

Copmarative Study of Electromagnetic Modeling of Dipole Antenna Radiated in Free Space and Inside a Rectangular Waveguide Using MoM-GEC

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Abstract—A theoretical and modulation study of a dipole antenna located in rectangular waveguide is presented. The aim is to determine the nature and dimensions which allow us to attain the simulations in free space. We compare the behavior of a dipole antenna modeled by MOM-GEC with those obtained in free space. A method of moment (MoM) approach combined to the generalized equivalent circuit (GEC) modeling is applied to compute the input impedance of a dipole antenna resonating at 9GHz. A parametric study is conducted to investigate the wall effects of the waveguide on the antenna performances. Also we study the convergence of the input impedance as a function of test function number. The current and the electric field distribution are simulated. The current values with integral method (MOM) are compared with the computed values obtained from numerical methods such as the combined MOM-GEC. The simulated values agree with values obtained from numerical methods.

Keywords—dipole antenna; MOM-GEC; infinite waveguide.

I. INTRODUCTION

In radio and telecommunications a dipole antenna is the simplest and most widely used class of antenna.

A new formulation based on the integral-equation formulation in the frequency domain is used to study the behavior of a dipole antenna with finite length. The MoM method, if well designed and carefully optimized, can be a highly efficient and reliable tool for the analysis and design of a many complex EM structures. The current distribution and the input impedance of a dipole are also presented in analytical as well as numerical method such as MoM.

Dipole antennas is designed using Hallen and Pocklington's integral equations approximations. Numerical techniques in solving electromagnetic problems are the most common methods, which are used, with the budding inventions of high-speed computers and powerful software's. Among these numerical techniques, Method of Moments (MOM) which is a powerful numerical technique to solve integral equations and this technique will be applied on Hallen and Pocklington's integral equations for a dipole antenna.

This kind of antenna is the simplest type of antenna from a theoretical point of view. The two "arms" of the dipole are the regions where currents can flow and charges accumulate. Consider a thin dipole antenna of 'L' length being fed at its center as shown below. The antenna is chosen for the purpose of this analysis to lie along the z-axis and to be centered at (0, 0, 0) [8, 9].

The aim of the developed method is to investigate the antenna parameters such as the input impedance and current distribution. Starting with a convergence study followed by a parametric study is conducted to investigate the wall effects of the wave-guide on the antenna performances.

In this paper, we first detail the integral equation formulation and the derivation of the necessary Green's function for an infinite waveguide with the MOM method. The MOM-GEC is then presented and we present our structure developed using an equivalent circuit. Followed by a comparative investigation of the electromagnetic behavior of dipole antenna in free space and inside an infinite rectangular waveguide as a function of the walls effect on the antenna performance. Such as a convergence study of the input impedance is done .

II. DESIGN OF DIPOLE ANTENNA

A. Antenna geometry

1) Dipole antenna dimensions:

The dipole antenna is one of the most commonly used types of RF antenna. The desired frequency that the dipole is desired to operate on is 9GHz. The length of the antenna is the parameter that controls the resonant frequency. As a result, the length is treated as half wave dipole, which is given by the following formula:

$$L = \frac{\lambda}{2}$$

$$\lambda = \frac{c}{f}$$

$$0.05\lambda \leq w \leq 0.1\lambda$$

Where L is dipole length, λ is the wavelength, c is the speed light in free space and f is the operating frequency.

2) *Thin antenna approximation, Pocklington equation:*

Considering the case of thin dipole antenna of length ' L ' and radius ' a ' placed in an infinite homogeneous medium, and subjected by an incident electric field E^i . The dipole antenna oriented along the z -axis, which is shown in Figure 1. The electric field at the surface of the antenna may be written as:

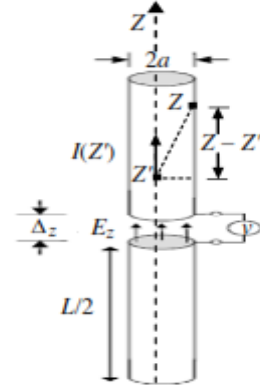


Figure (1) model of cylindrical dipole antenna [6]

$$E^i + E^r = 0 \quad (1)$$

We may simplify the expression using the approximation of thin antenna (Pocklington equation), the current equation is:

$$I(z') = 2\pi a J(z') \quad (2)$$

E^r and E^i are the radiated electrical field and incident electric field respectively given by:

$$E^r = \frac{1}{j\omega\epsilon} \left(k^2 + \frac{\partial^2}{\partial z^2} \right) \int_{-\frac{L}{2}}^{\frac{L}{2}} I(z') \frac{e^{-jkR}}{4\pi R} dz' \quad (3.a)$$

$$E^i = -\frac{1}{j\omega\epsilon} \left(k^2 + \frac{\partial^2}{\partial z^2} \right) \int_{-\frac{L}{2}}^{\frac{L}{2}} I(z') \frac{e^{-jkR}}{4\pi R} dz' \quad (3.b)$$

By substituting the radiated electrical field expression, we have the Pocklington integral equation:

$$\frac{1}{j\omega\epsilon} \left(k^2 + \frac{\partial^2}{\partial z^2} \right) \int_{-\frac{L}{2}}^{\frac{L}{2}} I(z') \frac{e^{-jkR}}{4\pi R} dz' + E^i = 0 \quad (4)$$

$$G(z-z') = \frac{e^{-jkR(z-z')}}{4\pi R(z-z')} \quad (5)$$

(5) was the green function in homogeneous and limited space with parameter ϵ and μ .

This integral equation is numerically solved using the MoM method [5].

The current distribution along the dipole antenna is expressed as the sum of the samples current I_m using a basis function $B(z)$:

$$I(z) = \sum I_m B(z-z')$$

B. *The Moment Method (MOM)*

An antenna structure is broken into "segments" and the currents on the segments are then evaluated. The "moment" is numerically the size of the currents times the vector, describing the little segment (length and orientation).

A set of "basis functions" are assumed into which the current distributions are decomposed. The "MoM" starts from deriving the currents on each segment, or the strength of each moment, by using a coupling Green's function [7]. This Green's function incorporates electrostatic coupling between the moments, by knowing the spatial change of the currents, buildup of charges at points on the structure is computed. The MoM was developed by R.F Harrington[5].

C. *Basic concept of MoM_GEC*

Since they write initial boundary conditions in form of integral equations defined on the obstacle surface, these methods permit the reduction of the problem's dimension. But, the resolution will become more complicated as the structure's complexity increases. In this context, the equivalent circuits are introduced for the development of integral method formulation based on the transposition of field problems in generalized equivalent circuit that are simpler to treat [1].

In fact, for alleviating the resolution of Maxwell's equations, the method of Generalized Equivalent Circuit (MGEC) was proposed [1,2,3,4] in order to represent integral equations by equivalent circuits that express the unknown electromagnetic boundary conditions. The equivalent circuit presents a true electric image of the studied structures for describing the discontinuity and its environment.

In the discontinuity plane, the electromagnetic state is described by generalized test functions that are modeled by virtual sources not storing energy. An impedance operator or admittance operator that represents boundary conditions on each side of discontinuity surface expresses the discontinuity environment. However, the wave exciting the discontinuity surface is represented by a real field source or a real current source because it delivers energy.

III. PROBLEM FORMULATION

The presented numerical analysis is based entirely on the rectangular waveguide modes and assumes propagation as $e^{-\gamma z}$. The waveguide is infinite in both $z > 0$ and $z < 0$ directions and the equivalent circuit of this structure is shown in Figure (2).

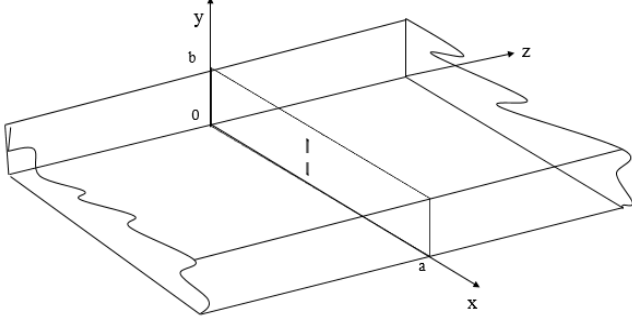


Figure (2): Dipole antenna in an infinite waveguide.

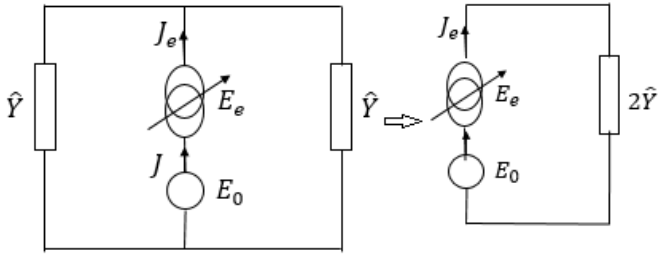


Figure (3): representation of a dipole antenna with an equivalent circuit.

$f_{mn}(m \in \{0,1,2,\dots,M\})$ is the local modal basis corresponding the waveguide with electric walls (EEEE) described in:

$$f_{mn}^{TE, TM} = \begin{cases} N_x^{TE, TM} \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) \\ N_y^{TE, TM} \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) \end{cases} \quad (1)$$

Where $N_x^{TE, TM}$ and $N_y^{TE, TM}$ the normalization constants.

E_0 is a source of electric field. It has an amplitude V_0 and depends on special variables through F_0 as in expression $E_0 = V_0 F_0$. The function F_0 acts as the shape function which ensures reliable expression for the voltage and current case of rectangular source $F_0 = \frac{1}{\delta}$.

J_e is the virtual source defined on the metallic domain of the discontinuity surface and it is the problem unknown.

J_e is expressed as a series of known test functions g_p weighted by unknown coefficients x_p :

$$J_e = \sum_{p=1}^{N_p} x_p g_p \quad (2)$$

Two sources, a virtual and a real one being involved in the formulation process. We treat the case where the virtual source is of current type and the real one is of electric field type. Then the equivalent circuit of the studied structure is completed by addition the terminating operator \hat{Y} :

$$\hat{Y} = 2\hat{Y}_{IWG} \quad (3)$$

When applied to the circuit depicted in Figure(3), the generalized Kirchhoff and Ohm laws lead to the equations system:

$$\begin{cases} J_e = J \\ E_e = \hat{Y}^{-1} J_e - E_0 \end{cases} \quad (4)$$

A formal relation between sources (real and virtual) and their dual is then deduced:

$$\begin{pmatrix} I_0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix} = \begin{pmatrix} 0 & \langle F_0, g_1 \rangle & \langle F_0, g_2 \rangle & \dots & \langle F_0, g_{N_e} \rangle \\ -\langle g_1, F_0 \rangle & \langle g_1, \hat{Y}^{-1} g_1 \rangle & \langle g_1, \hat{Y}^{-1} g_2 \rangle & \dots & \langle g_1, \hat{Y}^{-1} g_{N_e} \rangle \\ -\langle g_2, F_0 \rangle & \langle g_2, \hat{Y}^{-1} g_1 \rangle & \langle g_2, \hat{Y}^{-1} g_2 \rangle & \dots & \langle g_2, \hat{Y}^{-1} g_{N_e} \rangle \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -\langle g_{N_e}, F_0 \rangle & \langle g_{N_e}, \hat{Y}^{-1} g_1 \rangle & \langle g_{N_e}, \hat{Y}^{-1} g_2 \rangle & \dots & \langle g_{N_e}, \hat{Y}^{-1} g_{N_e} \rangle \end{pmatrix} \begin{pmatrix} V_0 \\ x_1 \\ x_2 \\ \vdots \\ x_{N_e} \end{pmatrix}$$

The admittance operator \hat{Y} is given by:

$$\hat{Y} = \sum |f_{mn}^\alpha \rangle y_{mn}^\alpha \langle f_{mn}^\alpha| \quad (5)$$

Where $\alpha = TE$ or TM

y_{mn}^α being modal admittance of the n^{th} mode of an infinite waveguide (IWG):

$$y_{mn}^{TE} = \frac{\gamma_{mn}}{j\omega\mu_0} = \frac{\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2 \mu_0 \epsilon_0}}{j\omega\mu_0} \quad (6.a)$$

$$y_{mn}^{TM} = \frac{j\omega\epsilon_0}{\gamma_{mn}} = \frac{j\omega\epsilon_0}{\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2 \mu_0 \epsilon_0}} \quad (6.b)$$

The current J is expressed in modal basis functions

$f_{mn}(m \in \{0,1,2,\dots,M\})$ weighted by unknown coefficients I_m :

$$J = \sum_{m=0}^M I_m f_m \quad (7)$$

Therefore, the application of the Galerkin method and Kirchhoff's theorem leads to obtaining the simplified matrix representation as follows:

$$\begin{pmatrix} I_0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 & [A] \\ -[A]^t & [B] \end{pmatrix} \begin{pmatrix} V_0 \\ X \end{pmatrix} \quad (8)$$

Where $[A] = \langle F_0, g_p \rangle$ and $[B] = \langle g_q, \hat{Y}^{-1} g_p \rangle$. Thus from equation (8), we obtain the equation system:

$$\begin{cases} I_0 = [A][X] \\ 0 = [X][B] - V_0[A]^t \end{cases} \quad (9)$$

The resolution of the equation system (9) leads to calculate the structure input admittance:

$$Y_{in} = 2[A]^t [B]^{-1} [A] \quad (10)$$

IV. RESULTS AND DISCUSSION

An important parameter that may affect the antenna response is the placement of the waveguide walls. We present in Figure (4) the current density distribution along the dipole antenna for different positions of electric walls. It's found that, the current distribution is strongly affected by the lateral walls position. The separation distance have not any significant effect for a value that exceed $a=150\text{mm}$. In the following, the lateral walls positions are fixed at $a=150\text{mm}$ and the $b=67\text{mm}$ to insure the impactless of this parameter to further results.

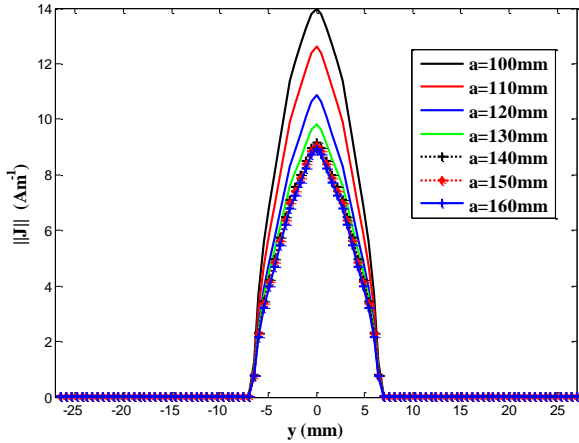


Figure (4): current density distribution as a function of a walls distance.

Figure (5) and Figure (6) illustrate the electric field and current density evaluated by the MOM-GEC and obtained at convergence conforms to the theory with consideration to the boundary conditions.

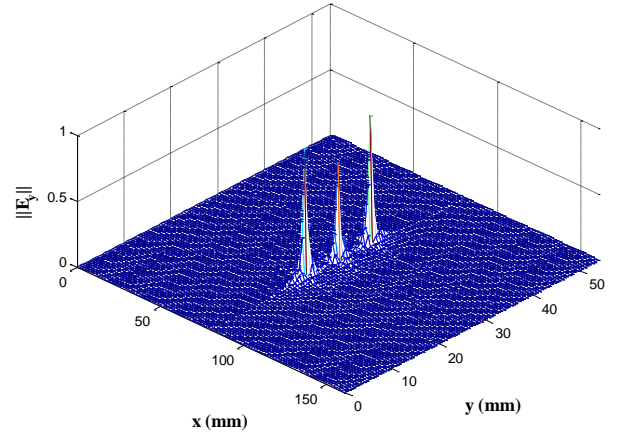


Figure (5): 2D representation of the electric field density ($A.m^{-1}$) at $f=9\text{GHz}$.

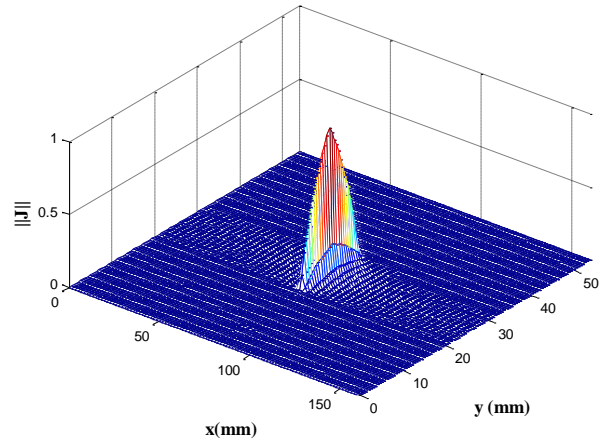


Figure (6): 2D representation of the current density ($A.m^{-1}$) at $f=9\text{GHz}$.

As it is shown in Figure (7), integral method and MOM-GEC gives nearly the same current distribution for a dipole antenna of length $16,7\text{mm}$ which corresponds to $L = \frac{\lambda}{2}$ for an operating frequency $f=9\text{GHz}$.

The current distribution with the integral method take approximately a triangular form, the current distribution with MOM-GEC is a half sinusoid for the same operating frequency.

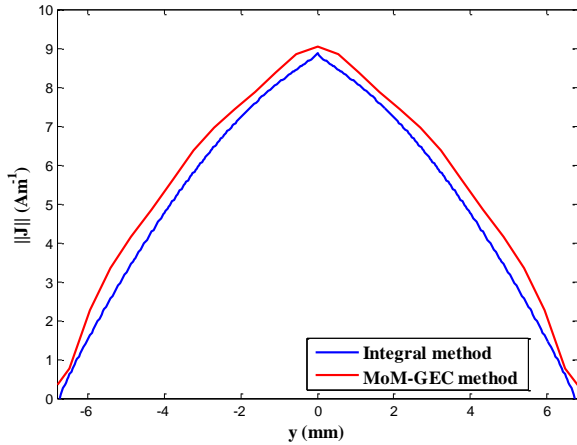


Figure (7): current distribution on a dipole antenna using integral method and MOM-GEC method for an antenna length $L=16,7\text{mm}$ and operating frequency $f=9\text{GHz}$

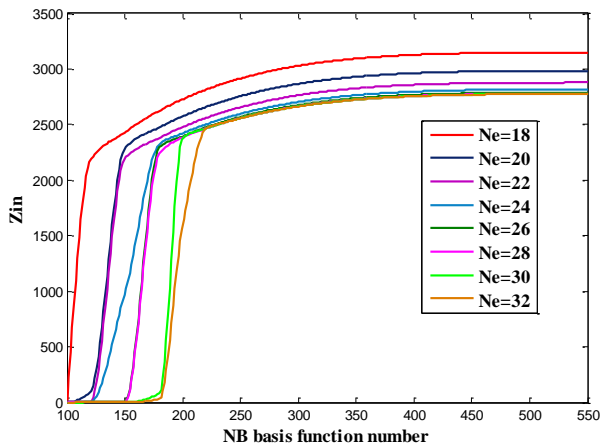


Figure (8): variation of the input impedance as a function of the guide modes number for different test functions number at $f=9\text{GHz}$.

We are interested to study the convergence of our problem. Figure (8) presents the input impedance variation as a function of the test function number N_e for different used basis function number N_b , which is fixed to 200 to ensure convergence.

V. CONCLUSION

Studying the behavior of a dipole antenna both in free space and inside a rectangular infinite waveguide, It's found that, MOM-GEC gives nearly the same results as that obtained in free space. Also a good study of convergence is elaborated to investigate the theoretical input impedance Z_{in} evaluated by the MOM-GEC.

The convergence was obtained for $N_b=200$ and $N_e=24$. The current and electric field distribution conforms to the theory and obey to the boundary conditions.

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