\partial T}{\partial y} = q_w = D \left(\frac{x}{l} \right)^2; \quad y = 0 \quad (5a)$$

$$T \rightarrow T_\infty; \quad \text{as } y \rightarrow \infty \quad (5b)$$

2. Analytical solution

The equation of continuity is satisfied if we choose a dimensionless stream function $\psi(x, y)$ such that

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x} \quad (6)$$

Introducing the similarity transformations

$$\eta = y \sqrt{\frac{\lambda}{\nu}}, \quad \psi(x, y) = x \sqrt{\nu \lambda} f(\eta) \quad (7)$$

Momentum Eq.2 becomes

$$f''(\eta) - ff''(\eta) = f'''(\eta) - K1(2ff'''(\eta) - f''^2(\eta) - ff^{IV}(\eta)) - Mn f'(\eta) \quad (8)$$

The boundary conditions Equation 4a and Equation 4b become:

$$K1 = \frac{K_0 \lambda}{\nu}, \quad Mn = \frac{\sigma B_0^2}{\rho \lambda} \quad (9)$$

The boundary conditions Equation 5a and Equation 5b become:

$$\eta = 0, \quad f(\eta) = 0 \text{ et } f'(\eta) = 1 \quad (10a)$$

$$\eta \rightarrow \infty, \quad f'(\eta) \rightarrow 0, \quad f''(\eta) \rightarrow 0 \quad (10b)$$

The solution of Eq.8, satisfying the boundary conditions Eq. 10a and Eq.10b is:

$$f'(\eta) = (e^{-a\eta}) \quad (a > 0) \quad \text{with } a = \sqrt{\frac{1+Mn}{1-K1}}$$

This gives the velocity component

$$u = \lambda x e^{-a\eta} \quad (11a)$$

$$v = -\sqrt{\nu \lambda} \frac{1}{a} (1 - e^{-a\eta}) \quad (11b)$$

Introducing the dimensionless temperature as:

$$\Theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \quad (12)$$

$$\text{Where } T_w - T_\infty = D \left(\frac{x}{l} \right)^2 \sqrt{\frac{v}{\lambda}} \quad (13)$$

Using Eqs.5a, b and Eq. 12-13, then the energy equation Eq. 3 becomes:

$$\Theta''(\eta) + \frac{Pr}{\alpha} (1 - e^{-a\eta}) \Theta'(\eta) - (2Pr e^{-a\eta} - \beta) \Theta(\eta) = 0 \quad (14)$$

Where $\beta = \frac{Qv}{\lambda k} Pr = \frac{C_p \mu}{k}$ are respectively the heat source/sink parameter and the Prandtl number

The corresponding thermal boundary conditions are:

$$\Theta'(\eta) = -1 \text{ at } \eta = 0 \quad (15a)$$

$$\Theta(\eta) \rightarrow 0 \text{ as } \eta \rightarrow \infty \quad (15b)$$

introducing the variable

$$\xi = -\frac{Pr}{\alpha^2} e^{-a\eta} \quad (16)$$

The solution of Eq 14 is written as:

$$\xi \Theta''(\xi) + \left(1 - \frac{Pr}{2} - \xi\right) \Theta'(\xi) + \left(2 + \frac{\beta}{\alpha^2 \xi}\right) \Theta = 0 \quad (17)$$

The boundary conditions Equation 5a and Equation 5b become:

$$\xi = -\frac{Pr}{\alpha^2} \Theta' \left(-\frac{Pr}{\alpha^2} \right) = -1 \quad (18a)$$

$$\xi = 0 \quad \Theta(0) = 0 \quad (18b)$$

The solution of Eq. (17) satisfying (18a) and (18b) is given by

$$\Theta(\eta) = \frac{-\left(\frac{-\alpha^2}{Pr} \xi\right)^{(a+b)} M(a+b-r, 2b+1, \xi)}{A_1 M(a+b-r, 2b+1, \frac{-Pr}{\alpha^2}) + A_2 M(a+b-r+1, 2b+2, \frac{-Pr}{\alpha^2})} \quad (19)$$

The solution of (19) in terms of η is written as

$$\Theta(\eta) = \frac{-e^{-\alpha(a+b)\eta} M(a+b-r, 2b+1, \frac{-Pr}{\alpha^2} e^{-a\eta})}{A_1 M(a+b-r, 2b+1, \frac{-Pr}{\alpha^2}) + A_2 M(a+b-r+1, 2b+2, \frac{-Pr}{\alpha^2})} \quad (20)$$

Where

$$A_1 = -\alpha(a+b), A_2 = \frac{Pr}{\alpha} \left(\frac{a+b-r}{2b+1} \right), \alpha = \frac{Pr}{2a^2}$$

$$b = \sqrt{\frac{Pr^2 - 4\beta a^2}{2a^2}}, M(a+b-r, 2b+1, \frac{-Pr}{\alpha^2}) \text{ is kummer function.}$$

III. SECOND LAW ANALYSIS

According to Woods [23] the local volumetric rate of entropy generation in the presence of a magnetic field is given by:

$$S_G = \frac{k}{T_0} \left[\left(\frac{\partial T}{\partial x} \right)^2 + \left(\frac{\partial T}{\partial y} \right)^2 \right] + \frac{\mu}{T_0} \left(\frac{\partial u}{\partial y} \right)^2 + \frac{\sigma B_0^2}{T_0} u^2 \quad (21)$$

Eq. (21) clearly shows contributions of three sources of entropy generation. The first term on the right-hand side of Eq. (21) is the entropy generation due to heat transfer across finite temperature difference; the second term is the local entropy generation due to viscous dissipation, whereas the third term is the local entropy generation due to the effect of the magnetic field. It is appropriate to define dimensionless number for entropy generation rate NS This number is defined by dividing the local volumetric entropy generation rate SG :

$$N_S = \frac{k T_0^2}{q^2} S_G \quad (22)$$

Using (11 a,b), (20) ,(21) then the Eq.22 can be written as

$$N_S = \frac{4}{X^2 Re_l} \Theta^2(\eta) + \Theta'^2(\eta) + \frac{Br Re_l}{\Omega} f'^2(\eta) + \frac{Br}{\Omega} (Ha^2) f'^2(\eta) \quad (23)$$

Where Re_l and Br are respectively the Reynolds number and the Brinkman number. $\Theta(\eta)$ and Ha are respectively the dimensionless temperature difference and the Hartman number. These number are given by the following relationships

$$X = x/l, \quad Re_l = u_1 l / \nu, \quad Ha = B_0 l \sqrt{\sigma / \mu}, \quad \Omega = \Delta T / T_0, \quad Br = \mu u_1^2 / k \Delta T.$$

IV. RESULTATS AND DISCUSSION

A boundary layer problem for momentum and heat transfer in a viscoelastic fluid under the influence of a transverse uniform magnetic field over stretching sheet prescribed has been solved analytically using Kummer's functions and analytic expressions of non-dimensional temperature profile for boundary condition namely prescribed wall heat flux. The velocity and temperature have been used to compute the entropy generation.

In all the figures of dimensionless temperature profiles plotted we notice that the temperature is maximum at the wall where heat flux is imposed and minimum at the free surface whatever the values of the all the parameter studied. It is clear from figure 2 and 3 that an increase of viscoelastic parameter and magnetic parameter results an increase of temperature this is due to the thermal boundary layer increases with the magnetic parameter.

Figure 4 depicts the temperature profiles $\Theta(\eta)$ as a function of η for different values of the Prandtl number Pr . As it can be noticed, temperature decreases with η whatever is the value of the Prandtl number, For a fixed value of η , the temperature $\Theta(\eta)$ decreases with an increase in Prandtl number, which means that the hydrodynamic boundary layer is thicker than the thermal boundary layer. Figure 5 shows the temperature profiles as function of η for various values of the heat source/sink parameter. For a fixed value of η , the temperature $\Theta(\eta)$ decreases with a decrease in heat source/sink. This is due to the increase of the heat generation inside the boundary layer leading to higher temperature profile.

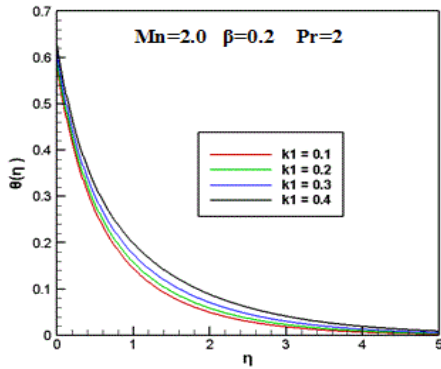


Fig. 2. Effect of the viscoelastic parameter on the temperature

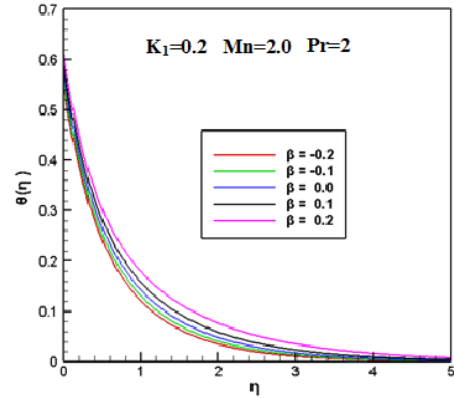


Fig.5. Effect of the heat source/sink parameter on the temperature.

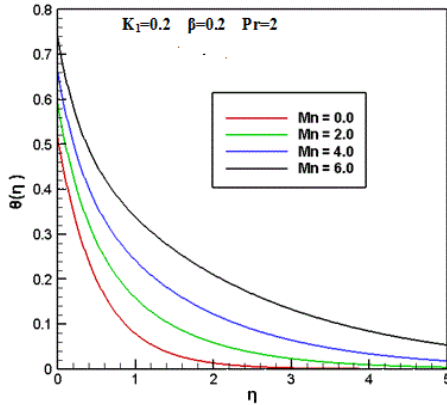


Fig. 3. Effect of the magnetic parameter on the temperature.

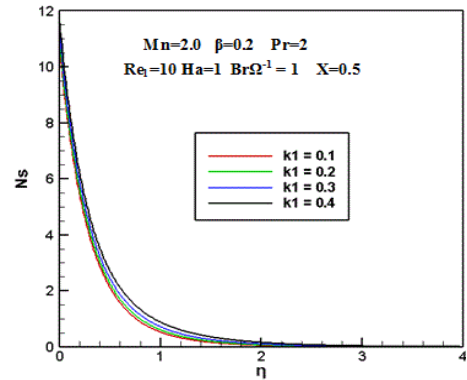


Fig.6. Effect of the viscoelastic parameter on the entropy generation number K_1

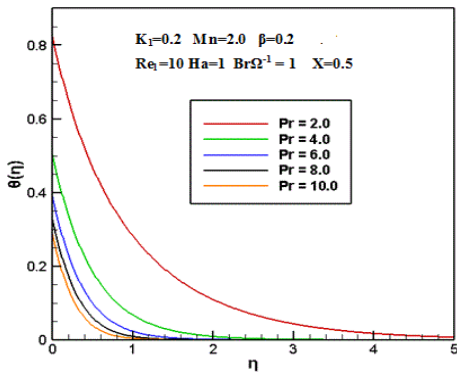


Fig. 4. Effect of the Prandtl number on the temperature.

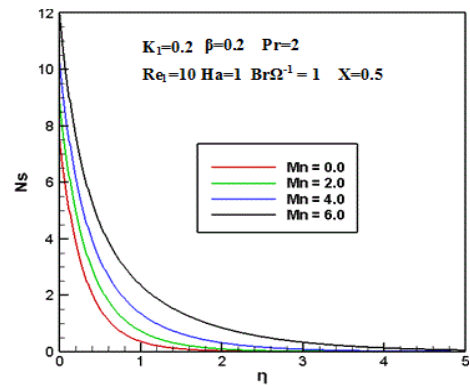


Fig.7. Effect of the magnetic parameter on the entropy generation number.

The influence of viscoelastic parameter, magnetic parameter, heat source/ heat sink on The entropy generation number N_s (Equation 23) are illustrates in Figure 6.7.8.9 we can conclude that an increase of all this parameters has increased the entropy generation number moreover this last is higher near the surface where the heat flux imposed and velocity are at their maximum values, this means that the surface acts as strong source of irreversibility.

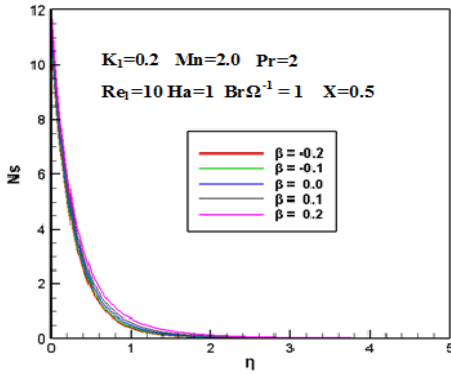


Fig.8. Effect of the heat source/ heat sink parameter on the entropy generation number.

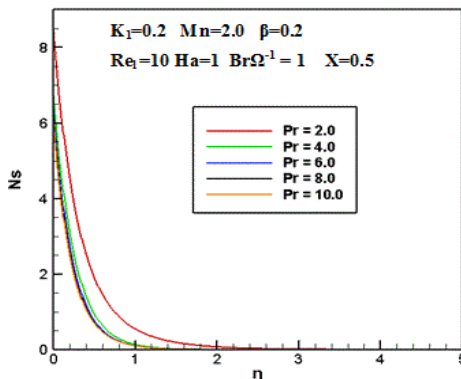


Fig.9. Effect of the Prandtl number on the entropy generation number.

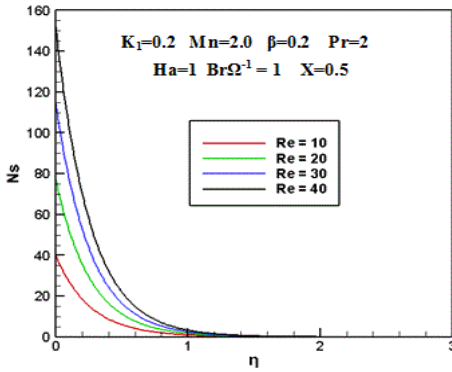


Fig.10. Effect of Reynolds number on the entropy generation number.

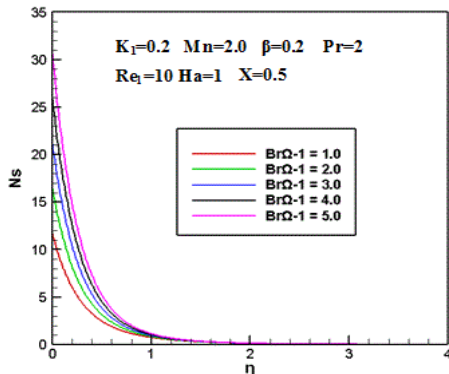


Fig.11. Effect of the dimensionless group on the entropy generation number.

The figure.9 illustrates the variations of on the entropy generation number N_s as a function of η for different values of the Prandtl number.

For a given η thickness, the entropy production decreases with the increase of the Prandtl number. This is due to the fact that the temperature decreases with the increase of the Prandtl number.

The influence of the Reynolds number Re_L on the entropy generation number N_s (first, third Equation 23) is plotted on Figure 10. For a given value of η , the entropy generation number increases as the Reynolds number increases. The augmentation of the Reynolds number increases the contribution of the entropy generation number due to fluid friction, heat transfer in the boundary layer. The effect of the dimensionless group parameter $Br\Omega^{-1}$ on the entropy generation number N_s (third and fourth term of Equation 24) is depicted in Figure 11. The dimensionless group determines the relative importance of viscous effect. For a given η , the entropy generation number is higher for higher dimensionless group. This is due to the fact that for higher dimensionless group, the entropy generation numbers due to the fluid friction.

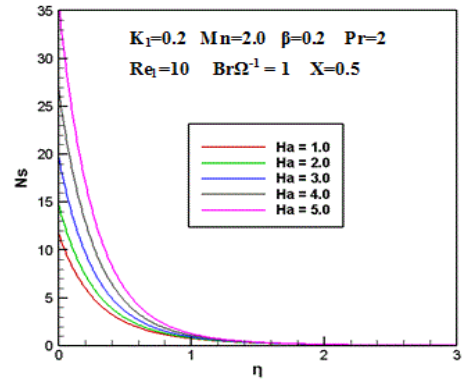


Fig.12. Effect of the Hartmann number on the entropy generation number.

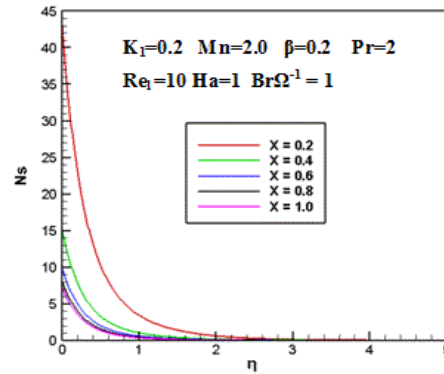


Fig.13. Effect of the characteristic length on the entropy generation number.

The effect of the Hartmann number Ha on the entropy generation number N_s (fourth term of Equation 23) is plotted in Figure 12. For a given η , as the Hartmann number increases, the entropy generation number increases. The entropy generation number is proportional to the Hartmann number which proportional to the magnetic field. The presence of the magnetic field creates additional entropy. The variation of the entropy generation with different values of characteristic length is shown on the fig.13, for fixed value an augmentation of characteristic length decreases the entropy generation this behavior can be interpreted by the energy lost inside the fluid flow.

V. CONCLUSION

The velocity and temperature profiles are obtained analytically and used to compute the entropy generation number in a viscoelastic fluid over a stretching sheet prescribed wall heat flux subject to a transverse magnetic field.

The effects of the magnetic parameter and the viscoelastic parameter on the longitudinal and transverse velocities are discussed. The influences of the Prandtl number, the magnetic parameter and the heat source/sink parameter on the temperature profiles are presented. The dependence of the magnetic parameter is also presented. As far as the entropy generation number is concerned, its dependence on the magnetic parameter, the Prandtl number, the Reynolds number, the dimensionless group, the Hartmann number, the ratio of the dimensionless concentration difference to the dimensionless temperature difference and the constant parameter are illustrated and analyzed.

From the results the following conclusions could be drawn:

(a) The longitudinal and the transverse velocities decrease as the magnetic parameter and the viscoelastic parameter increase.

(b) The temperature increases as the viscoelastic, the magnetic and heat source sink parameter increases, but it decreases as the Prandtl number increases.

(c) The entropy generation number increases as Hartman number, dimensionless group parameter and Reynolds number increase.

(d) The entropy generation number is slightly influenced by Prandtl number, viscoelastic parameter and heat source/heat sink parameter.

(e) The surface acts as a strong source of irreversibility.

Nomenclatures

a	constant
b	constant
B_0	uniform magnetic field strength
Br	Brinkman number
C_p	specific heat of the fluid
f	Dimensionless function
Ha	Hartman number
k	thermal conductivity of the fluid
K_1	Viscoelastic parameter
Mn	Magnetic parameter
N_s	Entropy generation number
Pr	Prandtl number
Q	heat generation coefficient
u	Axial velocity
l	Characteristic length
Ω	Dimensionless temperature difference
S_G	local volumetric rate of entropy

	generation
S_{G0}	Characteristic volumetric rate of entropy generation
T	Temperature
σ	Electric conductivity subscripts
Re_1	Reynolds number based on the characteristic length
u_p	Plate velocity
v	Transverse velocity
x	Axial distance
y	transverse distance
a	Positive constant
$Br\Omega^{-1}$	Dimensionless group parameter heat
β	Source/heat sink parameter
λ	Proportional constant
ξ	Dimensionless variable
η	Dimensionless variable
μ	Dynamic viscosity of the fluid
ν	Kinematic viscosity of the fluid
ΔT	Temperature difference
Θ	Dimensionless temperature
ρ	Density of the fluid
∞	Far from sheet
u_1	Plate velocity based on the characteristic length
P	Plate

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