

Importance of Noise Reduction and Suppression of Artifacts in Restoration Techniques: A State-of-the-Art.

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Abstract— The Removal of noise and restoration of signals has been one of the most interesting researches in the field of signal processing in the past few year.

In this paper, we have tested various deconvolution algorithms proposed in literature, using denoised signal (by wavelets techniques in our case) instead of measured one which is the real signal degraded by measurement procedure. It is very difficult to compare algorithms because the results obtained depend heavily on signal quality (signal-to-noise ratio, sampling), and on algorithm parameters and optimizations. Which criteria should be used to compare signals?

Our algorithm which based on Tikhonov-Miller regularization and a model of solution, is a iterative algorithm, gives best results without artifacts and oscillations related to noise, and achieves higher-quality denoising and a high restoration ratio for noisy signal than the existing methods.

Keywords— Resoration technique, SIMS depth profiling, DWT, Wavelet tresholding, Denoising signal, Regularization tools, Multiresolution deconvolution.

I. INTRODUCTION

The Deconvolution methods are used in several electron spectroscopies to improve the experimental results which are masked by instrumental effects and by physical processes involved in the measurements. In Secondary Ion Mass Spectrometry depth profiles, deconvolution methods have been employed to approximate the measured composition profiles to the supposedly original profiles.

Secondary ion mass spectrometry (SIMS) is widely used for the measurement of doping, impurity and matrix profiles in semiconductors. In this application, the concentration data required may span 10 orders of magnitude overall, and 4-6 orders for a particular species [1-3].

As the SIMS data do not directly represent the true element profile, a data quantification procedure is necessary. Various methods from simple linear mapping (ion dose to depth, signal to concentration) to deconvolution using response functions of various types [3-5]. Due to the complexity of the dynamic SIMS profile process and the large record dynamic range required, a very careful and unbiased treatment of the measured SIMS data is vital to the success of a deconvolution method to be used.

The deconvolution of depth profiling data in SIMS analysis amounts to the solution of an appropriate ill-posed problem in that any random noise in data leads to no unique and no stable solution (oscillatory signal with negative components, which are physically not acceptable in SIMS analysis). Thus, the results must be regularized. Our algorithm based on the Tikhonov-Miller regularization [6].

In this study, we evaluate a few well-known deconvolution algorithms and their modifications. specific attention is given to the comparaison of the deconvolution based on measured profiles and other based on denoised profiles.

The simplist approach to deconvolution is the inverse filtering in which the discrete Fourier Transform (DFT) of the true signal is estimated. This approach leads to excessive noise amplification. Another linear approach is based on the linear Wiener Filtering. Therefore, other classes of iterative deconvolution techniques will studied.

II. EXPERIMENTAL

Secondary-ion mass spectrometry (SIMS) is a technique used to analyze the composition of solid surfaces and thin films by sputtering the surface of the specimen with a focused primary ion beam and collecting and analyzing ejected secondary ions.

When acquiring a depth profile, the secondary ions are emitted discontinuously. It is the control electronics that manages the counting of the secondary ions striking the detector, and this is done discretely over time. The SIMS signals are thus discrete signals of finite duration. In this case, the transformation of a continuous signal into a discrete signal, that is to say the sampling problem, does not arise. The SIMS

signals will therefore be treated as discrete signals their continuous equivalent does not exist.

In the most frequent case, the linear system which degrades the source signal $x(t)$ is a low-pass filter. The high attenuation of the high frequencies of this signal makes the observation of noise not negligible. This noise by additive hypothesis, is introduced by the sensors and electronic circuits of amplification. Like any measurement system, SIMS has an equation that governs the system, which is as follows:

$$y_n(t) = h(t) * x(t) + n(t) = y(t) + n(t) \quad (1)$$

Where $x(t)$ is the original signal, $h(t)$ is the blur operator, n is a vector representing the unknown perturbations such as noise or measurement error, and $y_n(t)$ is the observed signal, respectively.

By going through a Fourier transform, convolution becomes a product, so the convolution equation becomes as follows:

$$Y_n(f) = H(f) \cdot X(f) + N(f) = Y(f) + N(f) \quad (2)$$

With H , X , B represent the Fourier transforms of h , x , and n respectively.

However, in some cases, the source signal $x(t)$ is also subjected to a multiplicative noise, in this case the noise can not be dissociated from the source signal $x(t)$ in the deconvolution problem, because it is an integral part of it.

Under these conditions, an a priori denoising procedure is necessary before the implementation of a deconvolution procedure, otherwise the result will be aberrant. This point is the core of our digital processing of SIMS signal.

A. Resolution of the convolution equation in presence of noise: Wiener Filtering

In the case where the noise is absent in the convolution equation, the term $Y(f) / H(f)$ is strictly equal to $X(f)$ if the inverse of convolution obviously exists. With the presence of noise, an inversion could also be performed in the Fourier domain, but the starting data has changed. Indeed, the system $h(t)$ is a low-pass filter and the multiplication by its convolution inverse will therefore have the same effect as the application of a high-pass filter, which will cause the amplification of the high frequencies of Noise ratio resulting in a signal embedded in the noise.

By dividing the two members of equation (2) by $H(f)$, we obtain :

$$\tilde{X}(f) = X(f) + \frac{N(f)}{H(f)} \quad (3)$$

$\tilde{X}(f)$ is an estimate of $X(f)$ obtained by dividing $Y_n(f)$ by $H(f)$. The equation above shows that noise is important here: $\tilde{X}(f)$ is composed of the real profile $X(f)$ to which is added the noise $N(f)$ strongly amplified by the term $H^{-1}(f)$. This equation shows us that the noise takes on its importance here. Therefore, $\tilde{X}(f)$ has a "saturated" noise spectrum in high frequencies, and its image $\tilde{x}(t)$ in the time domain is a highly oscillatory and unusable signal.

The solution is therefore the application of a filter on the deconvolved signal, such that this amplification of the high frequencies will be minimized. It is then appropriate to find an

adequate filter in such a way that a deconvolved profile is found as near as possible to the input signal $X(f)$ sought. Therefore, we search the filter $F(f)$ such as :

$$\tilde{X}(f) = \frac{Y_n(f)}{H(f)} F(f) \quad (4)$$

$\tilde{X}(f)$ is the estimate of the signal sought and which must be close to the real signal $X(f)$ in the least squares sense, which leads us to minimize the quantity:

$$\int_{-\infty}^{+\infty} |\tilde{X}(f) - X(f)|^2 df \quad (5)$$

Finally, the optimal Wiener filter:

$$F(f) = \frac{|Y(f)|^2}{|Y(f)|^2 + |B(f)|^2} \quad (6)$$

Note that in the expression of the Wiener filter, the resolution function H does not appear explicitly, which implies that it is not necessary to know it to define the optimal Wiener filter. However, it is necessary to discriminate the share of the useful signal and the share of noise, but this discrimination constitutes the core of the problem of deconvolution in the presence of noise. Since it is difficult to obtain the expression of the noise in the measured data, it is therefore necessary to employ noise estimation techniques. A first estimation consists in calculating the power spectral density of the noisy signal, based on the PSD plot, it is possible to estimate roughly the part of the noise and the part of the useful signal while trying several signal-to-noise ratios (SNR) And choose the one that gives a better solution. However, this estimate remains relative, because for some the parts of the signal considered as part of the noise are considered a useful signal for the others. So this method contains a certain degree of subjectivity.

The decomposition of the signal on a wavelet basis gives us a robust and objective estimate of the noise. Nevertheless, the Wiener filter is a fast method and is always a first approximation of the solution of the deconvolution problem.

B. Resolution of discrete convolution equation: direct deconvolution

After the implementation of the matrix formalism of the convolution equation, we will now see the resolution of this problem in its new context. The estimate of the solution will have the following expression:

$$\tilde{x} = H^{-1} y = x + H^{-1} n \quad (7)$$

If the matrix H has very small eigenvalues, which during the inversion will give large values thus amplifying the noise, which can lead to unacceptable solutions due to their instability.

Indeed, the noise of measurement only worsens the ill-posed character of the problem. Under these conditions the uniqueness of the solution will be difficult to guarantee. As a result, for the same measured signal, it is possible to construct several different input signals.

C. Reverse filtering and least squares solution

This approach is the basis of the other deconvolution methods: we seek a solution \tilde{x} such that $\tilde{y} = H \tilde{x}$ is as close as possible to y in the least squares sense. We must minimize the following quantity:

$$E = \|y - H\tilde{x}\|^2 \quad (8)$$

The least squares solution is:

$$\tilde{x} = (H^T H)^{-1} H^T y \quad (9)$$

However, if H is badly conditioned, $(H^T H)$ is even more so, the solution is therefore highly degenerate and may have no physical significance. There is not only one solution that satisfies the proximity condition of the reconstructed signal and the measured signal.

D. Filtering In least squares sense under constraints: Regularization

Different forms of regularization are proposed in the literature, each being more or less adapted according to the field of application [6-8]. We were interested, to the regularization of Tikhonov-Miller.

The matrix H characterizing the problem of deconvolution before regularization is replaced by the matrix H^+ better conditioned. This is done by modifying the eigenvalues of the system H . We see that the solution involves not $(H^T H)^{-1}$ which is unstable but $(H^T H + \alpha D^T D)^{-1}$.

The problem of solving a linear system is therefore made more stable, and the solution more acceptable [8].

E. Introduction of the solution model in regularization

Barakat et al [9] proposed an extension of the regularization Tikhonov-Miller by introducing a term x_{mod} representing an a priori information model. This model translates the local properties of the signal such as discontinuities, homogeneous zones, etc.

The solution sought becomes :

$$\tilde{x} = (H^T H + \alpha D^T D)^{-1} (H^T y + \alpha D^T D x_{mod}) \quad (10)$$

F. Iterative methods

The unstable character of the deconvolution in the presence of noise may also require the choice of a method which converges step by step towards a single solution and where it is possible to control at each step this convergence.

Van Cittert was one of the first to propose an iterative method to solve this problem [10]. Based on the fixed point method, the solution in step $k + 1$ is obtained from the solution obtained in step k :

$$\begin{cases} x^{(k+1)} = x^{(k)} + (y - Hx^{(k)}) \\ x^{(0)} = y \end{cases} \quad (11)$$

In our work, we adopted an iterative method based on the barakat's approach, with as a solution model a signal decomposed previously on a wavelet basis. It is a denoised signal and reconstructed by retaining only the approximation coefficients and the thresholded coefficients of detail.

For more information see [8].

The mathematical formulation of this approach, in the Fourier space, is as follows:

$$\begin{cases} \hat{X}_{n+1} = \frac{H^* Y + \alpha |D|^* X_{mod,n}}{|H|^2 + \alpha |D|^2} \\ X_{mod,0} = TF[\hat{x}^{(j-1)}] = TF[\hat{F}^{(j)} y_a^{(j)} + \hat{G} \hat{y}_d^{(j)}] \\ \hat{x}_n = TF^{-1}[\hat{X}_n] \end{cases} \quad (12)$$

With $y_a^{(j)}$ and $\hat{y}_d^{(j)}$ are, respectively, the approximation signal and the detail thresholded signal, α : regularization parameter, D : regularization operator, H : matrix constructed from the impulse response h , Y : vector constructed from the measured signal.

III. RESULTS AND DISCUSSION

The results of the deconvolution of the sample MD4 (four delta layer) in linear and logarithmic scales are illustrated in this section. These results clearly show the importance of the idea of denoising the measured signal before using it subsequently in signal processing techniques. We benefit of each technique proposed in the literature.

Really, this case of profile (delta layer profiles) is the most difficult in terms of restoration because the spectrum is rich in high frequencies that limit the quality of the final solution.

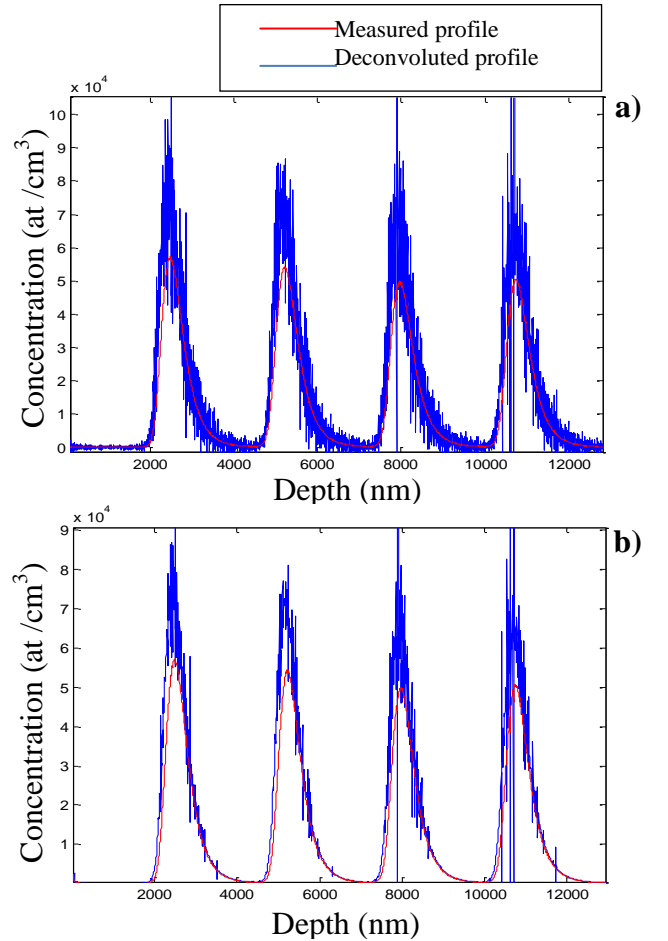


Fig. 1 Results of deconvolution of MD4 by direct inversion of the convolution equation, linear scale representation (DRF: $\lambda_d = 47.8$; $\lambda_u = 10.55$; $\sigma_g = 26.3$): a) Using measured signal, b) using denoised signal

Fig. 1 show an example of deconvolution by direct inversion of the convolution equation of a measured profile of boron in silicon. It shows that the shape of the deconvolved signal using a denoised signal is better than that of the measured signal, as well as a minimization of the oscillations, especially in the junctions between the delta doping.

Fig. 2 illustrates results of the solution of Reverse filtering and least squares solution. There is still a good improvement in the shape of the deconvolved signal and a minimization of the oscillations and artifacts in the second case (Fig. 2-b), thus improving the gain in resolution and peak's maximum, but it is not sufficient to best define the actual profile.

We must therefore define criteria on the solution sought in order to limit the number of solutions to those that are physically acceptable. This modification of the specification of the problem of deconvolution is widely used in signal processing, and is designated under the name of regularization or least squares filtering under constraints.

In the regularization of Tikhonov-Miller, the regularization term refers only to the solution sought, and this in a global way, that is to say that the property that one wishes to impose on x applies throughout its interval of definition, Without distinguishing the different zones of the signal.

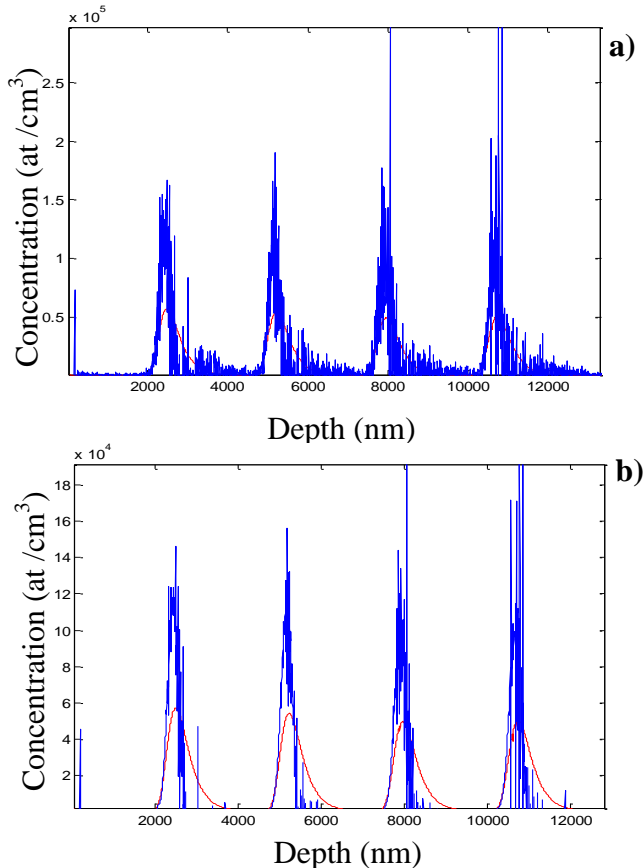


Fig. 2 Results of the solution of reverse filtering and least squares solution, linear scale representation: a) Using measured signal, b) using denoised signal

Fig. 3 show result of deconvolution using Barakat's approach. In this case, we have chosen as a solution model a signal previously decomposed on a wavelet basis. It is

concluded that noise is totally eliminated, As well as a modest improvement in the gain in resolution. The precision of the solution will depend directly on the precision of the model brought.

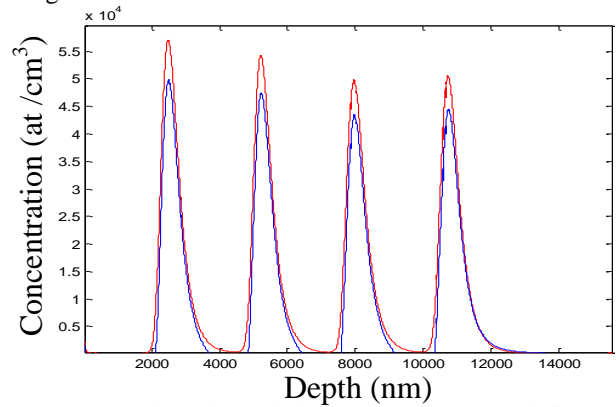
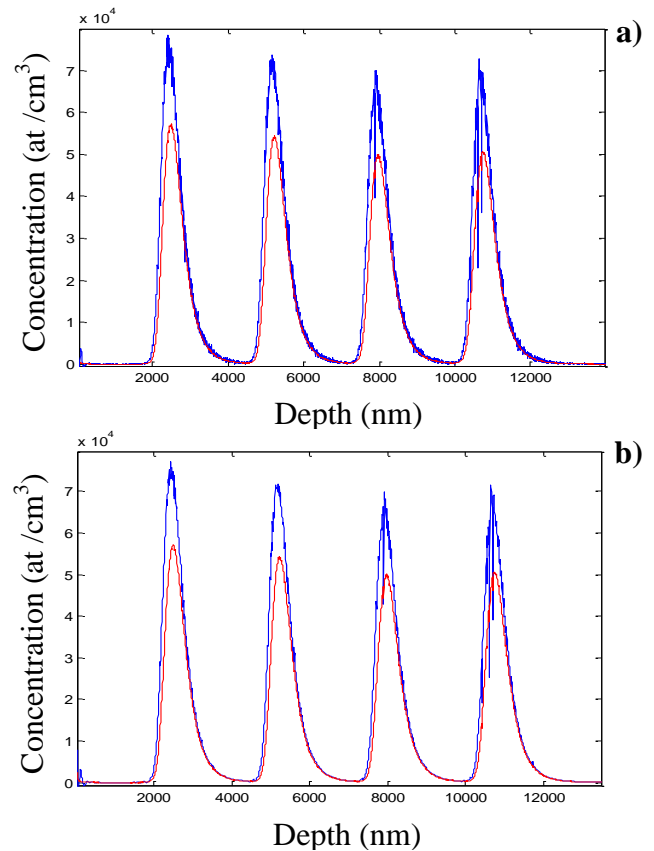


Fig. 3 Results of deconvolution using Barakat's approach, linear scale representation, (DRF: $\lambda d = 47.8$; $\lambda u = 10.55$; $\sigma g = 26.3$)

As any method discussed in this work, the Van Cittert algorithm know a remarkable development of the form resulting mainly to the low level of the resulting signal (Fig. 4).

Our algorithm is the result of several algorithms proposed in past years in this field, it is a iterative algorithm, based on Tikhonov-Miller regularization. Where a priori model of solution is included. The latter is a denoisy and pre-deconvolved signal obtained by wavelets shrinkage algorithm. It is shown that this new algorithm gives best results without artifacts and oscillations related to noise.



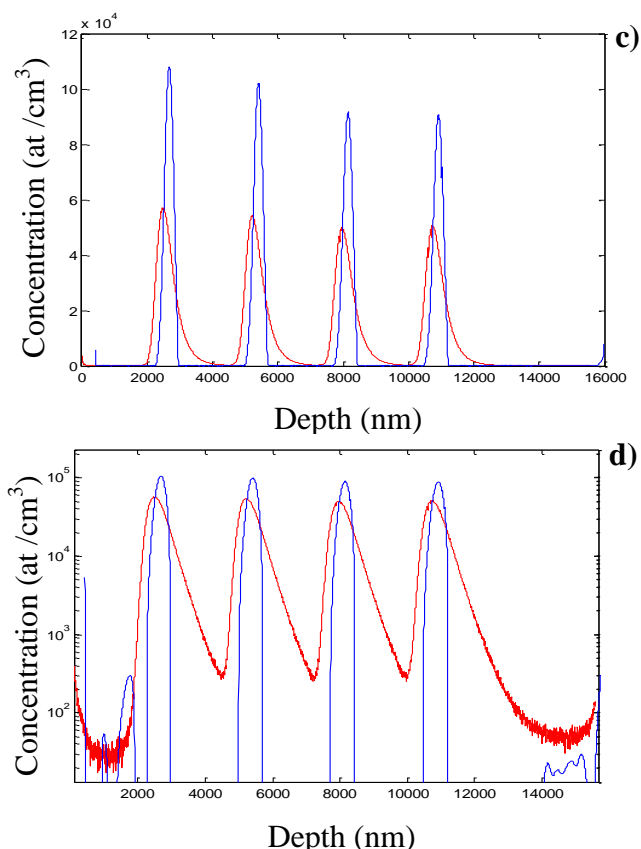


Fig. 4 Results of iterative methods deconvolution, (DRF: $\lambda_d = 47.8$; $\lambda_u = 10.55$; $\sigma_g = 26.3$): a) By Van-cittert Algorithm & using measured signal, linear scale representation, b) By Van-cittert Algorithm & using denoised signal, linear scale representation, c) multiresolution deconvolution, linear scale representation, d) multiresolution deconvolution, logarithmic representation

When studying a phenomenon using a wide range of values, the linear scale is ill suited. It is preferred to use a logarithmic scale that separates the low values and approximates the strong values. It may be preferable in this case since it shows oscillations and artefacts in a remarkable way. Deconvolution of this sample (Fig. 4-d) resulted a great improvement of the depth resolution and recovery of the original shape of the signal. The exponential slopes have been completely removed giving symmetrical peaks and well separated.

IV. CONCLUSIONS

This paper presents a state of the art review of published research papers and reports that interested in signal processing techniques. We show the utility of each one. we are interested in the idea of denoising, of the signal from the measure, as a first step of treatment before applying other techniques of digital signal processing.

In the SIMS analysis, deconvolution will not make an instrument of poor quality excellent, we can refine the physical resolution of it. In other words, better have a "good instrument" than correct a bad one! In any case, it is not desirable to apply the deconvolution to a concentration profile acquired under poor conditions. The quality and reliability of the initial measurement remains the essential condition for a

good result, just as the improvement of the instrumentation remains a privileged way towards the ultimate improvement of the depth resolution. The deconvolution is in addition to the instrumental performances and allows to derive the maximum resolution of an experimental profile. Moreover, when deconvolution is implemented, every element involved in the calculations must be taken into account, since deconvolution is a delicate operation and should not be implemented without precautions, otherwise, we can obtain Aberrant results, such as non-existent structures in reality or results that are mathematically correct, but physically unrealistic. This necessitates a good control of the restitution process and its mechanisms. Among the elements to be taken into account, we find the following:

- Application a denoising technique such as wavelets which is powerful tool in signal processing.
- Choice of the form that fit the DRF as well as the choice of the parameters of the DRF itself.
- Application of hard constraints (constraint of positivity, support, amplitude ...) and soft (regularization), as well as the choice of regulation parameters. As well as the right choice of the solution model, if it exists to the deconvolution algorithm.
- Type of the applied algorithm (iterative or non-iterative). The iteration number in the case of an iterative method is very important (criterion for stopping the algorithm).

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