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Digital Wiener Filtering for Signal and Contour Processing

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***Abstract*-- In this paper, the digital equivalent of wiener filter is used for the filtering of image contours represented in X, Y Cartesian coordinates, using different fast transforms. The Wiener filter is a linear estimator and minimizes the mean-squared error between the original and filtered signal. Each coordinate is assumed to be disturbed by a white noise at different SNR levels and treated separately. The paper describes the procedure to find relative spectrum of each transform. The relative spectrum is obtained using a generalised fourier series related to the harmonic basis functions and the transform functions. The filter matrix G_m is found for each of the considered transforms. Mean square error is the criterion of performance. Results show that transforms with sinusoidal basis functions give much better results.**

***Keywords*-- Wiener filter, Signal filtering, Spectral filtering, Contour processing, Image analysis, Fast transforms.**

I. INTRODUCTION

It is well known that spectral filtering of a discrete signal x (vector of dimension N of samples) can be accomplished using the fast Fourier transform (FFT). The type of filtering considered here is the digital equivalent of Wiener filtering [1]. Assuming that the input data to the filter X(n) is a discrete random process composed of a useful part S(n) and noise Z(n).

In the generalised Wiener filtering process a type of transformation, such as Fourier transform, is performed on the data [2]. The transformed data is then modified (filtered) by the filter function G_m, and finally the inverse transformation is performed to get the new spectrally filtered signal as shown in Fig.1, G_m is a diagonal NxN gain matrix.

The transforms considered for filtering in this investigation are Fourier, Hartley [3], Walsh-Hadamard, and Periodic Walsh Piecewise-Linear (PWL) transforms [4]. Two factors are affecting the choice of the last three transforms. The first is their ability of fast computation, where, computation time is largely governed by the required number of multiplications, this fact made Walsh and PWL transforms faster than Fourier and Hartley [4],[5]. The second factor is for using three different types of basis functions, namely, Sinusoidal, stepwise, and Piecewise-linear basis functions.

For many applications which deal with digital image processing, the details given by gray scale images are not required, only the contours of objects are needed [6], [7], getting undistorted contours is the first step for object recognition and analysis. The image, itself, may get distorted, and even if the contour was extracted from a pure image, the coordinates of the contour may get distorted during storage or transmission.

Data **x** is assumed to be composed of a random sequence **s** corrupted by white noise **z** components. Both **s** and **z** are assumed to have zero mean and to be uncorrelated [5].

$$X(n) = S(n) + Z(n) \quad for\ n = 0,1, \dots, N-1 \qquad (1)$$

Where $X(n)$ -- Input signal to the filter,
$S(n)$ -- Useful (original) signal,

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$Z(n)$ -- White noise.

The filter function is to provide the best mean square estimate of the signal in the input data. The elements of the diagonal filtering matrix $\boldsymbol{G_m}$ are selected using the mean square error as a criterion, the best elements of G are those which give minimum mean square error, where:

$$MSE = E\{[G_m(n,n).X_\emptyset(n) - S_\emptyset(n)]^2\} \qquad (2)$$

Differentiating Eq.(2) with respect to $G_m(n,n)$, gives a formula with which, we can determine the optimal values of the diagonal elements of the filter matrix $\boldsymbol{G_m}$ for minimising the error. By assuming that the signal S(n) and noise Z(n) are uncorrelated and using Fourier transform we get:

$$G_{mF}(n,n) = \frac{E[S_F^2(n)]}{E\left[S_F^2(n)+E[Z_F^2(n)]\right]} \qquad (3)$$

The terms $E[S_F^2(n)]$ *and* $E[Z_F^2(n)]$ are interpreted as power spectral density of the original signal and noise, respectively. The power spectral density of a stationary random process can be estimated numerically by computing the squared magnitude of the discrete Fourier transform (DFT) and dividing by the number of data samples [5].

$$P_F(n) = \frac{|S_F(n)|^2}{N} \qquad (4)$$

The power spectral density function, in frequency domain, $P_F(n)$ describes the distribution of power versus frequency. Power spectrum may be directly obtained using the harmonic basis functions (Fourier transform) or, with respect to other transforms, using the so called relative spectrum which to be derived in the next section.

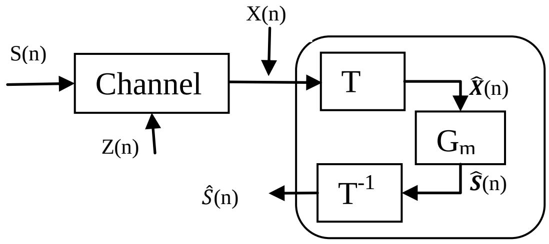

Fig.1. Block diagram of the filtering process

II. RELATIVE SPECTRUM

The filter matrix $\boldsymbol{G_{m\emptyset}}$ is to be found for each of the considered transforms, using the relative spectrum of the transform. The relative spectrum is obtained using a generalised Fourier series related to the harmonic basis functions and the transform functions $\emptyset$. The generalised Fourier series has the form:

$$X(n) = \sum_{k=0}^{N-1} S_F(k).e^{wkn} = \sum_{i=0}^{N-1} S_\emptyset(i).\emptyset(i,n) \qquad (5)$$

For $w = \frac{j2\pi}{N}$ *and* $n = 0,1, \dots \dots N-1$

Eq.(5) says that, having two different transforms for recovering a signal from its related spectrum give the same result, where;

$$S_F(k) = \frac{1}{N}\sum_{n=0}^{N-1} x(n).e^{-wkn} \qquad (6a)$$

And

$$S_\emptyset(i) = \frac{1}{N}\sum_{n=0}^{N-1} x(n).\emptyset(i,n) \qquad (6b)$$

substituting the relation in Eq.(5) in to Eq.(6b) gives

$$S_\emptyset(i) = \frac{1}{N}\sum_{n=0}^{N-1}[\sum_{k=0}^{N-1} S_F(k).e^{wkn}].\emptyset(i,n) \qquad (7a)$$

Reordering of this equation gives

$$S_\emptyset(i) = \frac{1}{N}\sum_{k=0}^{N-1} S_F(k).\sum_{n=0}^{N-1} e^{wkn}.\emptyset(i,n) \qquad (7b)$$

And by defining the relative spectrum:

$$d_\emptyset(i,k) = \sum_{n=0}^{N-1} e^{wkn}.\emptyset(i,n) \qquad (8)$$

Eq.(7b) can be rewritten as:

$$S_\emptyset(i) = \frac{1}{N}\sum_{k=0}^{N-1} S_F(k).d_\emptyset(i,n) \qquad (9)$$

Using Eq.(9) instead of Eq.(6b) in the relation of Eq.(3), where spectral elements of $S_\emptyset(n)$ are not correlated, gives the following general equation;

$$G_{m\emptyset}(i,i) = \frac{\sum_{k=0}^{N-1}|d_{\emptyset}(i,k)|^2 E|S_F(k)|^2}{\sum_{k=0}^{N-1}|d_{\emptyset}(i,k)|^2\{|S_F(k)|^2 + E|Z_F(k)|^2\}} \qquad (10)$$

By using the relative spectrum in Eq.(8) we find the elements of the filter matrix $\boldsymbol{G}_{m\emptyset}$ as defined above. The relative spectrum $d_{\emptyset}(i,k)$ can be found for each of the selected transforms as follows:

A. Hartley Domain Filtering

The Hartley transform has orthogonal sinusoidal basis functions as with Fourier.

$$d_{Hrt}(i,k) = \sum_{n=0}^{N-1} e^{wkn}.Hrt(i,n) \qquad (11)$$

Where $w = \frac{j2\pi}{N}$ and $i,k = 0,1,\ldots,N-1$

B. Walsh Domain Filtering

The Walsh transform has orthogonal stepwise basis functions.

$$d_{Wal}(i,k) = \sum_{n=0}^{N-1} e^{wkn}.Wal(i,n) \qquad (12)$$

for w,i,k as above.

C. PWL Domain Filtering

From the definition of the Periodic Walsh Piecewise-Linear transform [3], the basis functions of this transform are piecewise linearly independent but not orthogonal.

$$S_{pwl}(i) = \frac{-1}{2^{m+1}} \sum_{n=0}^{N-1} x(n).W\hat{al}(i,n) \qquad (13)$$

where m is the group number, therefore

$$d_{pwl}(i,k) = \frac{-1}{2^{m+1}} \sum_{n=0}^{N-1} e^{wkn}.W\hat{al}(i,n) \qquad (14)$$

for w,i,k as in the previous case.

The above equations, of relative spectrum, are used to find the power spectrum in the related transform domain, applying any one of these equations in Eq.(10) gives the matrix $\boldsymbol{G}_{m\emptyset}$ of the related transform filter.

III. RESULTS ANALYSIS

In fig.2, one of the tested contours, which was extracted from an image of a dog, is shown with its distorted case for SNR = 14 dB.

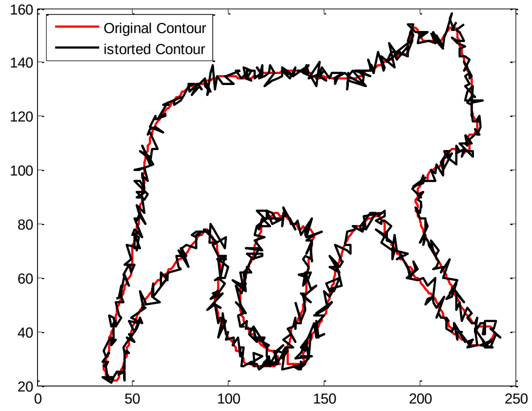

Fig.2. The Original and distorted contours

The x and y vectors are contaminated by different samples of white noise as shown in fig.3.

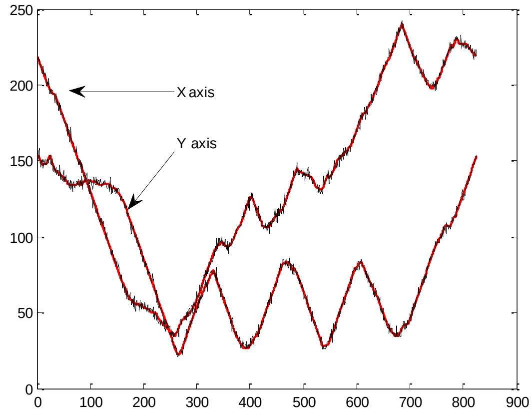

Fig.3. Original and distorted coordinates of contour dog

Combining the filtered x and y coordinates yields the filtered contour. For the case of SNR = 14 dB, the filtered contour and the original one are shown in Fig.4. using Hartley transform .

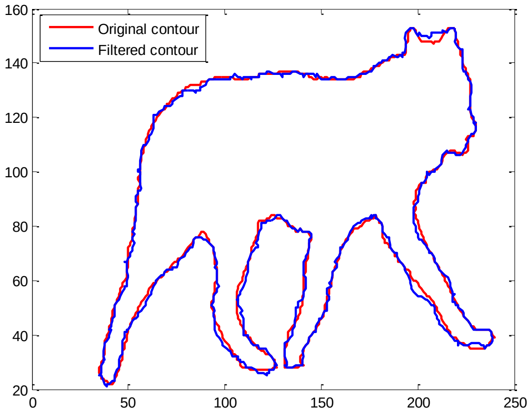

Fig.4. Original and filtered contour

The performance results of using Fourier, Hartley and Walsh-Hadamard (orthogonal) transforms and that of using PWL (non-orthogonal) transform, are depicted in Figs.5 and 6. In both figures, the MSE using Walsh and PWL transforms are close to each other, while they are almost twice that of Fourier and Hartley transforms.

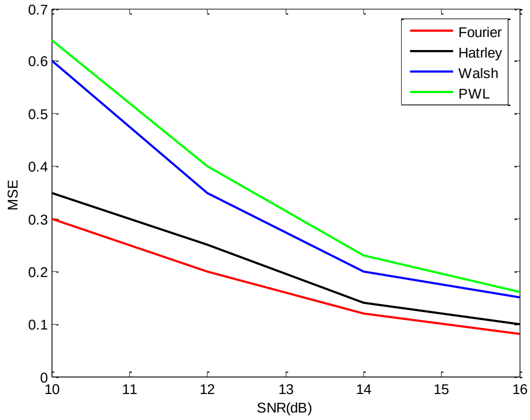

Fig.5. MSE versus SNR(dB) for filtering of vector x

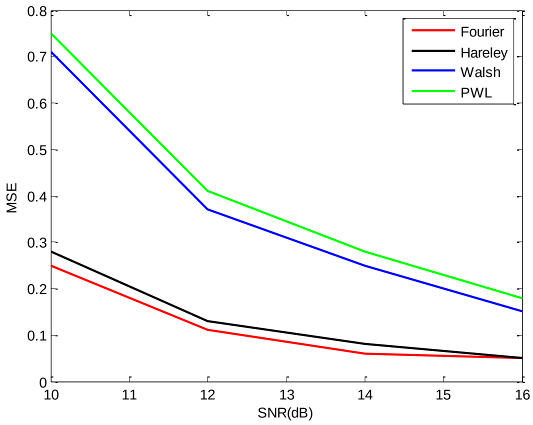

Fig.6. MSE versus SNR(dB) for filtering of vector y

This is the mean square error (MSE) of the two vectors (x and y). The next step is to find the mean square error of the filtered contour itself, where each point of the contour is affected by the change of its (x, y) Cartesian coordinates.

The distance of shift can be found by applying the Cartesian coordinates of the contour to the following well known formula:

$$PD(i) = \sqrt{[x_o(i) - x_f(i)]^2 + [y_o(i) - y_f(i)]^2} \qquad (15)$$

for i=1,2,…..N

where subscript (o) for original signal.

and (f) for filtered signal.

From the knowledge of the shift error $PD(i)$, the mean square error (MSE) of the filtered contour can be found by

$$MSE = \frac{1}{N} . \sum_{i=1}^{N} [PD(i)]^2 \qquad (16)$$

The results of the filtered contour, for different transforms and different values of signal-to-noise ratio (SNR) are shown in Fig.7. Comparison of the results of this figure with that of Figs.5 and 6 describes the relation between the contour and its Cartesian coordinates.

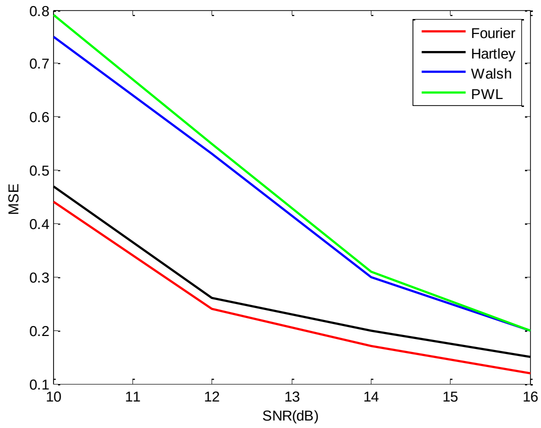

Fig.7. MSE versus SNR(dB) for contour filtering

IV. CONCLUSIONS

From the obtained results, it is clear that Fourier and Hartley transforms give better results than Walsh and PWL transforms in the sense of mean square error. As mentioned above, both Fourier and Hartley transforms have sinusoidal basis functions. In general, the MSE of the filtered contour is a bit larger than the MSE of each of its combined coordinates. The results show that digital filtering depends to a high degree on the type of applied transformation determining the spectrum domain of filtering.

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