3rd International Conference on Automation, Control, Engineering and Computer Science (ACECS'16)

Proceedings of Engineering & Technology (PET)
pp. 282-287

Nonlinear Discrete High-Gain Observer: Application to Bioreactor Model

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***Abstract*—An adaptive observer is proposed in this paper for joint estimation of unmeasurable states and unknown parameters of nonlinear systems. The formulation of discrete-time nonlinear observer design problem is realized by using Euler approximate discretization of the continuous observer.**

A nonlinear discrete high gain observer is constructed, which can be chosen to make each estimated state and parameters converge faster to its real values. Performances of the proposed approach are illustrated through simulation results related to a typical bioreactor.

I. Introduction

The construction of observers is very interesting and there are some available works have been introduced in literatures by numerous researchers [1]-[12].

Many different classes of systems have been considered and a number of papers and studies have focused on just are nonlinear observers design deal with the continuous time measurements [1][2][4]. Another observers approaches have been proposed for a particular classes of continuous-time systems with discrete-time outputs measurements [5]-[14]. However the study of the nonlinear discrete-time observers is important. Of those , design of discrete-time nonlinear observers has been paid a lot of attention in recent years and many methods of discretization are considered [4][19][21] and [22], examples include the methods using the exact discretization [18][23] and the Euler approximation [24][25], but the exact discretization of continuous-time nonlinear model is usually not possible to obtain. However, the Euler approximation is very important because not only it is easy to derive but also maintains the structure of the original nonlinear model.

The objective of this paper, is the design a adaptive nonlinear discrete-time observer for joint estimation of the unmeasurable state vector and the unknown parameters of a typical bioreactor model. A high gain discrete observer is constructed by using the Euler approximation of continuous high gain nonlinear observer proposed by Farza et al.[8].

In this theme, many different observers for state and parameters estimation in bioreactor have been treated. Indeed, Gauthier et al. [15] proposed a simple observer for nonlinear system application to bioreactor under general technological assumptions.

Martinez et al. [16] proposed joint parametric and state estimation using high gain nonlinear observers applied to a bioreactor.

In Leher et al. [19], a novel parameter estimate for a class of discrete-time nonlinearity parametrized systems was developed, the application of this method is applied to highly nonlinear system of a bioreactor.

Farza et al. [8] proposed a high gain adaptive observer for a class of uniformly observable nonlinear systems with nonlinear parametrization and sampled outputs for state and parameters estimation in a typical bioreactor.

An outline of this paper is as follows:

In the next section, the design of nonlinear continuous high gain observer proposed by Farza et al.[8] is briefly reminded. Section 3, includes the main results of this paper, in which a new time-discretization method via the Euler approximate for nonlinear system and the new discrete high gain observers are introduced. In section 4, the effectiveness of the proposed adaptive observer in discrete-time is shown through simulation results applied to a bioreactor system. Finally, section 5 concludes the paper.

II. The continuous-time output adaptive observer design

This part briefly summaries the high-gain nonlinear observer for a class of continuous-time systems, which is studied by Farza et al. [8]. Consider the class of nonlinear systems described by the following set of equations:

$$\begin{cases} \dot{x}(t) = Ax(t) + \Phi(u, x, \rho) \\ y(t) = Cx(t) = x_1(t) \end{cases} \tag{1}$$

where $x = [x_1 \cdots x_n] \in \Re^n$ and $\rho = [\rho_1 \cdots \rho_m] \in \Re^m$ respectively denote the state and the parameters estimates, $u \in \Re$ and $y \in \Re$ are respectively the input and the output of

ISSN: 2356-5608

system (1),

$$A = \begin{bmatrix} 0 & I_{n-1} \\ 0 & 0 \end{bmatrix}, \mathrm{C} = \begin{pmatrix} 1 & 0 & \cdots & 0 \end{pmatrix}$$

The sampled output adaptive observer has the following form:

$$\begin{cases} \dot{\hat{x}}(t) = A\hat{x}(t) + \Phi(u(t), \hat{x}(t), \hat{\rho}(t)) \\ \qquad - \theta{\Delta_\theta}^{-1}(S^{-1} + \Upsilon(t)P(t)\Upsilon^T(t))C^T C\tilde{x}(t) \\ \dot{\hat{\rho}}(t) = -\theta{\Omega_\theta}^{-1}P(t)\Upsilon^T(t)C^T C\tilde{x}(t) \\ \dot{\Upsilon}(t) = \theta(A - \theta S^{-1}C^T C)\Upsilon(t) + \theta\Phi(u(t), \hat{x}(t), \hat{\rho}(t)) \\ \text{with} \qquad \Upsilon(0) = 0 \\ \dot{P}(t) = -\theta P(t)\Upsilon^T(t)C^T C\Upsilon(t) + \theta P(t) \\ \text{with} \qquad P(0) = P^T(0) = 0 \end{cases} \tag{2}$$

where $\tilde{x}(t) = x(t) - \hat{x}(t)$ and $\tilde{\rho}(t) = \rho(t) - \hat{\rho}(t)$

Δ_θ, S ,Ω_θ and $\Phi(u, x, \rho)$ are respectively given by the following form:

The diagonal matrix Δ_θ is defined by:

$$\Delta_\theta = diag \begin{bmatrix} I_p & \frac{1}{\theta}I_p & \cdots & \frac{1}{\theta^{q-1}}I_p \end{bmatrix} \tag{3}$$

where θ is a positive scalar. One can easily check that the following identities hold.

$$\Delta_\theta A{\Delta_\theta}^{-1} = \theta A, \qquad C{\Delta_\theta}^{-1} = C \tag{4}$$

The unique solution S of the algebraic Lyapunov equation:

$$S + A^T S + SA - C^T C = 0 \tag{5}$$

Now, let Ω_θ be the following $m \times m$ diagonal matrix:

$$\Omega_\theta = diag \begin{bmatrix} I_p & \frac{1}{\theta^{\upsilon_1}} & \cdots & \frac{1}{\theta^{\upsilon_m - 1}} \end{bmatrix} \tag{6}$$

Φ is a continuous nonlinear function with triangular structure in the state, i.e.

$$\Phi(u, x, \rho) = \Phi_i(u, x_1, \cdots, x_i, \rho) \tag{7}$$

III. The discrete-time output adaptive observer design

The proposed discrete observer is obtained using Euler approximate discretization of the continuous observers (2),

$$\begin{cases} x(k+1) = (I_n + T_e A)x(k) + T_e\Phi(u_k, x_k, \rho_k) \\ y(k) = Cx(k) = x_1(k) \end{cases} \tag{8}$$

where T_e is the sampling time of the discrete observer.

The choice of T_e is such that the discrete-time nonlinear system as stated above provides a good approximation of the corresponding continuous-time.

The proposed discrete observer is obtained by using Euler approximata discretization of the continuous observer (2) where T_e is the sampling time of the discrete observer:

$$\begin{cases} \hat{x}(k+1) = (I_n + T_e A)\hat{x}(k) + T_e\Phi(u(k), \hat{x}(k), \hat{\rho}(k)) \\ \qquad - T_e\theta{\Delta_\theta}^{-1}(S^{-1} + \Upsilon(k)P(k)\Upsilon(k)^T)C^T C\tilde{x}(k) \\ \hat{\rho}(k+1) = \hat{\rho}(k) - T_e\theta{\Omega_\theta}^{-1}P(k)\Upsilon(k)^T C^T C\tilde{x}(k) \\ \Upsilon(k+1) = (I + T_e\theta(A - S^{-1}C^T C))\Upsilon(k) \\ \qquad + T_e\theta\Phi(u(k), \hat{x}(k), \hat{\rho}(k)) \\ P(k+1) = (I - T_e\theta P_k\Upsilon(k)^T C^T C\Upsilon_k + T_e\theta)P_k \end{cases} \tag{9}$$

where $\hat{x} = (\hat{x}_1 \ldots \hat{x}_n)^T \in \Re^n$ and $\hat{\rho} = (\hat{\rho}_1 \ldots \hat{\rho}_m)^T \in \Re^m$ respectively denote the state and parameter estimates, and are respectively the input and the output of system (41), Let us introduce some useful notations and definitions related to high gain observer design.

A, C , Δ_θ, S ,Ω_θ and $\Phi(u, \hat{x}, \hat{\rho})$ are respectively given by (2), (5), (7), (9) and (42) and θ is a positive scalar.

The nonlinear descritization requires an appropriate reformulation of the assumption 1,2,3 and 4 , namely

A1. The state $x(k)$, the control $u(k)$ and the unknown parameters ρ are bounded, i.e $x(k) \in X$, $u(k) \in U$ and $\rho \in \Omega$ where $X \subset \Re^n$, $U \subset \Re$ and $\Omega \subset \Re^m$ are compacts sets.

A2. $\Phi : \Re^n \times \Re^p \rightarrow \Re^n$ is a discrete nonlinear function. Morever, it is Lipschitz with respect to x and ρ and uniformily in u for all $(u, x, \rho) \in U \times X \times \Omega$; its Lipschitz constant is denoted L_d*.

A3. the function $\Phi^j_{vj}(u, x, \rho), j = 1, \ldots, m$, satisfy the following condition $\forall(u, \hat{x}) \in U \times X, \forall(\hat{\rho}, \rho) \in \Omega^2$

$$\left\| \Phi^j_{vj}(u, \hat{x}, \hat{\rho}) - \Phi^j_{vj}(u, \hat{x}, \rho) \right\| \leq T_e.\upsilon\sqrt{\lambda_M(P)/_{(n\lambda_M(S))}} \tag{10}$$

Based on (8), the Lipschitz constant of the Euler approximation is $L^*_d = T_e.\upsilon$. Again, we assume $L^*_d < 1$. This is even less restrictive than in section 2, because here T_e directly multiplies υ and can be chosen sufficiently small.

where $\|.\|$ is the induced 2-norm, T_e is a sampling time, S and P are respectively given by (2) and (5) and υ is a positive scalar satisfying $L^*_d = T_e.\upsilon < 1$.

A4. The input u is such that for any trajectory of system (8) starting from $(\hat{x}(0), \hat{\rho}(0)) \in X \times \Omega$, the matrix $C\Upsilon(t)$ is persistently exciting i.e.

$\exists\delta_1, \delta_2, \exists T > 0; \forall t = k.T_e \geq 0$

$$\delta_1 I_m \leq \sum_{t=k.T_e}^{k.T_e+T} \Upsilon^T(\tau)C^T C\Upsilon(\tau)d(\tau) \leq \delta_2 I_m \tag{11}$$

IV. A typical bioreactor Model

In this section, we will illustrate the discrete-time high gain observer by means of an example concerning a typical bioreactor. We consider a simple microbial culture which involves a single biomass of concentration $x_2(t)$ growing on a single substrate of concentration $x_1(t)$. The bioprocess is supposed to be continuous with a scalar dilution rate $D(t)$ and an input substrate concentration S_{in}.

A. System description

The mathematical dynamical model of this process is hence constituted by the following two mass balance equations:

$$\begin{cases} \dot{x}_1(t) = -\mu^* x_1(t)x_2(t)/(k_c x_2(t) + x_1(t)) \\ \qquad + D(k)(S_{in} - x_1(t)) \\ \dot{x}_2(t) = \mu^* x_1(t)x_2(t)/(k_c x_2(t) + x_1(t)) - D(k)x_2(t) \end{cases} \tag{12}$$

where:

- $x_1(t)$: cell density of inhibitor resistant species,
- $x_2(t)$: cell density of inhibitor sensitive species,
- $D(t)$: dilution rate,
- $S_{in}(t)$: the input substrate concentration,
- μ^*: the Contois law parameter,
- $k_c(t)$: the input substrate concentration.

The objective of this paper is to estimate $x_2(t)$ together with the Contois law parameters μ^*, $k_c(t)$ and the input substrate concentration S_{in}.

System (12) has been already considered in Farza et al. [8], where the transformation:

$$\begin{array}{c} f:(x_1,x_2)^T \in X \\ \downarrow \\ Z:\left(z_1 = x_1, z_2 = \frac{-\mu^* x_1(t)x_2(t)}{(k_c x_2(t)+x_1(t))}\right)^T \end{array} \quad (13)$$

where f is a diffeomorphism from X, the system (12) can be input under the following form:

$$\begin{cases} \dot{z}_1(t) = z_2 + D\left(\rho_1 - z_1\right) \\ \dot{z}_2(t) = -\rho_3(z_2/z_1)^2(z_2 + D\left(\rho_1 - z_1\right)) \\ +\rho_2 z_2(1+\rho_3(z_2/z_1))^2 - Dz_2\left(1+\rho_3(z_2/z_1)^2\right) \\ y = z_1 \end{cases} \quad (14)$$

where $\rho_1 = S_{in}$, $\rho_2 = \mu^*$ and $\rho_3 = k_c/\mu^*$.

After the transformation, the system (14) is described under the form:

$$\begin{cases} \dot{z}(t) = Az(t) + \Phi(u,z,\rho) \\ y(t) = Cz(t) = z_1(t) \end{cases} \quad (15)$$

with

$$\Phi(u,z,\rho) \triangleq \begin{pmatrix} D\left(\rho_1 - z_1\right) \\ \\ -\rho_3(z_2/z_1)^2(z_2 + D\left(\rho_1 - z_1\right)) \\ \ldots \quad \ldots + \rho_2 z_2(1+\rho_3(z_2/z_1))^2 \\ \ldots \quad \ldots \quad - Dz_2\left(1+\rho_3(z_2/z_1)^2\right) \end{pmatrix} \quad (16)$$

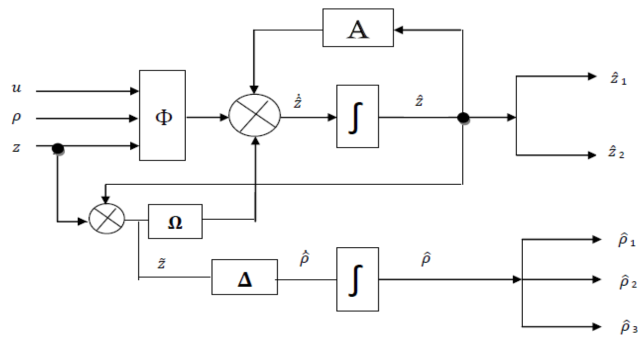

Fig. 1. Nonlinear continuous high gain observer design proposed by Farza et al. [8].

where

$$\Delta(t) = \theta\Delta_\theta{}^{-1}(S^{-1} + \Upsilon(t)P(t)\Upsilon(t)^T)C^T C\tilde{z}(t)$$

and

$$\Omega(t) = \theta\Omega_\theta{}^{-1}P(t)\Upsilon(t)^T C^T C\tilde{z}(t)$$

B. Discrete-time system

Consider the following system representing a typical bioreactor, discretized using a Euler approximation method, we have:

$$\begin{cases} z_{1,k+1} = z_{1,k} + T_e z_{2,k} + T_e D\left(\rho_1 - z_{1,k}\right) \\ z_{2,k+1} = z_{2,k} - T_e\rho_3(z_{2,k}/z_{1,k})^2\left(z_{2,k} + D\left(\rho_1 - z_{1,k}\right)\right) \\ \qquad + T_e\rho_2 z_{2,k}(1+\rho_3(z_{2,k}/z_{1,k}))^2 \\ \qquad - T_e D z_{2,k}\left(1+\rho_3(z_{2,k}/z_{1,k})\right) \\ y_k = z_{1,k} \end{cases} \quad (17)$$

The model (17) can be written as follows:

$$\begin{cases} z(k+1) = (I_n + T_e A)z(k) + T_e\Phi(u,z,\rho) \\ y(k) = Cz(k) = z_1(k) \end{cases} \quad (18)$$

Consider the proposed discrete high gain observer described by the following form:

$$\begin{cases} \hat{z}(k+1) = (I_n + T_e A)\hat{z}(k) + T_e\Phi(u(k),\hat{z}(k),\hat{\rho}(k)) \\ \qquad - T_e\theta\Delta_\theta{}^{-1}(S^{-1} + \Upsilon(k)P(k)\Upsilon(k)^T)C^T C\tilde{z}(k) \\ \hat{\rho}(k+1) = \hat{\rho}(k) - T_e\theta\Omega_\theta{}^{-1}P(k)\Upsilon(k)^T C^T C\tilde{z}(k) \\ \Upsilon(k+1) = (I + T_e\theta(A - S^{-1}C^T C))\Upsilon(k) \\ \qquad + T_e\theta\Phi(u(k),\hat{z}(k),\hat{\rho}(k)) \\ P(k+1) = (I - T_e\theta P_k\Upsilon(k)^T C^T C\Upsilon_k + T_e\theta)P_k \end{cases} \quad (19)$$

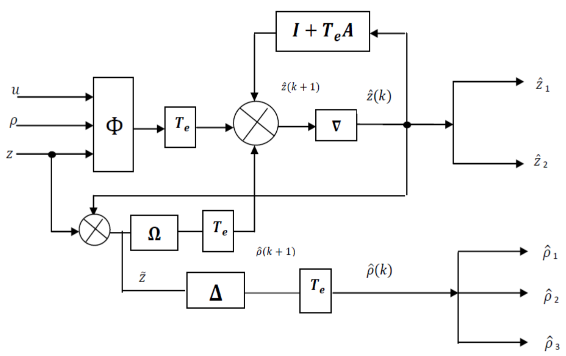

Fig. 2. Nonlinear discrete high gain observer design.

where

$$\Delta(k) = \theta\Delta_\theta{}^{-1}(S^{-1} + \Upsilon(k)P(k)\Upsilon(k)^T)C^T C\tilde{z}(k)$$

and

$$\Omega(k) = \theta\Omega_\theta{}^{-1}P(k)\Upsilon(k)^T C^T C\tilde{z}(k)$$

The estimation of the state z and the unknown parameters ρ_1, ρ_2 and ρ_3 can then be achieved using an observer of the form (19). One can check that the values of the characteristic indices associated to the unknown parameters are:

- $\upsilon_1 = 1$, $\upsilon_2 = \upsilon_3 = 2$,

- $\Omega_\theta = diag(1/\theta, 1/\theta^2, \theta^2)$

The following theorem shows that how the proposed design observer proposed in (9) using Euler approximate discrete-time model guaranteing observer practical converge when applied to the exact model.

C. Simulation results

To verify the effectiveness of the proposed observer, simulations have been done with the discrete high-gain observer. The observer are used to estimate system states and the unknown parameters in the typical bioreactor.

The numerical values used for this simulation follow the work of Farza et al. [8] and are reproduced in the table 1

$Parameters$	$Values$	$Units$
S_{in}	20	(g/L)
μ^*	1.064	$(1/h)$
k_c	4.39	(g/g)
ρ_1	20	(g/L)
ρ_2	1.064	$(1/h)$
ρ_3	4.1259	(h)

TABLE I
THE NUMERICAL VALUES FOR SIMULATION

The resulting values of the parameters ρ_i are:

$\rho_1 = 20(g/L)$, $\rho_2 = 1.064(1/h)$ and $\rho_3 = 4.1259(h)$,

The above system is also studied in [6]. Notice that the equilibrium point is $(x_1(0); x_2(0)) = (15; 7.5)(g/L)$;

while the observer was initialized with

$\hat{z}_1(0) = z_1(0) = 1.5z_2(0)$, $\hat{\rho}_2(0) = \hat{\rho}_3(0) = 0$

and $\hat{\rho}_1(0) = 0.5\rho_1 = 10(g/L)$.

All the entries of the matrix $\Upsilon(0) = 0$ were set to zero and $P(0)$ was set to the 3×3 identity matrix.

In this simulation, we consider a constant sampled time $T_e = 0.1s$. Indeed, the choice of T_e is such that the discrete-time nonlinear system as stated above provides a good approximation of the corresponding continuous-time dynamics of the bioreactor.

Simulation results are reported in figures 3,4,5 and 6. Figure 3 shows the parameter estimation $\hat{\rho}_1$, $\hat{\rho}_2$ and $\hat{\rho}_3$ and the true values ρ_1, ρ_2 and ρ_3. Since the parameter estimation error $\tilde{\rho}(k) = \rho(k) - \hat{\rho}(k)$ are given in Figure 4. The Figure 5 and 6, shows the evaluations of the state variables $z_1(k)$, $z_2(k)$, their estimates variables $\hat{z}_1(k)$, $\hat{z}_2(k)$ and the state estimation errors $\delta_1(k)$, $\delta_2(k)$.

Based on simulation results, one can conclude the following:

- From Figure 3, it is clear that the parameter estimates converge to their true values,
- From Figure 4, it is shown that the parameter estimation error converges to a neighborhood around zero,
- The Figure 5 and Figure 6, it is clearly seen that the state variables converges into the state estimate variables as fast as required and the estimation errors remain the neighbor of zero.

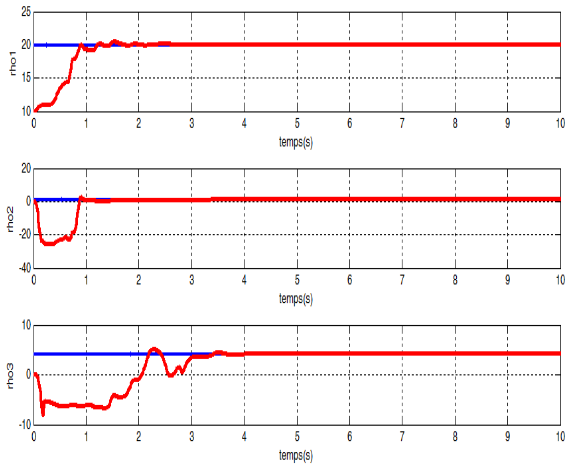

Fig. 3. Estimation of the unknown parameters ρ_1,ρ_2 and ρ_3.

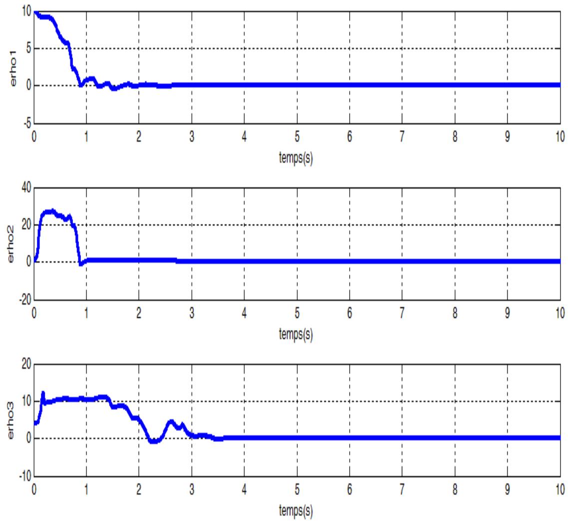

Fig. 4. Error estimation of the unknown parameters ρ_1,ρ_2 and ρ_3.

The high gain observer attained good performances and agreed well with the experimental results.

V. CONCLUSION

A discrete high gain adaptive observer has been proposed in this paper to calculate the state and the unknown parameters.

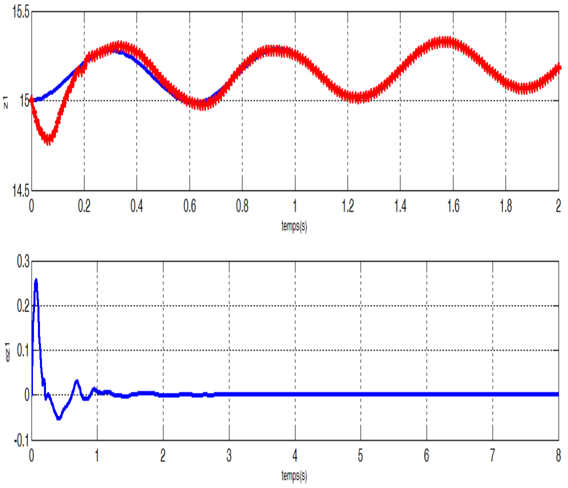

Fig. 5. Evolution of the state variable z_1, their estimate $\hat{z}_1$ and their error $z_1 - \hat{z}_1$.

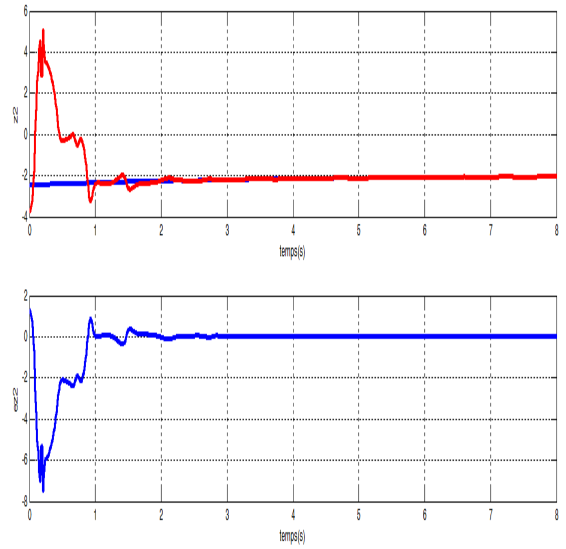

Fig. 6. Evolution of the state variable z_2, their estimate $\hat{z}_2$ and their error $z_2 - \hat{z}_2$.

The new discrete-time observer is obtained after discretization using the Euler approximation of the observer introduced by Farza et al. [8]. Of a typical bioreactor model, simulation results have been given and demonstrated the good performances with smaller error and faster estimation.

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