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The Bayesian formulation for radiographic image segmentation of welding defect

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***Abstract*—In this paper, we propose to use the method of piecewise constant level set with the Mumford-Shah model.For image segmentation, the Mumford-Shah model needs to find the regions and the constant values inside the regions for the segmentation. For this purpose, we need to use a variational approach generally used in image segmentation which allow the extraction of the regional information (mean and variance) in an adaptive manner. Finally, we validate the proposed models by numerical results for radiographic image segmentation.**

I. INTRODUCTION

The limitations of the active contour based on borders have directed the research towards solutions where the contour is built from all the information contained in the image. In most of the cases, a stage of pre-processing is necessary to extract the relevant information. Various types of information can be extracted for example: the average and the variance of every region in the image.

The application of the radiographic image segmentation on weld defects still remains a vast field of research. In this paper, we propose a variational approach which is a generalization of the model of Chan Vese [1] while estimating in a adaptive way the regions information (mean and variance). We suggested to use the variance to distinguish the various regions which have the same mean, this one is estimated during the evolution of the curve which bounds regions.

II. MUMFORD AND SHAH METHOD

Being a natural extension of geometrical active contour, the geometrical flow is obtained by the minimization of functional one given. Those are the active contour based regions. More strong to the noise and less sensitive to the position of the initial curve, the models based on regions have for general principle to develop a curve that, in order to in the convergence, realizes a partition of the image in two homogeneous regions. It is here two regions because a single curve bounds only two domains in image. If in the contour approaches, we calculate the gradient norm, the active contour regions base generally on statistical modellings. Besides, certain models take into account at the same time the local information situated along the contour and the statistical characteristics of regions defined by the realized partition. In 1989, Mumford and Shah proposed the first segmentation approach based on active contour [2]. The basic idea of this method is to deform a curve in an image where some conditions of regularity are imposed. The basic idea of Mumford and Shah is to minimize a function of energy. So if we note $u(x,y)$ our image with value of Ω limited of $\Re^2$ in $\Re$ and Γ_i the contour of every region R_i on which the image is approached by a function g_i, then the functional to be minimized can be written as:

$$E(\Gamma_i, g_i, u) = \lambda \int\int_{\Omega} (u(x,y) - g_i(x,y))^2 dxdy + \int\int_{R_i} |\nabla g_i(x,y))|^2 dxdy + \mu \int_{\Gamma_i} dl \tag{1}$$

Where λ and μ are positive real parameters for weighting the data fidelity term and long-term contours respectively. This problem has no general solution but it has been rigorously proven that in the simple case where the functions g_i are constants, the solution always exists. It can then be shown that the value of the g_i on region R_i is the average denoted c_i of u restreinte R_i. In this framework, energy can therefore be rewritten as follows:

$$E(\Gamma_i, u) = \lambda \int\int_{R_i} (u(x,y) - c_i)^2 dxdy + \mu \int_{R_i} dl \tag{2}$$

with

$$c_i = \frac{\int\int_{\Omega} (u(x,y))dxdy}{\int\int_{\Omega} dxdy}$$

In our case, there are only two areas, limited by a curve. If we denote $C \subset \Omega$ this deformed curve, the two zones are inside C and outside C.

A. *Formulation bayesian formulation*

The Bayesian formulation for the segmentation of image is based on the assumption that the intensities of pixel in every region are the realization of a "random" process with a given density function. Based on the work of Leventon [3], we introduce the statistical parameters as unknowns. We first begin with the simplest case of two partitions. Therefore, the image consists of two regions Ω_1 and Ω_2. Let p_i be the probability distribution in the region Ω_i:

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$$p_i(u \mid \mu_i, \sigma_i) = \frac{1}{\sigma_i \sqrt{2\pi}} e^{-\frac{(u-\mu_i)^2}{2\sigma_i^2}} \quad (3)$$

By using a level set ϕ as an implicit representation of the interfaces Ω_1 and Ω_2, The corresponding energy equation (1) can be written as:

$$E(\mu_{1,2}, \sigma_{1,2}^2, \phi) = \mu \int_\Omega \delta_0(\phi(x,y))|\nabla(\phi(x,y))|dxdy + \int_\Omega (log(2\pi\sigma_1^2) + \frac{(u(x,y)-\mu_1)^2}{2\sigma_1^2})H(\phi(x,y))dxdy + \int_\Omega (log(2\pi\sigma_2^2) + \frac{(u(x,y)-\mu_2)^2}{2\sigma_2^2})(1 - H(\phi(x,y)))dxdy \quad (4)$$

This energy is minimized by an algorithm of minimization, the optimal statistical parameters are estimated, and the energy is minimized with regard to ϕ. Both stages are repeated until the convergence. For a function of level set, the form of the optimal statistical parameters can be easily obtained from the following equations:

$$\mu_1 = \frac{\int_\Omega u(x,y)H(\phi(x,y))dxdy}{\int_\Omega H(\phi(x,y))dxdy} \qquad \mu_2 = \frac{\int_\Omega u(x,y)(1-H(\phi(x,y)))dxdy}{\int_\Omega (1-H(\phi(x,y)))dxdy} \quad (5)$$

$$\sigma_1^2 = \frac{\int_\Omega (u(x,y)-\mu_1)^2 H(\phi(x,y))dxdy}{\int_\Omega H(\phi(x,y))dxdy} \qquad \sigma_2^2 = \frac{\int_\Omega (u(x,y)-\mu_2)^2(1-H(\phi(x,y)))dxdy}{\int_\Omega (1-H(\phi(x,y)))dxdy} \quad (6)$$

B. Equation d'Euler Lagrange

To minimize $E(c_1, c_2, \phi)$ we write the Euler-Lagrange equation of ϕ [4], [5]. This amounts to cancel the differential of energy at ϕ We obtain:

$$\frac{\partial\phi}{\partial t} = \delta_0(\phi)[\mu div(\frac{\nabla\phi}{|\nabla\phi|}) - \frac{(u-\mu_1)^2}{2\sigma_1^2} + \frac{(u-\mu_2)^2}{2\sigma_2^2} - log\frac{\sigma_1^2}{\sigma_2^2}] \quad (7)$$

III. Bayesian formulation in the case multiphase

By using two functions of level set, we can identify four "colors" by the following regions on Ω :

$$\{\phi_1 > 0, \phi_2 > 0\}, \{\phi_1 > 0, \phi_2 < 0\}, \{\phi_1 < 0, \phi_2 > 0\}, \{\phi_1 < 0, \phi_2 < 0\}.$$

The link between the function u and the four phases are [6], [7]:

$$u(x,y) = \begin{cases} u_{00} = \{(x,y),\ \phi_1(x,y) > 0 \quad and \quad \phi_2(x,y) > 0\} \\ u_{01} = \{(x,y),\ \phi_1(x,y) > 0 \quad and \quad \phi_2(x,y) < 0\} \\ u_{10} = \{(x,y),\ \phi_1(x,y) < 0 \quad and \quad \phi_2(x,y) > 0\} \\ u_{11} = \{(x,y),\ \phi_1(x,y) < 0 \quad and \quad \phi_2(x,y) < 0\} \end{cases} \quad (8)$$

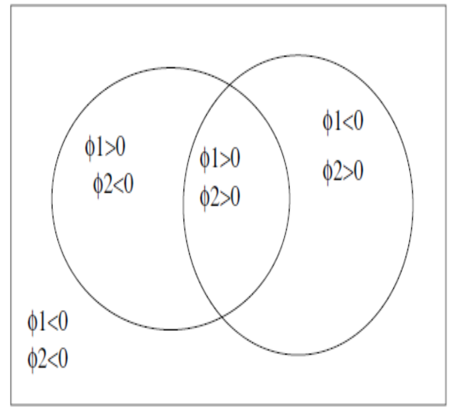

Fig. 1. Two initial curves of evolution which share the image in four regions

$$\mu_{00} = \frac{\int_\Omega u(x,y)H(\phi_1(x,y))H(\phi_2(x,y))dxdy}{\int_\Omega H(\phi_1(x,y))H(\phi_2(x,y))dxdy}$$
$$\mu_{01} = \frac{\int_\Omega u(x,y)H(\phi_1(x,y))(1-H(\phi_2(x,y)))dxdy}{\int_\Omega H(\phi_1(x,y))(1-H(\phi_2(x,y)))dxdy}$$
$$\mu_{10} = \frac{\int_\Omega u(x,y)(1-H(\phi_1(x,y)))H(\phi_2(x,y))dxdy}{\int_\Omega (1-H(\phi_1(x,y)))H(\phi_2(x,y))dxdy}$$
$$\mu_{11} = \frac{\int_\Omega u(x,y)(1-H(\phi_1(x,y)))(1-H(\phi_2(x,y)))dxdy}{\int_\Omega (1-H(\phi_1(x,y)))(1-H(\phi_2(x,y)))dxdy} \quad (9)$$

$$\sigma_{00}^2 = \frac{\int_\Omega (u(x,y)-\mu_{00})^2\ H(\phi_1(x,y))H(\phi_2(x,y))dxdy}{\int_\Omega H(\phi_1(x,y))H(\phi_2(x,y))dxdy}$$
$$\sigma_{01}^2 = \frac{\int_\Omega (u(x,y)-\mu_{01})^2\ H(\phi_1(x,y))(1-H(\phi_2(x,y)))dxdy}{\int_\Omega H(\phi_1(x,y))(1-H(\phi_2(x,y)))dxdy}$$
$$\sigma_{10}^2 = \frac{\int_\Omega (u(x,y)-\mu_{10})^2\ (1-H(\phi_1(x,y)))H(\phi_2(x,y))dxdy}{\int_\Omega (1-H(\phi_1(x,y)))H(\phi_2(x,y))dxdy}$$
$$\sigma_{11}^2 = \frac{\int_\Omega (u(x,y)-\mu_{11})^2\ (1-H(\phi_1(x,y)))(1-H(\phi_2(x,y)))dxdy}{\int_\Omega (1-H(\phi_1(x,y)))(1-H(\phi_2(x,y)))dxdy} \quad (10)$$

The resolution by the associated Euler-Lagrange equation leads to evolution equations. By using the notation $e_i(x) = log \mid \Sigma_i \mid + (I(x) - \mu_i)^T \Sigma_i^{-1}(I(x) - \mu_i)$, we obtained the following Euler Lagrange equation [8]

$$\frac{\partial\phi_1}{\partial t}(x) = \delta_0(\phi_1(x))[\mu div(\frac{\nabla\phi_1(x)}{|\nabla\phi_1(x)|}) + (e_1 - e_2)H(\phi_2) + (e_3 - e_4)(1 - H(\phi_2))] \quad (11)$$

$$\frac{\partial\phi_2}{\partial t}(x) = \delta_0(\phi_2(x))[\mu div(\frac{\nabla\phi_2(x)}{|\nabla\phi_2(x)|}) + (e_1 - e_3)H(\phi_1) + (e_2 - e_4)(1 - H(\phi_1))] \quad (12)$$

IV. Results

Figures (2) and (3) represent the results on a synthetic image, by applying the bayesian formulation of Mumford Shah for two types of initialization. The model manages to detect the contour.

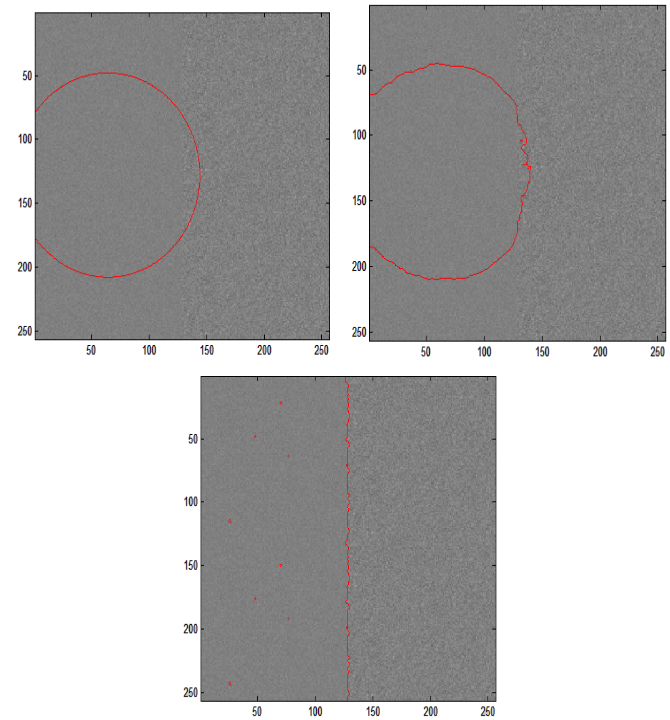

Fig. 2. Segmentation of a synthetic image by using a circle as an initial contour. μ = 4, size=256×256, Number of iterations = 10000.

In the Figure (3), we consider an initialization compound of small circles. The figure (4) represents an example of application for the case multiphase, the experience became more sensitive to the stage of initialization and more complicated regarding calculation.

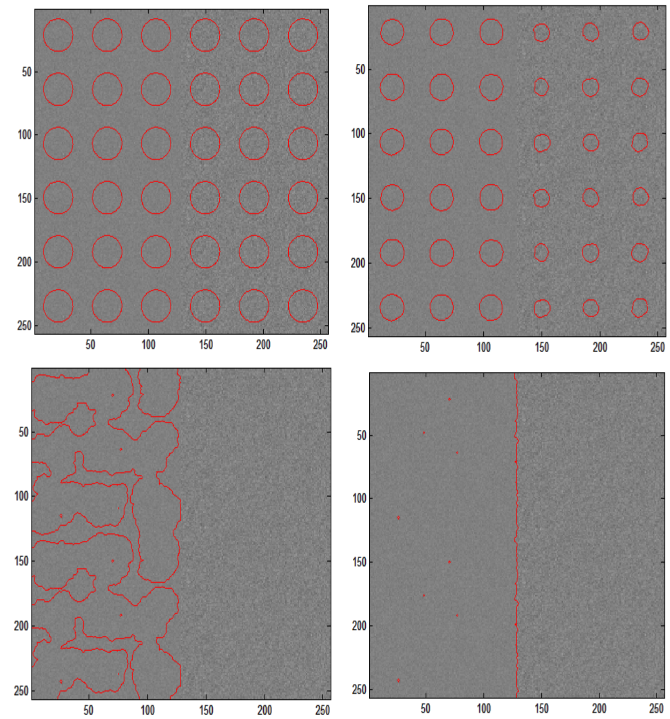

Fig. 3. Segmentation of a synthetic image by using a multicircle as an initial contour. μ = 2, size=256×256, Number of iterations=1500.

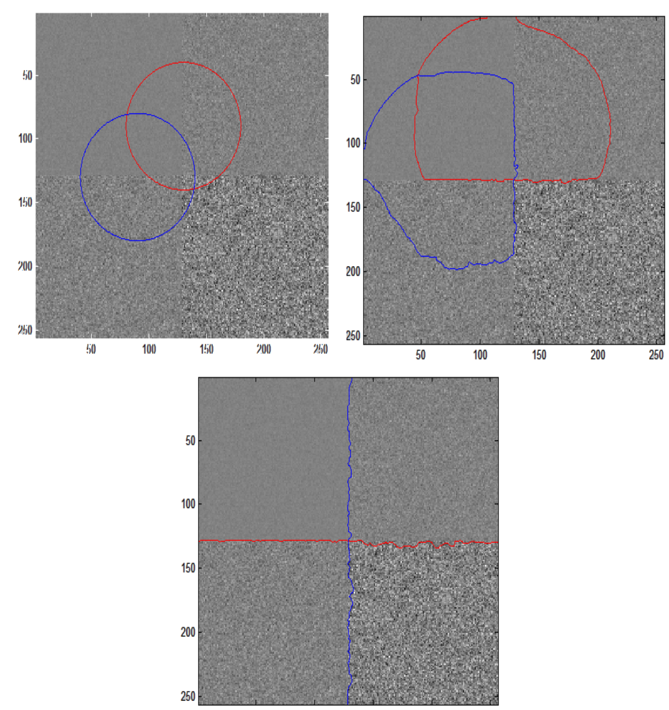

Fig. 4. Segmentation of a synthetic image by using the multiphase case. μ = 2, size=256×256, Number of iterations=1500.

The figure (5) show the results of real noisy images, by applying the above algorithm(biphase segmentation), the model gets to detect edges.

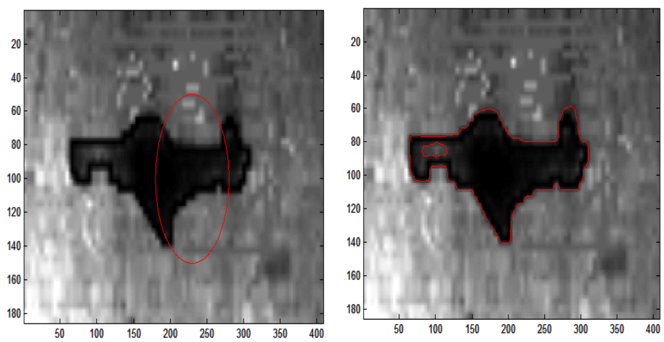

Fig. 5. Segmentation of a real radiographic image. μ = 10, size=256×256, Number of iterations=800.

To study the case of the multiphase level sets method using the formulation of Mumford and Shah, we apply the algorithm on radiography welding image. Figure (6) shows the result of the segmented image

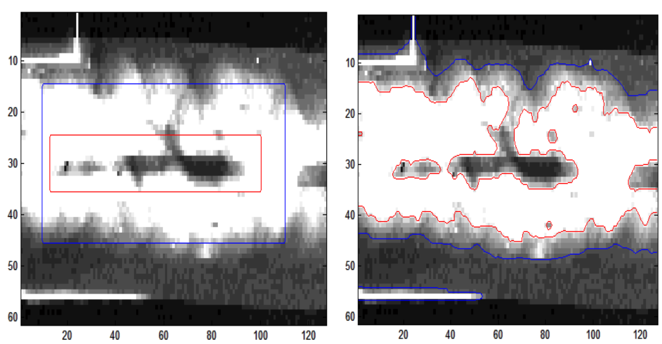

Fig. 6. Segmentation of a real radiographic image by using the case multiphase.

V. Conclusion

The Bayesian frame is more and more popular to solve numerous badly put problems such as the segmentation of the images. We have used a formulation obtained from the Bayesian model of Mumford-Shah. The representation by level set was introduced to define the partition, what leads naturally to a frontally implicit evolution. In this work, We introduced an evolution of the front based region stemming from a Bayesian formulation which allows to integrate statistical models into a geometrical approach.

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