Proceedings of Engineering & Technology (PET)
pp. 406-412


# Modeling and Stabilizing Control of an UAV for Easy Taking-off and Hovering.

NASR Salah
CEM-Lab ENIS Sfax,
National Engineering School
of Sousse,
University of Sousse, Tunisia.
nasrsalah.doc@gmail.com

BOUALLEGUE Kais
Department of Electrical Engineering,
Higher Institute of Applied Sciences
and Technology of Sousse,
University of Sousse, Tunisia.
kais_bouallegue@yahoo.fr

MEKKI Hassen
CEM-Lab ENIS Sfax,
National Engineering School
of Sousse,
University of Sousse, Tunisia.
mekki.hassen@gmail.com

*Abstract* — A synthesis robust stabilisation of a small unmanned aerial vehicle type quadrotor with vertical taking-off and hovering motion is proposed for control the position and altitude tracking. Analytical relations for dynamic modeling, physical phenomena and control of the actions on four rotors that ensure quadrotor motion over a prescribed trajectory with desired values of position and altitude is presented. The quadrotor is controlled with classical proportional integral derivative controller in three state of system: position control, angle control and inputs control. Numerical simulation results are provided to show the good performances of stabilisation and control strategy.

***Keywords*** — **PID controller, Quadrotor UAV, Hovering, model, simulation.**

## I. INTRODUCTION

The subject of unmanned aerial vehicle (UAV) for military, industrial and in urban environment tasks acquired a keen interest in recent years where the UAV will have to operate in enclosed spaces. A number of important applications will require this, e.g. fire and natural disaster search and rescue, police and security services, inspection and surveillance dangerous tasks that put human integrity at risk [1-3]. In civilian sectors, UAV can be used for many applications such as low enforcement, traffic report, aerial photography and more [4]. Many research teams are working on the control and stabilization for the surveillance of land, exploration ground for explosives or hazardous materials and saving human victims at the scene of the disaster.

UAV are known to be inherently unstable, nonlinear and coupled. Different methods for model-based autonomous UAV control have been presented in literature. Dynamic modeling and configuration of quadrotor were proposed by Mc Kerrow [5] and Hamel et al. [6]. Nonlinear control problems for hovering quadrotor such as feedback linearization control and back-stepping control laws were studied by Altug et al. [7] and Mistler et al. [8]. Therefore, based on the dynamic model of a PVTOL aircraft, Castillo et al. [9], Hamel and al. [10] designed controllers for Yaw angular displacement, Pitch and Roll movements of a hovering quadrotor. In [11], Xiong and Zheng are proposed a novel robust terminal sliding mode control algorithm.

In this work, our approach is to use a simple nested proportional integral derivative (PID) controller to track a desired trajectory for take-off and hovering motion.

The rest of this paper is organized as follows: The dynamic model of quadrotor is introduced in the next section. Then, the proposed method of control is given in section III. In section IV, simulation results are presented. Our concluding remarks are contained in the final section.

## II. DYNAMIC OF QUADROTOR

The quadrotor have four rotors as shown in Fig. 1, the two pair of motors (1,3) are running in the same direction, whereas the others (2,4) in the opposite direction to eliminate the anti-torque.

By increasing/decreasing the 2 and 4 rotor's speed conversely produces the Roll angle $\phi$ allows to move the quadrotor in lateral direction x. Pitch angle $\theta$ produced by varying the 1 and 3 rotor's speed allows the quadrotor to move in lateral direction y, and by varying all rotor's speed together with the same values, can change the lift force, affecting the altitude z of quadrotor then Yaw angle $\psi$ produced and enable the motion to take-off. The forces and moments of control the altitude and position of the system provides by the Euler angle orientation ($\phi,\theta,\psi$),under the conditions ($-\pi \prec \psi \prec \pi$) for Yaw angle, ($-\pi/2 \prec \theta \prec \pi/2$) for Pitch angle, and ($-\pi/2 \prec \phi \prec \pi/2$) for Roll angle.

To describe the motion of a 6 degree of freedom (DOF) rigid body it is usual to define two reference frames as shown in Fig. 2.

• the earth inertial frame $E(0,X,Y,Z)$, and

ISSN: 2356-5608

• the body-fixed frame Q(0,x,y,z).

The equations of motion are more conveniently formulated in the Q-frame because of the following reasons:

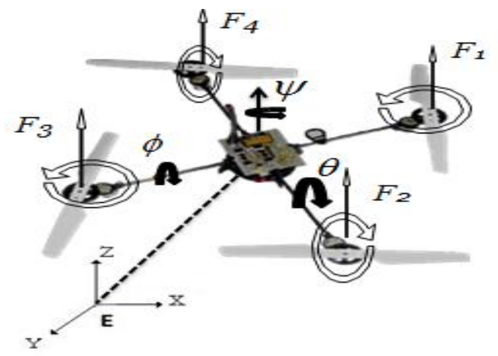


Fig. 1 The quadrotor UAV.

• the inertia matrix is time-invariant;
• advantage of body symmetry can be taken to simplify the equations;
• measurements taken on-board are easily converted to body-fixed frame;
• control forces are almost always given in body-fixed frame.

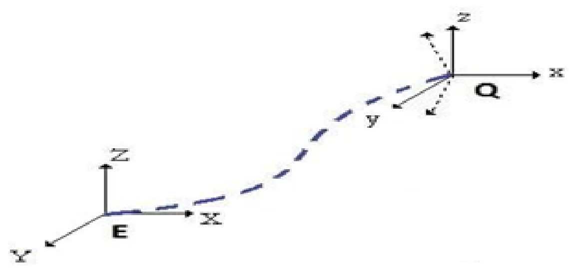


Fig. 2 General coordinate system

The linear position of the quadrotor $(X, Y, Z)$ is determined by the coordinates of the vector between the origin of the Q-frame and the origin of the E-frame according to the equation of motion. The angular position (or attitude) of the quadrotor $(\phi, \theta, \psi)$ is defined by the orientation of the Q-frame with respect to the E-frame. This is given by three consecutive rotations about the main axes which take the E-frame into the Q-frame.

The rotation matrix between the E and Q frames has the following form [12] :

$$R = \begin{bmatrix} c\psi c\theta & s\phi s\theta c\psi - s\psi c\phi & c\phi s\theta c\psi + s\psi s\phi \\ s\psi c\theta & s\phi s\theta s\psi + c\psi c\theta & c\phi s\theta s\psi - s\phi c\psi \\ -s\theta & s\phi c\theta & c\phi c\theta \end{bmatrix} \quad (1)$$

Where c and s indicate the trigonometrically functions **cos and sin** respectively.

Let the vector $[p, q, r]^t$, denotes the quadrotor's angular velocity in the Q-frame. The corresponding transformation matrix from $[\phi, \theta, \psi]^t$ to $[p, q, r]^t$ is given by:

$$\begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} 1 & 0 & -\sin\theta \\ 0 & \cos\phi & \sin\phi\cos\theta \\ 0 & -\sin\phi & \cos\phi\cos\theta \end{bmatrix} \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} \quad (2)$$

J is a symmetric positive definite constant inertia matrix of the quadrotor.

$$J = \begin{bmatrix} I_x & 0 & 0 \\ 0 & I_y & 0 \\ 0 & 0 & I_z \end{bmatrix} \quad (3)$$

- For the force, we note the gravity force $\vec{F}_g$ and the resultant of lift $\sum_{i=1}^{4} \vec{T}_i$ created by the four rotors:

$$\vec{F} = \vec{F}_g + \sum_{i=1}^{4} \vec{T}_i \quad (4)$$

Where ,

$$\vec{F}_g = \begin{bmatrix} 0 \\ 0 \\ -mg \end{bmatrix} \quad (5)$$

And $$\vec{T}_i = \begin{bmatrix} T_{ix} \\ T_{iy} \\ T_{iz} \end{bmatrix} = lb \begin{bmatrix} \Omega^2{}_{ix} \\ \Omega^2{}_{iy} \\ \Omega^2{}_{iz} \end{bmatrix} \quad (6)$$

$\Omega_i$ is the angular speed of rotor i , b denotes the lift coefficient, m denotes the total mass, g represents the acceleration of gravity and l is the distance between the quadrotor center of mass and the rotation axis of propeller.

- For the moment, M is the moments developed by the quadrotor according to the Q-frame. It is described by the following matrix:

$$M = \begin{bmatrix} T_4 - T_2 \\ T_3 - T_1 \\ -D_1 + D_2 - D_3 + D_4 \end{bmatrix} \quad (7)$$

$$D_i = d\Omega_i^2 \quad (8)$$

Where d is the drag coefficient.

We can finally derive the equations governing the dynamics model of quadrotor as described by the following equations [13-16]:

$$\begin{cases} \ddot{\phi} = \frac{(I_y - I_z)}{I_x}\dot{\theta}\dot{\psi} - \frac{I_r}{I_x}\Omega_r\dot{\theta} - \frac{K_{ax}}{I_x}\dot{\phi}^2 + \frac{l}{I_x}u_2 \\ \ddot{\theta} = \frac{(I_z - I_x)}{I_y}\dot{\phi}\dot{\psi} + \frac{I_r}{I_y}\Omega_r\dot{\phi} - \frac{K_{ay}}{I_y}\dot{\theta}^2 + \frac{l}{I_y}u_3 \\ \ddot{\psi} = \frac{(I_x - I_y)}{I_z}\dot{\theta}\dot{\phi} - \frac{K_{az}}{I_z}\dot{\psi}^2 + \frac{1}{I_z}u_4 \\ \ddot{x} = -\frac{K_{tx}}{m}\dot{x} + \frac{1}{m}(\cos\phi\cos\psi\sin\theta + \sin\phi\sin\psi)u_1 \\ \ddot{y} = -\frac{K_{ty}}{m}\dot{y} + \frac{1}{m}(\cos\phi\sin\psi\sin\theta - \sin\phi\cos\psi)u_1 \\ \ddot{z} = -\frac{K_{tz}}{m}\dot{z} - g + \frac{\cos(\phi)\cos(\theta)}{m}u_1 \end{cases} \quad (9)$$

Where $k_{tx}, k_{ty}$ and $k_{tz}$ are the translation drag coefficients and $k_{ax}, k_{ay}$ and $k_{az}$ are frictions aerodynamics coefficients.

The system's inputs are posed $u_1$, $u_2$, $u_3$, $u_4$ and $\Omega$ is a disturbance, obtained as:

$$\begin{cases} u_1 = b(\Omega_1^2 + \Omega_2^2 + \Omega_3^2 + \Omega_4^2) \\ u_2 = b(\Omega_4^2 - \Omega_2^2) \\ u_3 = b(\Omega_3^2 - \Omega_1^2) \\ u_4 = d(\Omega_1^2 - \Omega_2^2 + \Omega_3^2 - \Omega_4^2) \\ \Omega = \Omega_1 - \Omega_2 + \Omega_3 - \Omega_4 \end{cases} \quad (10)$$

The rotors are driven by DC motors with the well-known equations [17]:

$$\begin{cases} V = Ri + L\frac{di}{dt} + k_e\omega \\ k_m i = I_r\frac{d\omega}{dt} + C_s + k_r\omega^2 \end{cases} \quad (11)$$

Where, v is the motor input, $k_e$, $k_m$: are respectively the electrical and mechanical torque constant , $C_s$ : is the solid friction and $k_r$ is the load torque constant.

## III. Stabilizing and Control STRATEGY

The objective of this part of stabilization and control is to develop a method that calculates the voltages of four motors from the two main entrances. These are the desired yaw angle ($\psi_d$ ) , spatial desired position ( $x_d, y_d, z_d$ ) and real values that are provided by the sensors (accelerometer, gyro and altimeter).

Our stabilization strategy and control is divided into three blocks of controllers ( $C_1, C_2$ and $C_3$ ) as shown in Fig.3. The first block controller $C_1$ contains three controllers PID: control of x position, control of y position and control of z altitude with invoking Eqs. (13), Eqs.(14) and Eqs.(15). The second block controller $C_2$ used to control of Yaw ( $\psi_d$ ), Roll ( $\phi_d$ ) and Pitch ( $\theta_d$ ) (generated from the state block) by using Eqs. ( 16), Eqs.(17) and Eqs. (18).

The last block C3 controllers include the control inputs of system generated by the force and torque block (illustrate by Eqs.(19).

The force and torque block include all computed and transformation of the force and torque injected as inputs system with invoking the matrix (1), (2) and (8).

The state block discloses using to generate the Euler angles $\phi_d$ and $\theta_d$ defined as follows:

$$\begin{cases} \phi_d = \arcsin\left(\frac{(\ddot{x}\sin\psi_d - \ddot{y}\cos\psi_d)}{\sqrt{\ddot{x}^2 + \ddot{y}^2 + \ddot{z}^2}}\right) \\ \theta_d = \arctan\left(\frac{(\ddot{x}\cos\psi_d + \ddot{y}\sin\psi_d)}{z_d}\right) \end{cases} \quad (12)$$

The dynamic of quadrotor block present the global dynamics of quadrotor as described in section II.

The general approach to stabilize our system is as follows:

The control algorithms for the position and angle, roll, pitch And yaw, of the UAV are designed based on PID controllers as shown in Fig. 3.

We calculate the difference between the desired value and the actual value and a PID controller is used to minimize this error.

The control input $u$ to controlling the position and angle of the UAV respecting to the reference input designed as follows:

$$u(t) = K_p\, e(t) + K_i\int_0^t e(t)dt + K_d\frac{d}{dt}e(t) \quad (13)$$

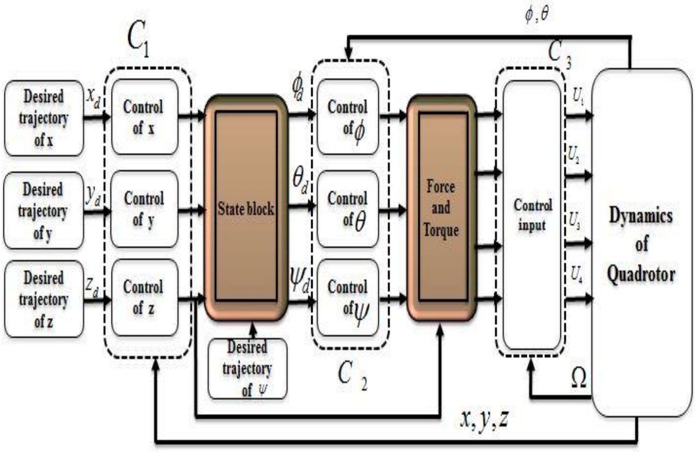


Fig. 3 Synoptic scheme of control strategy.

This principle is applied in our model to control the spatial position (x, y and z) as follows:

$$u_x = k_p(x - x_d) + k_i\int_0^t (x - x_d)dt + k_d\frac{d(x - x_d)}{dt} \quad (14)$$

$$u_y = k_p(y - y_d) + k_i\int_0^t (y - y_d)dt + k_d\frac{d(y - y_d)}{dt} \quad (15)$$

$$u_z = k_p(z - z_d) + k_i\int_0^t (z - z_d)dt + k_d\frac{d(z - z_d)}{dt} \quad (16)$$

Where kp, ki, kd are PID controller gains for the position control.

The angles ( $\phi$ , $\theta$ and $\Psi$ ) are controlled as described:

$$u\phi = kp_a(\phi - \phi_d) + ki_a\int_0^t (\phi - \phi_d)dt + kd_a\frac{d}{dt}(\phi - \phi_d) \quad (17)$$

$$u\theta = kp_a e_\theta(\theta - \theta_d) + ki_a\int_0^t (\theta - \theta_d)dt + kd_a\frac{d}{dt}(\theta - \theta_d) \quad (18)$$

$$u\psi = kp_a(\psi - \psi_d) + ki_a\int_0^t (\psi - \psi_d)dt + kda\frac{d}{dt}(\psi - \psi_d) \quad (19)$$

With $kp_a$ , $ki_a$ , $kd_a$ are parameters of PID controller for the control of Roll angle , Pitch angle and Yaw angle.

And the four inputs rotors are controlled as follows [17]:

$$u_i(t) = kpe_{ui}(t) + ki\int_0^t e_{ui}(t)dt + kd\frac{d}{dt}e_{ui}(t) \quad (20)$$

The desired behaviour consists of two phases: taking off and hovering.

## IV. SIMULATION RESULTS

The mathematical model described by equation (9) and (10) was simulated on MATLAB / Simulink, with motor speed and basic system parameters (listed in TABLE I) as inputs.

The desired/reference value of Yaw angle and altitude used in simulation tests are as follows: $\psi_d = \pi / 6.28$ rad and $z_d = 25$ m.

TABLE I . MODEL PARAMETER

| Parameter | Value | Unit |
|---|---|---|
| m | 1 | Kg |
| Ix | 8.1e-3 | Kg. $m^2$ |
| Iy | 8.1e-3 | Kg. $m^2$ |
| Iz | 14.2e-3 | Kg. $m^2$ |
| Ir | 104e-6 | Kg. $m^2$ |
| b | 54.2e-6 | N/rad/s |
| l | 0.24 | m |
| g | 9.81 | m/ $s^2$ |
| d | 1.1e-6 | N.m/rad/s |

The PID parameters can be set as{kp=6, ki= 0, kd = 9} for the position x,y and z.

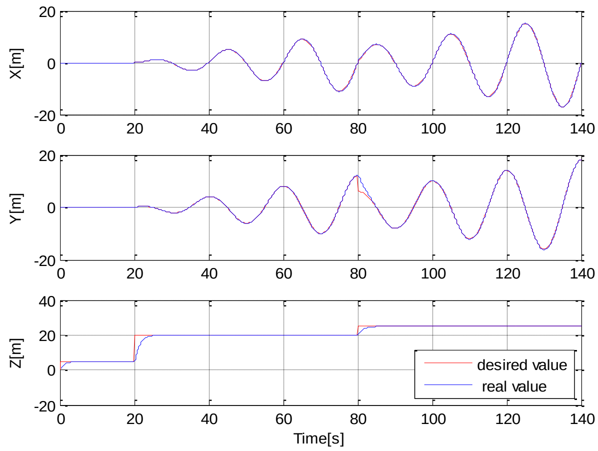


Fig. 4 Tracking Simulation results of trajectories along (X,Y,Z).

The PID parameters for the Roll, Pitch and Yaw Angle are the same because of the symmetry of dynamics quadrotor

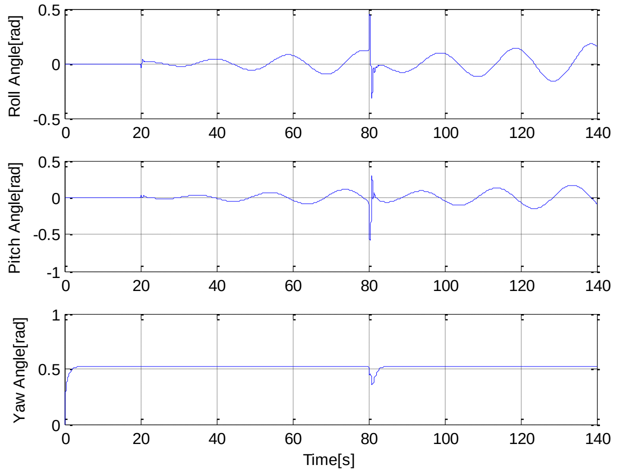


Fig. 5 Simulation results of trajectories along the Roll( $\phi$ ), Pitch ( $\theta$ ) and Yaw angle ( $\psi$ ) .

, it can be set as { $kp_a$ =10, $ki_a$ =0, $kd_a$ =15}.

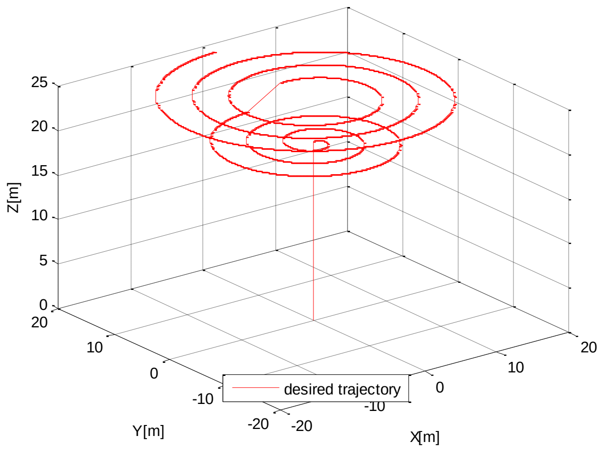


Fig. 6 Simulation result of desired trajectory in 3D.

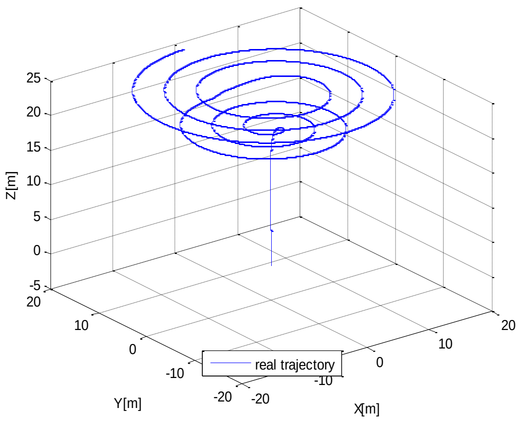


Fig. 7 Simulation result of real trajectory in 3D.

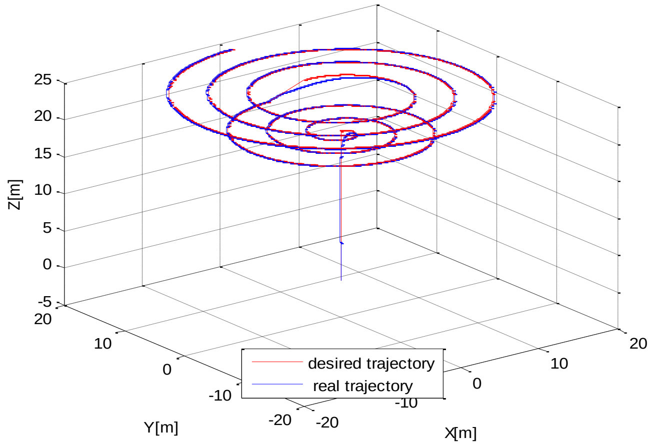


Fig. 8 Tracking Simulation results of global trajectories in 3D.

Fig. 4 represents the quadrotor positions. We can see well ,from this figure, a very good tracking of the desired trajectories.

Fig. 5 shows roll, pitch, and yaw angles during the motion.

Fig. 6 represents the trajectory of the desired movements for quadrirotor. It describes perfectly the movement of take-off and hover.

As shown in the Fig. 7, the performance of the real trajectory control is very satisfactory.

Fig. 8 describes the 3D position of quadrotor during the flight. This figure shows a good robustness towards stability and tracking for desired trajectory. Which explains the efficiency of stabilizing and control strategy developed in this paper.

Simulation results presented at the end, confirm that the proposed stabilisation and control strategy could be succefully used UAV. The PID controller proved to be well adapted to the quadrotor when flying and hovering.

## V. Conclusions

This presented work studies the stabilization and control for easy taking-off and hovering of a small quadrotor UAV using the proposed control strategy based on three state PID control method, this PID controller is based on nested loops. The result of simulation proves that the adopted method of control is simple, fast and effective for taking-off and hovering.

The take-off and hovering tasks is still challenging while the cover area varied from an environment to others. Next we will focus on new and more effective control methods for UAV [18], and how to implement control algorithms in more complicated environments is another challenging issue in our future work.

## References

[1] Raffo GV, Ortega MG, Rubio FR. An integral predictive/nonlinear H1 control structure for a quadrotor helicopter. Automatica 2010;46:29–39.

[2] Derafa L, Benallegue A, Fridman L. Super twisting control algorithm for the attitude tracking of a four rotors UAV. J Frankl Inst 2012;349:685–99.

[3] Garcia-Delgado L, Dzul A, Santibáñez V, Llama M. Quadrotors formation bansed on potential functions with obstacle avoidance. IET Control Theory Appl 2012;6(12):1787–802.

[4] Book :Unmanned Aircraft Systems ,International Symposum On Unmanned Aerial Vehicles,UAV'08 Springer.

[5] McKerrow, P.: Modelling the Draganflyer four-rotor helicopter, in: Proc. of IEEE International Conference on Robotics and Automation, 2004, pp. 3596–3601.

[6] Hamel, T., Mahony, R., Lozano, R., and Ostrowski, J.: Dynamic modeling and configuration stabilization for an X4-Flyer, in: Proc. of International Federation of Automatic Control Symposium, 2002.

[7] Altug, E., Ostrowski, J. P., and Mahony, R.: Control of a quadrotor helicopter using visual feedback, in: Proc. of IEEE International Conference on Robotics and Automation, 2002, pp. 72–77.

[8] Mistler, V., Benallegue, A., and M'Sirdi, N. K.: Exact linearization and non-interacting control of a 4 rotors helicopter via dynamic feedback, in: Proc. of IEEE International Workshop on Robot and Human Interactive Communication, 2001, pp. 586–593.

[9] Castillo, P., Dzul, A., and Lozano, R.: Real-time stabilization and tracking of a four-rotor mini rotorcraft, IEEE Trans. Control Syst. Technol. 12 (2004), 510–516.

[10] Chern, J. S., Hong, Z. C., and Chen Y. T.: Approximate chattering arc for minimum time flight, Acta Astronaut. 35 (1995), 623–631.

[11] Jing-Jing Xiong n, En-Hui Zheng. Position and attitude tracking control for a quadrotor UAV. ISA Transactions 53 (2014) 725–731

[12] Tommaso Bresciani, Modelling, Identification and Control of a Quadrotor Helicopter, Department of Automatic Control, Lund University, ISSN 0280-5316, ISRN LUTFD2/TFRT/5823.SE, October 2008

[13] Alexis K, Nikolakopoulos G, Tzes A. Model predictive quadrotor control:attitude, altitude and position experimental studies. IET Control TheoryAppl 2012;6(12):1812–27.

[14] Xu R, Özgüner Ü. Sliding mode control of a class of underactuated systems.Automatica 2008;44:233–41.

[15] David hyuchul shim,Jae-Suptan,Hong-tae Yea.A development of Unmanned helicopters for industrial applications, 2009,chapter pages 407-421.

[16] H.Khebbache, B.Sait, F.Yacef, "Modeling and stabilizing Control Laws Design Taking Into Account the Actuator Faults for an UAV Type-Quadrotor", International conference on Information Processing and Electrical Engineering , pages 345, 2012.

[17] A. Manecy, N. Marchand, F. Ruffier, S. Viollet, X4-MaG : A Low-Cost Open-Source Micro-Quadrotor and its Linux-based Controller, International Journal of Micro Air Vehicles,2015.

[18] H.Duan, P. Li, Progress in control approaches for hypersonic vehicle, Sci. China Technol. Sci. 55 (10) 2965-2970 ,2012.