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Fixed-Time Tracking Control of Chained-form Nonholonomic System with External Disturbances

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***Abstract*—This paper studies the fixed-time tracking problem for chained-form nonholonomic systems under matched external disturbances. The proposed method is to construct a tracking controller such that the tracking errors converge to zero for any arbitrary initial tracking error at fixed-time. First of all, the resulting tracking error dynamics is transformed into two second-order coupled subsystems. Then, the two subsystem are studied and fixed-time control laws are designed. An upper bound of the settling time, which only depends on the controller parameters is estimated regardless of the initial conditions. Finally, some simulation results are given to show the effectiveness of the proposed controller.**

I. Introduction

Control of nonholonomic systems has been an active research topic during the last decades due to its large number of applications (mobile robots [1], bicycle [2], underactuated ship [3], [4], mobile manipulator [5], hovercraft [6], etc). Indeed, control of these systems presents significant challenges due to the corresponding differential constraints [7], [8]. From the Brockett's theorem [9], nonholonomic system cannot be stabilized at an equilibrium point by pure smooth (or even continuous) state feedback controller [10]. Works on stabilization and trajectory tracking for such systems have been mainly divided into two directions: smooth time varying feedbacks [11], [12] and discontinuous controllers [13]. An interesting transformation of mechanical systems with nonholonomic constraints to chained-form systems was discussed in [14].

An interesting research topic in the area of stabilization and trajectory tracking is the convergence rate analysis. Indeed, the convergence rate is a significant performance index to evaluate the effectiveness of the control algorithms. The aims is to obtain a fast convergence rate of the tracking errors. Most of the existing researches in trajectory tracking focus on asymptotic [15] or finite-time [16] convergence. In [17], a recursive terminal sliding mode strategy was proposed to solve the trajectory tracking problem for disturbed chained-form nonholonomic systems in finite-time. However, when the convergence is asymptotic, the tracking errors converge to zero when time approaches to infinity. When the convergence occurs in finite-time, the errors goes to zero in a finite time which depends on the initial conditions.

Fixed-time stability was recently proposed to define algorithms which guarantee that the settling time is upper bounded regardless to the initial conditions [18]. Many results were recently introduced to design fixed-time controllers and observer for some classes of linear systems [19], [20]. The fixed-time stabilization problem for nonholonomic systems in chained form was firstly studied in [21]. Based on sliding mode theory, a nonlinear switching controller was proposed to ensure the fixed-time convergence. Motivated by this work, we investigate the fixed-time trajectory tracking problem. It should be noted that the extension of the work in [21] to the trajectory tracking problem is not trivial due to the nonholonomic constraint.

In this paper, we will consider the fixed-time trajectory tracking problem for chained-form nonholonomic systems. A switching controller, based on two stages, is designed to track the desired trajectory in a prescribed time. It should be noted that an explicit expression of the switching time for the proposed controller is provided. Using the proposed controller, an upper bound of the settling time is provided regardless of initial conditions.

The paper is organized as follows. In Section 2, some preliminaries on fixed-time stability are given. In Section 3, the trajectory tracking problem is formulated. In Section 4, the controller design which solves the trajectory tracking problem is discussed for chained-form nonholonomic systems. In Section 5, some simulations results are given to show the effectiveness of the proposed controller.

II. Recalls on fixed-time stability

Let us consider system

$$\begin{cases} \dot{x}(t) &= F(t,x(t)) \\ x(0) &= x_0 \end{cases} \tag{1}$$

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where $x \in \mathbb{R}^n$ is the state, $F : \mathbb{R}^+ \times \mathbb{R}^n \to \mathbb{R}^n$ is a nonlinear function and $F(t,0) = 0$ for $t > 0$. The solution of (1) are understood in the Filippov sense [22].

***Definition** 1:* [23] The origin of system (1) is a globally finite-time equilibrium if there is a function $T : \mathbb{R}^n \to \mathbb{R}^+$ such that for all $x_0 \in \mathbb{R}^n$, the solution $x(t,x_0)$ of system (1) is defined and $x(t,x_0) \in \mathbb{R}^n$ for $t \in [0, T(x_0))$ and $\lim_{t \to T(x_0)} x(t,x_0) = 0$. $T(x_0)$ is called the settling time function.

***Definition** 2:* [18] The origin of system (1) is a globally fixed-time equilibrium if it is globally finite-time stable and the settling time function $T(x_0)$ is bounded by a positive number $T_{max} > 0$, i.e. $T(x_0) \leq T_{max}$, $\forall x_0 \in \mathbb{R}^n$

***Lemma** 1:* [18] Assume that there exists a continuously differentiable positive definite and radially unbounded function $V : \mathbb{R}^n \to \mathbb{R}^+$ such that

$$\dot{V}(x) \leq -\alpha V^p(x) - \beta V^q(x) \tag{2}$$

with $\alpha > 0$, $\beta > 0$, $0 < p < 1$ and $q > 1$. Then, the origin of system (1) is globally fixed-time stable with settling time estimate

$$T(x_0) \leq T_{max} = \frac{1}{\alpha(1-p)} + \frac{1}{\beta(q-1)} \tag{3}$$

***Remark** 1:* [24] If $p = 1 - \frac{1}{\mu}$ and $q = 1 + \frac{1}{\mu}$ with $\mu \geq 1$, the settling time can be estimated by a less conservative bound:

$$T(x_0) \leq T_{max} = \frac{\pi\mu}{2\sqrt{\alpha\beta}} \tag{4}$$

III. Problem Statement

Consider the nonholonomic system in chained-form dynamics

$$\begin{array}{rcl} \dot{x}_1(t) & = & x_2(t) \\ \dot{x}_2(t) & = & u_1(t) + d_1(t) \\ \dot{x}_3(t) & = & x_4(t)x_2(t) \\ \dot{x}_4(t) & = & u_2(t) + d_2(t) \end{array} \tag{5}$$

where $x = [x_1, x_2, x_3, x_4]^T \in \mathbb{R}^4$ (resp. $u = [u_1, u_2]^T \in \mathbb{R}^2$) is the state (resp. control input) of the chained-form nonholonomic system, $d = [d_1, d_2]^T \in \mathbb{R}^2$ represents the unknown disturbances of the chained-form dynamics.

The dynamics of the desired trajectory is generated using the following system:

$$\begin{array}{rcl} \dot{x}_{1,d}(t) & = & x_{2,d}(t) \\ \dot{x}_{2,d}(t) & = & u_{1,d}(t) \\ \dot{x}_{3,d}(t) & = & x_{4,d}(t)x_{2,d}(t) \\ \dot{x}_{4,d}(t) & = & u_{2,d}(t) \end{array} \tag{6}$$

where $x_d = [x_{1,d}, x_{2,d}, x_{3,d}, x_{4,d}]^T \in \mathbb{R}^4$ (resp. $u_d = [u_{1,d}, u_{2,d}]^T \in \mathbb{R}^2$) is the state (resp. control input) of the desired trajectory.

Here, the control objective is to design a control law u which makes the tracking errors become zero in a fixed time T where disturbances are considered. It means that there exists a constant T such that

$$\begin{array}{l} \lim_{t \to T} \|x(t) - x_d(t)\| = 0 \\ x(t) = x_d(t), \ \forall t \geq T \end{array} \tag{7}$$

In order to solve the fixed-time trajectory tracking problem, the following assumptions are set.

***Assumption** 1:* The unknown disturbance is bounded as follows

$$\begin{cases} |d_1(t)| & \leq & d_1^{max} \\ |d_2(t)| & \leq & d_2^{max} \end{cases} \tag{8}$$

***Assumption** 2:* It is assumed that the desired velocity $u_{1,d}$ is differentiable and the desired trajectory satisfies the following condition

$$x_{2,d} \neq 0 \tag{9}$$

***Remark** 2:* Assumption 1 is not restrictive since the upper bounds of perturbation can be obtained a priori for any physical system. Assumption 2 restricts the desired trajectory.

IV. Fixed-time trajectory tracking controller

In this section, a new fixed-time trajectory tracking controller is proposed for chained-form nonholonomic systems with external disturbances.

Let us define the tracking errors as

$$e(t) = x(t) - x_d(t) \tag{10}$$

with $e = [e_1, e_2, e_3, e_4]^T \in \mathbb{R}^4$. The tracking error dynamics satisfy the following differential equations:

$$\begin{array}{lrcl} (\Sigma_1) & \dot{e}_1(t) & = & x_2(t) - x_{2,d}(t) \\ & \dot{e}_2(t) & = & u_1(t) + d_1(t) - u_{1,d}(t) \\ (\Sigma_2) & \dot{e}_3(t) & = & x_4(t)x_2(t) - x_{4,d}(t)x_{2,d}(t) \\ & \dot{e}_4(t) & = & u_2(t) + d_2(t) - u_{2,d}(t) \end{array} \tag{11}$$

To simplify the controller design, dynamics (11) is divided into two second-order coupled subsystems. To solve the fixed-time trajectory tracking problem, two steps are defined:

- Stabilization of subsystem Σ_1 in a fixed time T_s using control u_1,
- After $t > T_s$, stabilization of subsystem Σ_2 in a fixed time T using control u_2.

To design the fixed-time consensus tracking algorithm for the second-order subsystems, the following theorem is derived.

***Theorem** 1:* Consider system (5) with the trajectory tracking control law defined as:

$$u_1 = u_{1,d} + \varphi_1(e_1, e_2)$$

$$u_2 = \begin{cases} 1 & ,\forall t < T_s \\ u_{2,d} - \frac{e_4 u_{1,d}}{x_{2,d}} + \frac{1}{x_{2,d}}\varphi_2(e_3, \zeta_4) & ,\forall t \geq T_s \end{cases} \quad (12)$$

with $\zeta_4 = e_4 x_{2,d}$.

The sliding mode controllers are as follows:

$$\begin{aligned} \varphi_1(e_1,e_2) &= -\tfrac{\alpha_1+3\beta_1 e_1^2+2d_{1,max}}{2} sign(s_1(e_1,e_2)) \\ &\quad -\lfloor \alpha_2 s_1(e_1,e_2) + \beta_2 \lfloor s_1(e_1,e_2) \rceil^3 \rceil^{\frac{1}{2}} \\ \varphi_2(e_3,\zeta_4) &= -\tfrac{\alpha_1+3\beta_1 e_3^2+2d_{2,max}}{2} sign(s_2(e_3,\zeta_4)) \\ &\quad -\lfloor \alpha_2 s_2(e_3,\zeta_4) + \beta_2 \lfloor s_2(e_3,\zeta_4) \rceil^3 \rceil^{\frac{1}{2}} \end{aligned} \quad (13)$$

with sliding surfaces

$$\begin{aligned} s_1(e_1,e_2) &= e_2 + \lfloor \lfloor e_2 \rceil^2 + \alpha_1 e_1 + \beta_1 \lfloor e_1 \rceil^3 \rceil^{\frac{1}{2}} \\ s_2(e_3,\zeta_4) &= \zeta_4 + \lfloor \lfloor \zeta_4 \rceil^2 + \alpha_1 e_3 + \beta_1 \lfloor e_3 \rceil^3 \rceil^{\frac{1}{2}} \end{aligned} \quad (14)$$

The switching time is $T_s = \frac{2}{\sqrt{\alpha_2}} + \frac{2}{\sqrt{\beta_2}} + \frac{2\sqrt{2}}{\sqrt{\alpha_1}} + \frac{2\sqrt{2}}{\sqrt{\beta_1}}$ and constants α_i, β_j $(i = 1,2\ j = 1,2)$ are positive.

Then, the origin of system (11) is globally fixed-time stable with settling time can be given by:

$$T = 2T_s \quad (15)$$

Hence, the fixed-time trajectory tracking problem is solved.

Proof. The proof is divided into two steps.

- Let us first consider the time interval $t \in [0, T_s]$. Using controller (12), subsystem Σ_1 becomes

$$\begin{aligned} \dot{e}_1 &= e_2 \\ \dot{e}_2 &= \varphi_1(e_1,e_2) + d_1(t) \end{aligned} \quad (16)$$

Following [18], let us consider the Lyapunov function candidate $V_1 = |s_1|$. Its upper right-hand Dini derivative along the system trajectories is for $s_1 \neq 0$,

$$D^*V_1 = \dot{e}_2 sign(s_1) + \frac{|e_2|\dot{e}_2 sign(s_1) + \frac{\alpha_1+3\beta_1 e_1^2}{2} e_2 sign(s_1)}{\lfloor \lfloor e_2 \rceil^2 + \alpha_1 e_1 + \beta_1 \lfloor e_1 \rceil^3 \rceil^{\frac{1}{2}}}$$

$$= (\varphi_1 + d_1) sign(s_1) + \frac{|e_2|(\varphi_1 + d_1) sign(s_1) + \frac{\alpha_1+3\beta_1 e_1^2}{2} e_2 sign(s_1)}{\lfloor \lfloor e_2 \rceil^2 + \alpha_1 e_1 + \beta_1 \lfloor e_1 \rceil^3 \rceil^{\frac{1}{2}}}$$

Since

$$\lfloor \alpha_2 s_1 + \beta_2 \lfloor s_1 \rceil^3 \rceil^{\frac{1}{2}} sign(s_1) = (\alpha_2 |s_1| + \beta_2 |s_1|^3)^{\frac{1}{2}}$$

we have

$$\dot{e}_2 sign(s_1) = -\frac{\alpha_1 + 3\beta_1 e_1^2}{2} - (\alpha_2|s_1| + \beta_2|s_1|^3)^{\frac{1}{2}} - (d_{1,max} - d_1 sign(s_1))$$

for $s_1 \neq 0$. Hence, using Assumption 1,

$$D^*V_1 \leq -(\alpha_2 V_1 + \beta_2 V_1^3)^{\frac{1}{2}} \quad (17)$$

From Lemma 1, one can conclude that $s_1 = 0$ for all $t \geq T_{s1} = \frac{2}{\sqrt{\alpha_2}} + \frac{2}{\sqrt{\beta_2}}$

In sliding mode, i.e. $s_1 = 0$, the dynamics become

$$\dot{e}_1 = -\left\lfloor \tfrac{\alpha_1 e_1 + \beta_1 \lfloor e_1 \rceil^3}{2} \right\rceil^{\frac{1}{2}}$$

Let us consider the Lyapunov function candidate $V_2 = |e_1|$. Its upper right-hand Dini derivative along the system trajectories is

$$D^*V_2 = -(\frac{\alpha_1}{2} V_2 + \frac{\beta_1}{2} V_2^3)^{\frac{1}{2}} \quad (18)$$

From Lemma 1, one can conclude that $e_1 = 0$ for all $t \geq T_s$. One should note if $e_1 = 0$ and $s_1 = 0$, then $e_2 = 0$.

- Let us now consider $t > T_s$. From previously, subsystem Σ_1 becomes

$$\begin{aligned} \dot{e}_1 &= e_2 = 0 \\ \dot{e}_2 &= u_1 + d_1 - u_{1,d} = 0 \end{aligned}$$

Hence, system Σ_2 can be written as:

$$\begin{aligned} \dot{e}_3 &= e_4 x_{2,d} \\ \dot{e}_4 &= u_2 + d_2 - u_{2,d} \end{aligned} \quad (19)$$

Setting $\zeta_4 = e_4 x_{2,d}$, system (19) can be expressed as:

$$\begin{aligned} \dot{e}_3 &= \zeta_4 \\ \dot{\zeta}_4 &= (u_2 + d_2 - u_{2,d}) x_{2,d} + e_4 u_{1,d} \end{aligned} \quad (20)$$

Using controller (12), system (20) becomes:

$$\begin{aligned} \dot{e}_3 &= \zeta_4 \\ \dot{\zeta}_4 &= \varphi_2 + d_2 x_{2,d} \end{aligned} \quad (21)$$

Using Assumption 1 and following the same procedure as in the first step, one can conclude that the origin of system (20) is globally fixed-time state with settling time T given by Eq. (15). From Assumption 2, it is clear that the origin of system (11) is globally fixed-time stable with settling time T.

■

***Remark** 3:* It should be highlighted that since the settling time T is independent of the initial system conditions and can be estimated a priori, global finite-time stability of the closed-loop system is guaranteed (contrary to many existing works which only guarantee semi-global finite-time stability)

V. Simulation Results

In this section, some simulation results are provided to verify the theoretical analysis.

We consider the nonholonomic system in chained-form (5) where the perturbations are $d_1 = \sin(20t)$ and $d_2 = 10\cos(10t)$. The desired trajectory is generated by (6) with $x_d(0) = [0,2,0,0]^T$, $u_{1,d}(t) = 2 + sin(t)$ and $u_{2,d}(t) = 1$. The control objective is that system (5) follows its derived trajectory x_d. It is clear that Assumptions 1-2 are verified. The control parameter are selected as: $\alpha_1 = 20$, $\alpha_2 = 10$, $\beta_1 = 20$, $\beta_2 = 10$ and $T_s = 2.5s$. For the simulation purpose, the initial conditions of system (5) are set as: $x(0) = [10,1,3,1]^T$. Using Theorem 1, the robust controller (12) solves the fixed-time trajectory tracking problem with an estimation of the settling time $T = 4.5s$. Figures 1-2 show that the origin of subsystem Σ_1 is globally fixed-time stable with a settling time less than $T_s = 2.5s$.

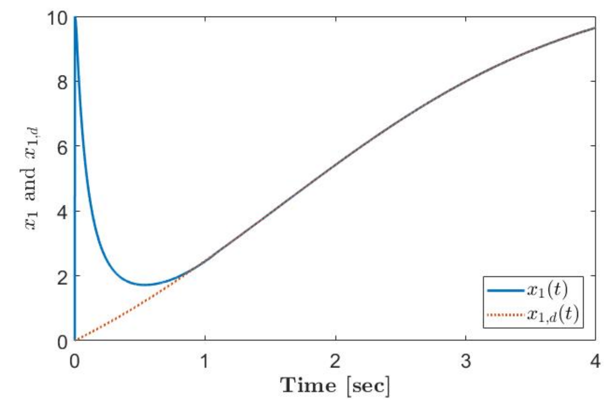

Fig. 1. Actual trajectory x_1 and desired state trajectories $x_{1,d}$.

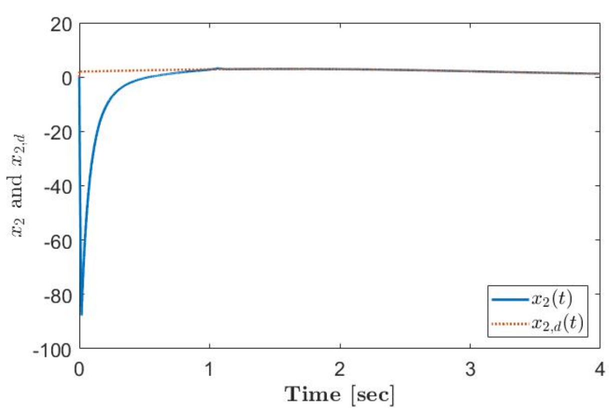

Fig. 2. Actual trajectory x_2 and desired state trajectories $x_{2,d}$.

Figures 3-4 show that the origin of subsystem Σ_2 is globally fixed-time stable with a settling time less than $T = 4.5s$.

Figure 5 displays the tracking errors. It can be seen that the trajectory tracking problem is solve in fixed-time.

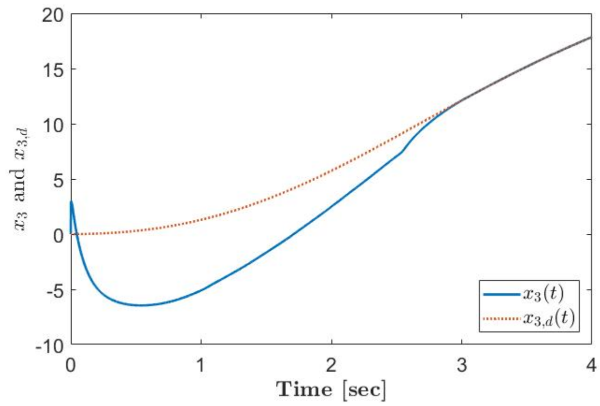

Fig. 3. Actual trajectory x_3 and desired state trajectories $x_{3,d}$.

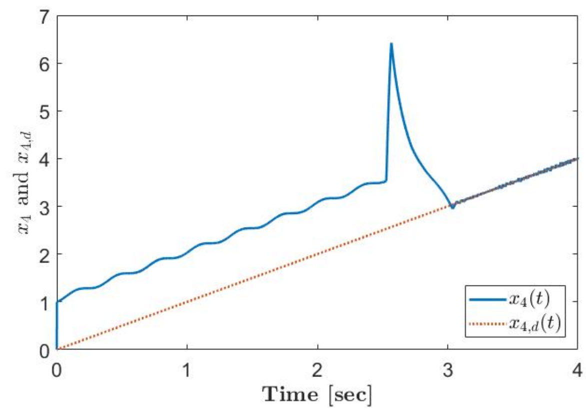

Fig. 4. Actual trajectory x_4 and desired state trajectories $x_{4,d}$.

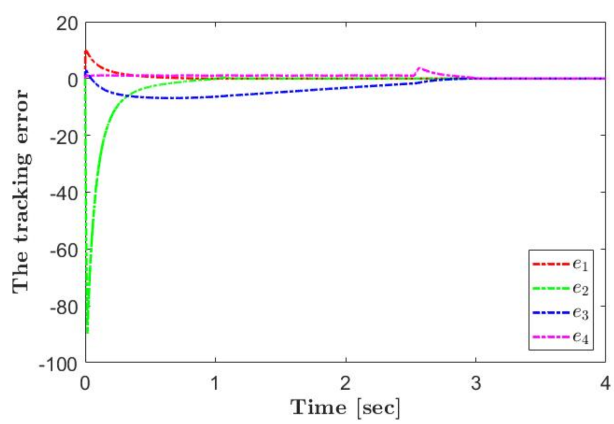

Fig. 5. Tracking errors e

VI. Conclusion

In this paper, the fixed-time trajectory tracking problem for chained-form nonholonomic systems has been considered. A switching controller has been proposed to solve this problem. An upper bound of the settling time, which only depends on the controller parameters has been estimated regardless of the initial conditions. Some simulation results have been given to show the effectiveness of the proposed controller.

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