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A discrete chaotic multimodel based on 2D Hénon maps

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***Abstract*—In this paper, we propose to design a new family of discrete-time chaotic systems based on the use of the multimodel approach. This approach is used to represent a way to generate new family of discrete-time chaotic systems, from p discrete time chaotic basis models. Our results are illustrated on the specific case of a new discrete-time chaotic multimodel based on two chaotic 2D Hénon maps with two different sets of parameters leading to different behaviors. The chaos characterization of obtained multimodels is performed using bifurcation diagrams.**

***Index Terms*—Chaos, multimodels, discrete-time, Hénon map, bifurcation diagram**

I. Introduction

In the recent years, there is a growing interest to the use of chaos-based techniques in the secure communication field. Chaotic systems proved that they are efficient to build robust cryptosystems due to their several features especially the noise-like time series and the sensitive dependence on initial conditions [1-4].

In order to have an efficient cryptosystem, some rules, detailed in [5], need to be applied where the complexity of the chaotic used system is considered as a fundamental issue for all types of cryptosystems.

In parallel, a global approach based on multiple Linear Time Invariant (LTI) models defined around different operating point has received a significant attention. This multimodel approach is a convex polytopic representation that can be obtained by the interpolation of LTI models. Every model represents a valid operating range. Three techniques are used to obtain the mutimodel either by identification [6-9] when input and output data are available or by linearization around different operating points or by polytopic transformation[6-9], if we have the analytic model. Numerous works was published concerning the mutimodel approach and its stability study [6-9].

In this paper, the mutimodel approach is used to build a new class of discrete-time chaotic systems which constitutes an extension of previous results of continuous chaotic processes using the mutimodel approach. In [10] Cherrier and Boutayeb have proposed to use the definition of multimodel to interpolate continuous chaotic subsystems. It's proven that the resulting system has a complex chaotic behavior.

The paper is organized as follows: Section II presents the way to design a multimodel based on p discrete time chaotic subsystems having different parameters and interpolate them using the appropriate activation function. The specific case of discrete time multimodels based on two 2D Hénon maps is presented in section III. It is also tested in this section; the chaotic behaviors through bifurcation diagrams and concluding remarks are given. In section IV, a case of interpolation of a chaotic and a non-chaotic 2D Hénon maps is considered.

II. Building a new chaotic discrete-time multimodel: Problem statement

Consider the n-dimensional discrete-time in Lurie systems as follows

$$x(k+1) = A_i x(k) + f_i(x(k)),\ i = 1, 2, ..., p \qquad (1)$$

kT is the discrete-time, T sampling time, $x \in R^n$ is the state vector A_i, $i = 1, 2, ..., p$, are $(n \times n)$ constant matrices and $f_i(x(k))$, $i = 1, 2, ..., p$, nonlinear vector.

The mutimodel approach proposed in [10] is extended to the case of discrete-time chaotic systems to interpolate p subsystems having different behaviors. The new multimodel, resulting from the interpolation of systems (1) with different sets of parameters, is described as following

$$\begin{cases} x(k+1) = \sum_{i=1}^{p} \mu_i(\xi)(A_i x(k) + f_i(x(k))) \\ y(k) = Cx(k) \end{cases} \qquad (2)$$

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where y is the output vector , C a constant matrix with an appropriate size and $\mu_i, i = 1, 2, ..., p$ activation functions modeling the weighting of the sub-model i, characterized in the global model, by A_i, $i = 1, 2, ..., p$ such us

$$\begin{cases} \sum_{i=1}^{p} \mu_i(\xi) = 1 \\ 0 \leq \mu_i(\xi) \leq 1 \quad \forall i = 1, p \end{cases} \tag{3}$$

Since the multimodel is built in order to be integrated in a cryptosystem and for the purpose of increasing security, the activation functions have to be chosen such as they ensure a kind of "mixing" between the different sub-models. It doesn't have to favor a model, but allows a real transition between them. This allows in one hand, to enhance the complexity of the system and, secondly, to ensure a continuous synchronization, in the sense that there is no loss of synchronization [10].

In the next section, are proposed two multimodels corresponding to (2) built from two subsystems having two different behaviors using an appropriate activation function.

The first mutimodel is a combination of two chaotic systems and the second a combination of a chaotic and a non-chaotic system. Bifurcation diagrams of the obtained multimodels are used to show if they are chaotic or not .

III. IMPLEMENTATION OF THE CHAOTIC 2D HÉNON MAPS

In this section, for this first example, we have chosen as base models two systems of 2D Hénon maps, with two different sets of parameters. Considered first discrete-time 2D Hénon subsystem, which is described as follows [10-14]

$$\begin{cases} x_1(k+1) = a_1 - x_1^2(k) + b_1 x_2(k) \\ x_2(k+1) = x_1(k) \end{cases} \tag{4}$$

where $x = \left[x_1(k), x_2(k)\right]$ is the state vector and a_1 and b_1 are bifurcation parameters of Hénon map.

To build the multimmodel, the system (4) is interpolated with the following Hénon map using two different sets of parameters characterizing by two different chaotic behaviors.

$$\begin{cases} x_1(k+1) = a_2 - x_1^2(k) + b_2 x_2(k) \\ x_2(k+1) = x_1(k) \end{cases} \tag{5}$$

The corresponding attractors are found respectively in Fig. 1 and Fig. 2.

The parameters chosen for the two basic models (4) and (5) are such as a_1 = 1.4, b_1 = 0.3, a_2 =1.15, b_2 =0.4 with initial values $x(0) = (0, 0, 0)$ [10-14].

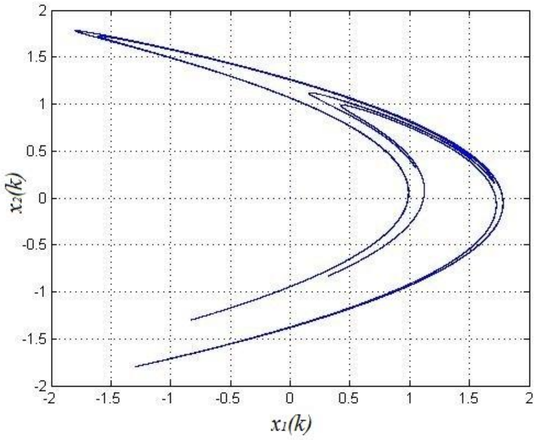

Fig 1. The chaotic attractor of the Hénon map for a_1 = 1.4, b_1 = 0.3 and $x(0)=(0,0)$

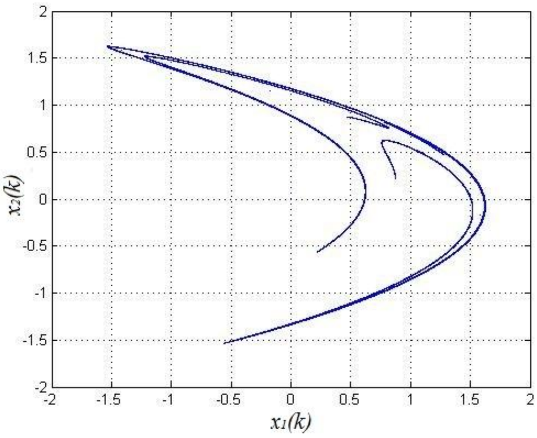

Fig 2. Chaotic attractor of Hénon map for a_2 =1.15, b_2 =0.4 and $x(0) = (0, 0)$

Once can note that the first subsystem Hénon map does not have a strange attractor for all values of the parameters a_1 and b_1. For example, by keeping b_1 fixed at 0.3, the bifurcation diagram of Fig. 3 shows that for $0.4 < a_1 < 1.1$ the Hénon map has a stable periodic orbit.

Besides, as presented in Fig. 4, for $b_2 = 0.4$ and $a_2 > 1$, the second subsystem discrete-time Hénon map (5) has a chaotic behavior, illustrated by the bifurcation diagram of Fig. 4.

The chosen activation function is described as following [10]

$$\mu(x_2) = (1 + tanh(\delta x_2)) / 2 \tag{6}$$

where δ is a parameter set so that the μ function performs a real transition between the two Hénon subsystems.
The resulting multi-model simulations are shown in Fig. 5 , δ was set at the value of 0.5.

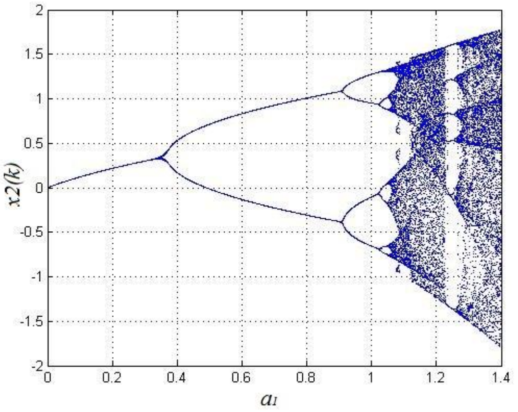

Fig 3. The bifurcation diagrammen of the Hénon mapfor a_1 variable, b_1 =0.3, and $x(0) = (0, 0)$

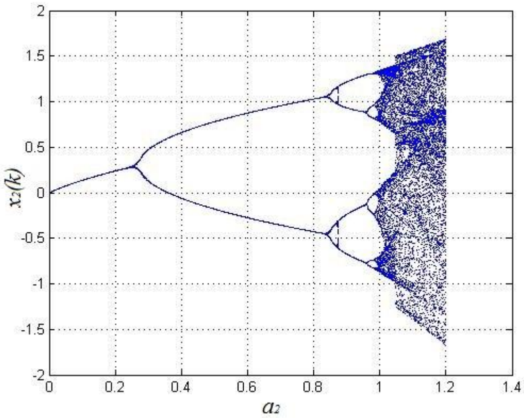

Fig 4. Bifurcation diagramme Hénon map for a_2 variable, b_2 =0.4 and $x(0) = (0, 0)$

Bifurcation diagram Fig. 6a shows that the chaotic behavior of the multimodel is obtained for $a_1 \in [0.92, 1.41]$, such as $b_1 = 0.3$, b_2 =0.4 and $a_2 \simeq 1.15$ while, for the same fixed values, the chaotic behavior of (4) is obtained for $a_1 \in [1.15, 1.41]$ as shown in Fig.3. The interval size of the multimodel's a_1 values originating chaos is larger than those of system (4). The same applies is obtained for the multimodel by varying the parameter a_2 and for the same fixed values. In fact, as it is shown in Fig.6b the chaotic behavior of the multimodel is obtained for $a_2 \in [1.1, 1.2]$. While the chaotic behavior of (5) is obtained for $a_2 \in [0.65, 1.2]$. The mutlimodel obtained from the combination of two chaotic systems (4) and (5) gives us a larger interval of parameters values which is advantageous to the security of the encrypting scheme [5].

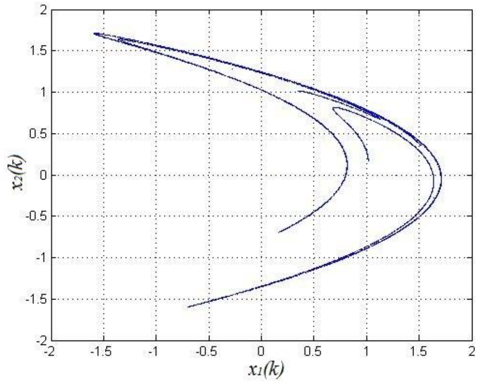

Fig 5. The chaotic attractor of the multimodel for $a_1 = 1.4$, b_1 =0.3, a_2 =1.15, b_2 = 0.4 and $x(0) = (0, 0)$

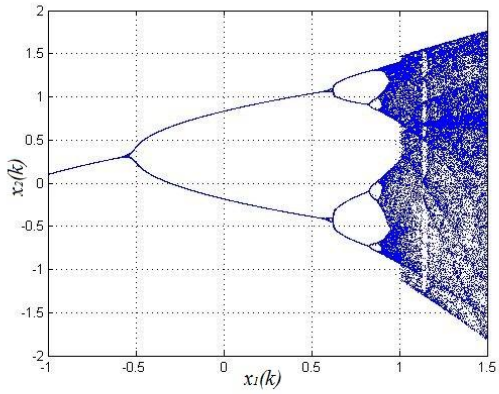

Fig 6a .Bifurcation diagram of the discrete-time multimodel for a_1 variable, b_1 =0.3, a_2 =1.15, b_2 =0.4 and $x(0) = (0, 0)$

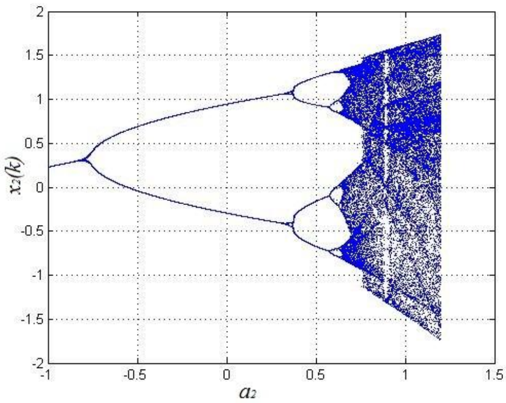

Fig 6b.Bifurcation diagram of the discrete-time multimodel for a_2 variable, $a_1 = 1.4$, $b_1 = 0.3$, b_2 =0.4 and $x(0) = (0, 0)$

IV. INTERPOLATION OF A CHAOTIC AND A NON-CHAOTIC 2D HÉNON MAPS

In this section, for this second example, we have chosen as base models two systems: a chaotic 2D Hénon subsystem (4) with fixed parameters $a_1 = 1.4,\ b_1 = 0.3$, and a non-chaotic 2D Hénon subsystem (5).

For the set parameter $a_2 = 0.3,\ b_2 = -0.9$ with initial values $x(0) = (0,0)$, the subsystem doesn't present a chaotic behavior as it is shown in Fig. 7. In fact, the figure doesn't illustrate a strange attractor and the bifurcation diagram Fig. 8 shows that chosen parameters doesn't lead to chaos.

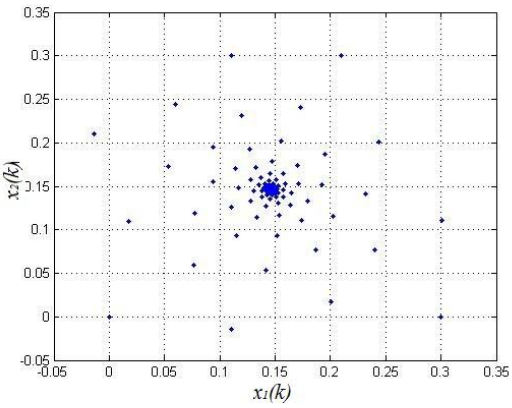

Fig 7. Attractor of the non chaotic Hénon map for $a_2 = 0.3,\ b_2 = -0.9$ and $x(0) = (0,0)$

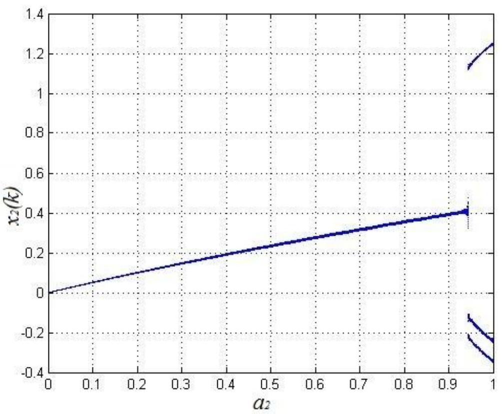

Fig 8. Bifurcation diagram of the Hénon map for a_2 variable, $b_2 = -0.9$ and $x(0) = (0,0)$

The simulation results of Fig. 9 don't give a clear illustration of the multimodel attractor obtained from the interpolation of the first subsystem (4) characterized by $a_1 = 1.4,\ b_1 = 0.3$ and the second subsystem (5) characterized by $a_2 = 0.3,\ b_2 = -0.9$ and for the chosen activation function (6).

However, bifurcation diagram of Fig. 10 shows that for the chosen parameter the multimodel doesn't have a chaotic behavior.

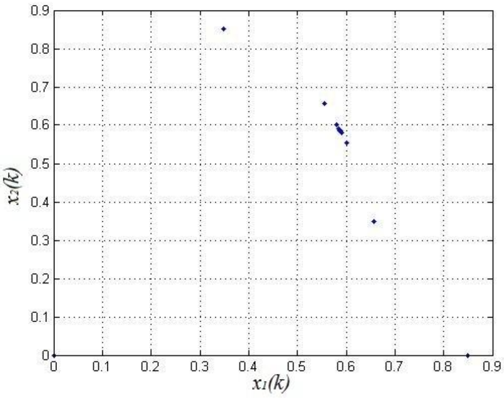

Figure 9. Attractor of the obtained multimodel for $a_1 = 1.4,\ b_1 = 0.3,\ a_2 = 0.3,\ b_2 = -0.9$ and $x(0) = (0,0)$

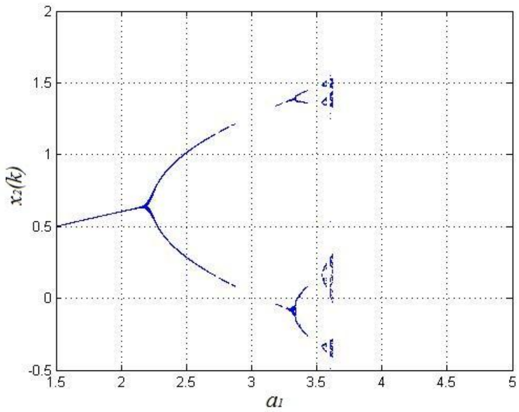

Figure 10. Bifurcation diagram of the multimodel for a_1 variable, $b_1 = 0.3,\ a_2 = 0.3,\ b_2 = -0.9$ and $x(0) = (0,0)$

V. CONCLUSION

The multimodel approach is used in this paper to build discrete-time chaotic multimodels. It has been shown, by the use of discrete time Hénon map 2D, that the interpolation of two chaotic systems can enhance the complexity of the chaos however mixing a non-chaotic system with chaotic one doesn't lead necessary to chaos. Bifurcation diagrams illustrate the range of possible parameters, giving to the multimodel, a chaotic behavior.

REFERENCES

[1] A. V. Oppenheim, K. M. Cuomo and S. H. Strogatz, "Synchronization of Lorenz-based chaotic circuits with applications to communications," IEEE Trans. on Circ. Syst. II, vol.40, no.10, pp. 626–633, 1993.

[2] H. Dedieu, M. P. Kennedy and M. Hasler, "Chaos shift keying: modulation and demodulation of a chaotic carrier using self-synchronizing Chua's circuits," IEEE Trans. on Circ. Syst. I: vol. 40, no.10, pp.634–642, 1993

[3] U. Parlitz, L. O. Chua, L. Kocarev K. S. Halle and A. Shang, "Trnasmission of digital signals by chaotic synchronization." Int. J. of Bifurcat. and Chaos, vol. 2, 1993.

[4] G. Millérioux and C. Mira, "Coding scheme based on chaos synchronization from noninvertible maps" Int. J. of Bifurcat. and Chaos, vol.8, no.10, pp.2019–2029, 1998.

[5] G. Alvarez and S. Li, "Some basic cryptographic requirements for chaos-based cryptosystems," Int. J. of Bifurcat. and Chaos, vol.16, no.8, pp. 2129–2151, 2006.

[6] M. Chadli, J. Ragot, and D. Maquin, "Multiquardratic stability and stabilization of continuous time multiple-model," 11th IFAC Symposium on Automation in Mining, Mineral and Metal processing, Nancy, France, 2004

[7] N. Elfelly, J-Y. Dieulot, M. Benrejeb and P. Borne, "A new approach for multimodel identification of complex systems based on both neural and fuzzy clustering algorithms," Eng. App. of Artificial Intelligence , vol. 23, n° 7, pp. 1064-1071, 2010

[10] E. Cherrier, M. Boutayeb J. Ragot, M.A. Aziz-Alaoui, "Chaotic multimodels: application to observer-based synchronization," IEEE Trans. on Cir. Syst. II.

[11] G. Baier, and M. Klein,"Maximum hyperchaos in generalized Hénon circuit," Phys. Lett, A., vol.151, no.67, pp. 281–284, 1990.

[8] N. Elfelly, J.-Y. Dieulot, M. Benrejeb and P. Borne," Multimodel control design using unsupervised classifiers," Studies in Informatics and Control, ISSN 1220-1766, vol. 21, no.1, pp. 101-108, 2012.

[9] N. Elfelly, J-Y. Dieulot, M. Benrejeb and P. Borne, "A Neural Approach of Multimodel Representation of Complex Processes," Int. J. of Computers, Communications & Control, vol. 3, pp.149-160, 2008.

[12] RL. Filali, M Benrejeb, P. Borne, "On observer-based secure communication design using discrete-time hyperchaotic systems," Communications in Nonlinear Science and Numerical Simulation, vol. 19, no.5, pp.1424-1432, 2014

[13] RL. Filali, S. Hammami, M. Benrejeb, "On Synchronization, Anti-Synchronization and Hybrid Synchronization of 3D Discrete Generalized Hénon Map," Nonlinear Dynamics and Systems Theory," vol.12, no.1, pp. 81-95, 2012

[14] E. Li, G. Li, G. Wen and H. Wang, Hopf, "Bifurcation of the third-order Hénon system based on an explicit criterion Nonlinear Analysis: Theory, Methods & Applications, vol.70, no.9, pp.3227–3235, 2009.

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