

Modeling and Optimal LQG Controller Design for a Quadrotor UAV

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Abstract—This paper deals with the modeling and optimal Linear Quadratic Gaussian (LQG) controller design for a Quadrotor Vertical Take-Off and Landing (VTOL) type of Unmanned Aerial Vehicle (UAV). Such a LQG-based control strategy is investigated to stabilize the attitude and altitude dynamics of the studied Quad rotorcraft. All aerodynamic forces and moments of the Quadrotor UAV are described within an inertial frame and a dynamical model is obtained thanks to the Newton-Euler formalism. An optimal LQG controller is then designed for the attitude and altitude stabilization of the plant, linearized around an equilibrium flight point. Several simulation results are carried out in order to show the effectiveness and robustness of the proposed LQG-based flight stabilization approach.

Keywords:- Quadrotor UAV, modeling, optimal LQG control, attitude and altitude stabilization.

I. INTRODUCTION

The Unmanned Aerial Vehicles (UAVs), particularly the Quadrotors, are flying robots without pilot which are able to conduct missions in autonomous or half-autonomous modes also in hostile and disturbed environments [1], [2]. Among the tasks to be conducted with these robots are found military acknowledgment, monitoring missions and civilian missions such as the inspection of dams and border monitoring, the prevention of forest fires and others [3], [4].

In recent years, these Quadrotors have seen a great evolution in terms of the miniaturization of these actuators and sensors, the modeling and especially the flight control design [5], [6], [7]. This explains the interest shown by many researchers to study the flight dynamics and the control laws of these kinds of vehicles. In [3], [8], the authors propose a PID controller to drive the position and the attitude of a Quadrotor. In [3], [9], [10], a Sliding Mode Control (SMC) approach, applied to a non-linear model of the Quadrotor, is used to stabilize its dynamics. The works in [7], [3] illustrate the Backstepping approach for a path tracking of a Quadrotor. In [11], [12], the authors developed a Model Predictive Control (MPC) strategy for the flight stabilization of such a vehicle.

So, a dynamical model of this type of rotorcraft UAVs, i.e. the Quadrotor, is established in this paper thanks to

the Newton-Euler formalism. All aerodynamics thrust and drag forces and torques, governing the VTOL flight of the Quadrotor, are described. Based on the linear model of this studied system, obtained around an equilibrium flight operating point, a Linear Quadratic Gaussian (LQG) based control structure is proposed for the position and attitude dynamics stabilization.

The remainder of this paper is organized as follows. Section II presents the aerodynamic forces and torques of the Quadrotor in VTOL flight. A dynamical model is then established thanks to the Newton-Euler formalism. In Section III, an optimal LQG controller is designed to stabilize the position and the attitude of the derived Linear Time-Invariant (LTI) system around an equilibrium operating point. All obtained simulation results are presented and discussed in Section IV. Section V concludes this paper.

II. MODELING OF THE QUADROTOR UAV

A. System description and aerodynamic forces

A Quadrotor is an UAV with four rotors that are controlled independently. The movement of the Quadrotor results from changes in the speed of the rotors. The structure of the Quadrotor in this paper is assumed to be rigid and symmetrical. The center of gravity and the body fixed frame origin are coincided. The propellers are rigid and the thrust and drag forces are proportional to the square of propellers speed.

The studied Quadrotor rotorcraft is detailed with their body and earth frames $\mathbf{R}_B(O, x, y, z)$ and $\mathbf{R}_E(o, e_x, e_y, e_z)$ respectively, as shown in Fig. 1.

Let consider the following model partitions naturally into translational and rotational coordinates:

$$\boldsymbol{\xi} = (x, y, z) \in \mathbb{R}^3, \quad \boldsymbol{\eta} = (\phi, \theta, \psi) \in \mathbb{R}^3 \quad (1)$$

where $\boldsymbol{\xi} = (x, y, z)$ denotes the position vector of the center of mass of the Quadrotor in the fixed inertial frame, $\boldsymbol{\eta} = (\phi, \theta, \psi)$ denotes the attitude of the Quadrotor given by the Euler angles ϕ, θ and ψ .

We note that ϕ is the roll angle around the x -axis, θ is the pitch angle around the y -axis and ψ are the yaw angle around the z -axis. All those angles are bounded as follows:

$$-\frac{\pi}{2} < \phi < \frac{\pi}{2} \quad (2)$$

$$-\frac{\pi}{2} < \theta < \frac{\pi}{2} \quad (3)$$

$$-\pi < \psi < \pi \quad (4)$$

Each motor M_i ($i=1, 2, 3$ and 4) of the Quadrotor produces the force F_i which is proportional to the square of the angular speed ω_i . Known that the motors are supposedly turning only in a fixed direction, the produced force F_i is always positive. The front and rear motors (M1 and M3) rotate counter-clockwise, while the left and right motors (M2 and M4) rotate clockwise. As given in [3], [12], the gyroscopic effects and aerodynamic torques tend to cancel in trimmed flight thanks to the mechanical design of the Quadrotor. The total thrust F is the sum of individual thrusts of each motor. Let denote by m the total mass of the Quadrotor and g the acceleration of the gravity. The orientation of the Quadrotor is given by the

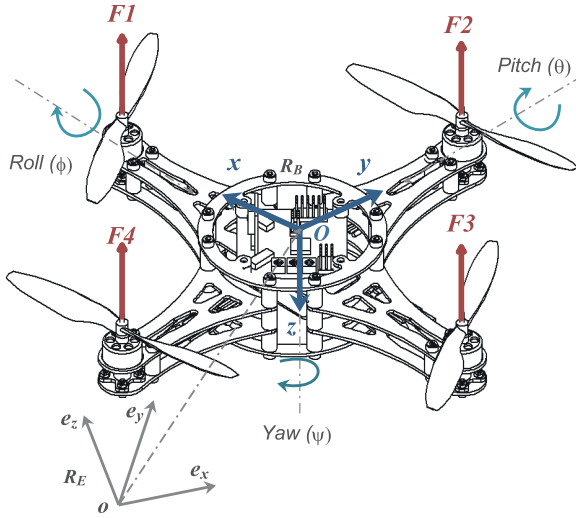


Fig. 1: Mechanical structure of the Quad rotorcraft.

rotation matrix $\mathbf{R} : \mathbf{R}_E \rightarrow \mathbf{R}_B$ which depends on the three Euler angles (ϕ, θ, ψ) and defined by the following equation:

$$\mathbf{R}(\phi, \theta, \psi) = \begin{bmatrix} c\psi c\theta & s\psi s\theta c\psi - s\psi c\theta & c\psi s\theta c\psi + s\psi s\phi \\ s\psi c\theta & s\psi s\theta s\psi + c\psi c\theta & c\psi s\theta s\psi - s\phi c\psi \\ -s\theta & s\phi c\theta & c\phi c\theta \end{bmatrix} \quad (5)$$

where $c(\cdot) = \cos(\cdot)$ and $s(\cdot) = \sin(\cdot)$.

During its flight, the Quadrotor is subjected to external forces like the gusts of wind, gravity, viscous friction and others which are self generated such as the thrust and drag forces. In addition, external torques are provided mainly by the thrust of rotors and the drag on the body and propellers. Moments generated by gyroscopic effects of motors are also

noted.

The thrust force generated by the i^{th} rotor of the Quadrotor is given by [5], [13]:

$$F_i = \frac{1}{2} \rho \Lambda C_T r^2 \omega_i^2 = b \omega_i^2 \quad (6)$$

where ρ is the air density, r and Λ are the radius and the section of the propeller respectively, C_T is the aerodynamic thrust coefficient.

The aerodynamic drag torque, caused by the drag force at the propeller of the i^{th} rotor and opposed the motor torque, is defined as follows [14]:

$$\delta_i = \frac{1}{2} \rho \Lambda C_D r^2 \omega_i^2 = d \omega_i^2 \quad (7)$$

where C_D is the aerodynamic drag coefficient.

The pitch torque is a function of the difference $(F_3 - F_1)$. The roll torque is proportional to the term $(F_4 - F_2)$ and the yaw one is the sum of all reactions torques generated by the four rotors and due to the shaft acceleration and propeller drag. All these pitching, rolling and yawing torques are defined respectively as follows [15], [3]:

$$\tau_\theta = l (F_3 - F_1) \quad (8)$$

$$\tau_\phi = l (F_4 - F_2) \quad (9)$$

$$\tau_\psi = C (F_1 - F_2 + F_3 - F_4) \quad (10)$$

where C is a proportional coefficient and l denotes the distance from the center of each rotor to the center of gravity.

Two gyroscopic effects torques, due to the motion of the propellers and the Quadrotor body, are additively provided. These moments are given respectively by [13], [10]:

$$\mathbf{M}_{gp} = \sum_{i=1}^4 \Omega \wedge [0, 0, J_r (-1)^{i+1} \omega_i]^T \quad (11)$$

$$\mathbf{M}_{gb} = \Omega \wedge \mathbf{J} \Omega \quad (12)$$

where Ω is the vector of the angular velocity in the fixed earth frame and $\mathbf{J} = \text{diag}[I_x, I_y, I_z]$ is the inertia matrix of the Quadrotor, I_x , I_y and I_z denote the inertias of the x -axis, y -axis and z -axis of the Quadrotor, respectively, J_r denotes the z -axis inertia of the propellers rotors.

The Quadrotor is controlled by independently varying the speed of their four rotors. Hence, these control inputs are defined as follows:

$$\begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} = \begin{bmatrix} F \\ \tau_\phi \\ \tau_\theta \\ \tau_\psi \end{bmatrix} = \begin{bmatrix} b & b & b & b \\ 0 & -lb & 0 & lb \\ -lb & 0 & lb & 0 \\ d & -d & d & -d \end{bmatrix} \begin{bmatrix} \omega_1^2 \\ \omega_2^2 \\ \omega_3^2 \\ \omega_4^2 \end{bmatrix} \quad (13)$$

where $b > 0$ and $d > 0$ are two parameters depending on the air density, the geometry and the lift and drag coefficients of

the propeller as given in Eq. (6) and Eq. (7), and $\omega_{1,2,3,4}$ are the angular speeds of the four rotors.

From Eq. (13), it can be observed that the input u_1 denotes the total thrust force on the Quadrotor body in the z -axis, the inputs u_2 and u_3 represent the roll and pitch torques, respectively. The input u_4 represents the yawing torque.

B. Modeling with the Newton-Euler formalism

While using the Newton-Euler method for modeling, the Newton laws lead to the following motion equations of the Quadrotor [15], [6], [3], [13]:

$$\begin{cases} m\ddot{\xi} = \mathbf{F}_{th} + \mathbf{F}_d + \mathbf{F}_g \\ \mathbf{J}\ddot{\Omega} = \mathbf{M} - \mathbf{M}_{gp} - \mathbf{M}_{gb} - \mathbf{M}_a \end{cases} \quad (14)$$

where $\mathbf{F}_{th} = R(\phi, \theta, \psi) \left[0, 0, \sum_{i=1}^4 F_i \right]^T$ denotes the total thrust force of the four rotors, $\mathbf{F}_d = \text{diag}(\kappa_1, \kappa_2, \kappa_3) \dot{\xi}^T$ is the air drag force which resists to the Quadrotor motion, $\mathbf{F}_g = [0, 0, mg]^T$ is the gravity force, $\mathbf{M} = [\tau_\phi, \tau_\theta, \tau_\psi]^T$ represents the total rolling, pitching and yawing torques, \mathbf{M}_{gp} and \mathbf{M}_{gb} are the gyroscopic torques and $\mathbf{M}_a = \text{diag}(\kappa_4, \kappa_5, \kappa_6) \left[\dot{\phi}^2, \dot{\theta}^2, \dot{\psi}^2 \right]^T$ is the torque resulting from the aerodynamic frictions.

Substituting the position vector and the forces expressions into Eq. (14), we obtain the following translational dynamics of the Quadrotor [12], [10], [5]:

$$\begin{cases} \ddot{x} = \frac{1}{m} (c\phi c\psi s\theta + s\phi s\psi) u_1 - \frac{\kappa_1}{m} \dot{x} \\ \ddot{y} = \frac{1}{m} (c\phi s\psi s\theta - s\phi c\psi) u_1 - \frac{\kappa_2}{m} \dot{y} \\ \ddot{z} = \frac{1}{m} c\phi c\theta u_1 - g - \frac{\kappa_3}{m} \dot{z} \end{cases} \quad (15)$$

From the second part of Eq. (14), and while substituting each moment by its expression, we deduce the following rotational dynamics of the rotorcraft [5], [14], [13]:

$$\begin{cases} \ddot{\phi} = \frac{(I_y - I_z)}{I_x} \dot{\theta} \dot{\psi} - \frac{J_r}{I_x} \bar{\Omega}_r \dot{\theta} - \frac{\kappa_4}{I_x} \dot{\phi}^2 + \frac{1}{I_x} u_2 \\ \ddot{\theta} = \frac{(I_z - I_x)}{I_y} \dot{\phi} \dot{\psi} - \frac{J_r}{I_y} \bar{\Omega}_r \dot{\phi} - \frac{\kappa_5}{I_y} \dot{\theta}^2 + \frac{1}{I_y} u_3 \\ \ddot{\psi} = \frac{(I_x - I_y)}{I_z} \dot{\theta} \dot{\phi} - \frac{\kappa_6}{I_z} \dot{\psi}^2 + \frac{1}{I_z} u_4 \end{cases} \quad (16)$$

where $\kappa_{1,2,\dots,6}$ are the drag coefficients and $\bar{\Omega}_r = \omega_1 - \omega_2 + \omega_3 - \omega_4$ is the overall residual rotor angular velocity.

Taking $X = (\phi, \dot{\phi}, \theta, \dot{\theta}, \psi, \dot{\psi}, x, \dot{x}, y, \dot{y}, z, \dot{z})^T \in \mathbb{R}^{12}$ as state vector, the following state-space representation of the

studied Quadrotor is obtained as follows:

$$\dot{X} = f(X, u) = \begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = a_1 x_4 x_6 + a_3 \bar{\Omega}_r x_4 + a_2 x_2^2 + b_1 u_2 \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = a_4 x_2 x_6 + a_6 \bar{\Omega}_r x_2 + a_5 x_4^2 + b_2 u_3 \\ \dot{x}_5 = x_6 \\ \dot{x}_6 = a_7 x_2 x_4 + a_8 x_6^2 + b_3 u_4 \\ \dot{x}_7 = x_8 \\ \dot{x}_8 = a_9 x_8 + \frac{1}{m} (c\phi c\psi s\theta + s\phi s\psi) u_1 \\ \dot{x}_9 = x_{10} \\ \dot{x}_{10} = a_{10} x_{10} + \frac{1}{m} (c\phi s\psi s\theta - s\phi c\psi) u_1 \\ \dot{x}_{11} = x_{12} \\ \dot{x}_{12} = a_{11} x_{12} + \frac{c\phi c\theta}{m} u_1 - g \end{cases} \quad (17)$$

where:

$$\begin{aligned} a_1 &= \frac{I_y - I_z}{I_x}; a_2 = -\frac{\kappa_4}{I_x}; a_3 = -\frac{J_r}{I_x}; a_4 = \frac{(I_z - I_x)}{I_y}; \\ a_5 &= -\frac{\kappa_5}{I_y}; a_6 = -\frac{J_r}{I_y}; a_7 = \frac{(I_x - I_y)}{I_z}; \\ a_8 &= -\frac{\kappa_6}{I_z}; a_9 = -\frac{\kappa_1}{m}; a_{10} = -\frac{\kappa_2}{m}; \\ a_{11} &= -\frac{\kappa_3}{m}; b_1 = \frac{1}{I_x}; b_2 = \frac{1}{I_y}; b_3 = \frac{1}{I_z} \end{aligned}$$

III. OPTIMAL LQG CONTROLLER DESIGN

A. Basic concepts of the LQG control

In order to design an optimal LQG controller for the Quadrotor, a linearized model is derived from the nonlinear system of Eq. (17). The state-space form, used in this control approach, is on a stochastic system and given by:

$$\begin{cases} \dot{x} = \mathbf{A}x + \mathbf{B}u + v \\ y = \mathbf{C}x + w \end{cases} \quad (18)$$

where w and v are the disturbance process and measurement noise inputs, respectively, $x(t)$ is the system state, $u(t)$ denotes the control input and $y(t)$ is the system output. The variables w and v are usually assumed to be Gaussian stochastic processes with constant covariance matrices \mathbf{W} and \mathbf{V} given by [16], [17]:

$$E\{vv^T\} = \mathbf{V} \geq 0 \text{ and } E\{ww^T\} = \mathbf{W} > 0 \quad (19)$$

The LQG control approach is based on the minimization of the following quadratic optimization criterion [8]:

$$J_{LQG} = \lim_{h \rightarrow \infty} E \left\{ \frac{1}{h} \int_0^h (x^T \mathbf{Q}x + u^T \mathbf{R}u) dt \right\} \quad (20)$$

where \mathbf{Q} and \mathbf{R} are the weighting matrices of the Linear Quadratic (LQ) control [18], [16], such as $\mathbf{Q} = \mathbf{Q}^T \geq 0$ and $\mathbf{R} = \mathbf{R}^T > 0$, $E\{\cdot\}$ denotes the expectation operator.

The resolution of this above problem is achieved according to the well known Separation Theorem [17], which consists to:

- Determine a KALMAN estimator allowing to reconstitute the estimated \hat{x} of the state x ;
- Calculate a state feedback control law expressed as $u = -K\hat{x}$, where K is the gain of the state feedback which is calculated by considering the classical LQ problem.

So, according to this theorem the state-space representation of this observer-based controller is given as follows:

$$\begin{cases} \dot{\hat{x}} = A\hat{x} + Bu + L(y - \hat{y}) \\ \hat{y} = C\hat{x} \\ u = -K\hat{x} \end{cases} \quad (21)$$

where L denotes the gain of the KALMAN estimator and is defined as follows:

$$L = PC^T W^{-1} \quad (22)$$

with P is the positive semi-definite solution of the following algebraic RICCATI equation :

$$PA^T + AP - PC^T W^{-1} CP + V = 0 \quad (23)$$

B. LQG controller design for the Quadrotor

The state and input matrices of the linearized state-space form (18) are given respectively by the following Jacobian expressions:

$$A = \begin{bmatrix} \frac{\partial f_1}{\partial x_1|_{x=x^0}} & \frac{\partial f_1}{\partial x_2|_{x=x^0}} & \cdots & \frac{\partial f_1}{\partial x_{12}|_{x=x^0}} \\ \frac{\partial f_2}{\partial x_1|_{x=x^0}} & \frac{\partial f_2}{\partial x_2|_{x=x^0}} & \ddots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f_{12}}{\partial x_1|_{x=x^0}} & \cdots & \cdots & \frac{\partial f_{12}}{\partial x_{12}|_{x=x^0}} \end{bmatrix} \in \mathbb{R}^{12 \times 12} \quad (24)$$

$$B = \begin{bmatrix} \frac{\partial f_1}{\partial u_1|_{u=u^0}} & \frac{\partial f_1}{\partial u_2|_{u=u^0}} & \cdots & \frac{\partial f_1}{\partial u_4|_{u=u^0}} \\ \frac{\partial f_2}{\partial u_1|_{u=u^0}} & \frac{\partial f_2}{\partial u_2|_{u=u^0}} & \ddots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f_{12}}{\partial u_1|_{u=u^0}} & \cdots & \cdots & \frac{\partial f_{12}}{\partial u_4|_{u=u^0}} \end{bmatrix} \in \mathbb{R}^{12 \times 4} \quad (25)$$

where (x_0, u_0) is an equilibrium operating point of the non-linear system of Eq. (15) and Eq. (16) given by:

$$(x_0, u_0) = \begin{cases} x_{1,2,3,4,5,6,8,10,12}^0 = 0 \\ x_{7,9,11}^0 = \text{constant} \\ u_1^0 = mg \\ u_{2,3,4}^0 = 0 \end{cases} \quad (26)$$

The LQG design for the Quadrotor altitude and attitude stabilization problem is solved under the MATLAB/Simulink environment. Through a trial-error process, we choose the weighting matrices Q and R as follows:

$$Q = 2 \times 10^{-1} I_{12} \quad (27)$$

$$R = \begin{bmatrix} 10^{-2} & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 \\ 0 & 0 & 10 & 0 \\ 0 & 0 & 0 & 10 \end{bmatrix} \quad (28)$$

where I_{12} is the 12×12 identity matrix.

After that the noise covariance matrices are determined, we solved such a problem to find the state feedback gain matrix K as well as the KALMAN estimator gain L . These two gains are given by Eq. (29) and Eq. (30):

$$L = \begin{bmatrix} 0.29 & 0.99 & 0 & 0 & 0 & 0 & 0 & 0 & -0.03 & -0.95 & 0 & 0 \\ 0.99 & 826.13 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.23 & 0 & 0 \\ 0 & 0 & 0.29 & 0.99 & 0 & 0 & 0.03 & 0.95 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.99 & 825.91 & 0 & 0 & 0 & 0.02 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0.99 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.99 & 413.01 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.03 & 0 & 0 & 0 & 0.99 & 0.95 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.95 & 0.02 & 0 & 0 & 0.95 & 0.6.11 & 0 & 0 & 0 & 0 \\ -0.03 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.99 & 0.95 & 0 & 0 \\ -0.95 & -0.02 & 0 & 0 & 0 & 0 & 0 & 0 & 0.95 & 6.11 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.99 & 0.86 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.86 & 6.44 \end{bmatrix} \quad (29)$$

$$K = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1.40 \\ 0.620 & 0.098 & 0 & 0 & 0 & 0 & 0 & -0.070 & -0.096 & 0 & 0 & 0 \\ 0 & 0 & 0.620 & 0.098 & 0 & 0 & 0.070 & 0.096 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.070 & 0.078 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (30)$$

IV. SIMULATION RESULTS AND DISCUSSION

In order to stabilize the position and the attitude of the studied Quadrotor, we choose the following desired setpoints for the controlled outputs:

$$X_{ref} = \begin{bmatrix} x_{ref} \\ y_{ref} \\ z_{ref} \\ \psi_{ref} \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 1 \\ 1 \end{bmatrix} \quad (31)$$

After implementation, the simulation results are summarized in Fig. 2, Fig. 3 and Fig. 4. For this purpose, recall that we retain the values $v = 0.01$ and $w = 0.1$ for the noises on the states and outputs.

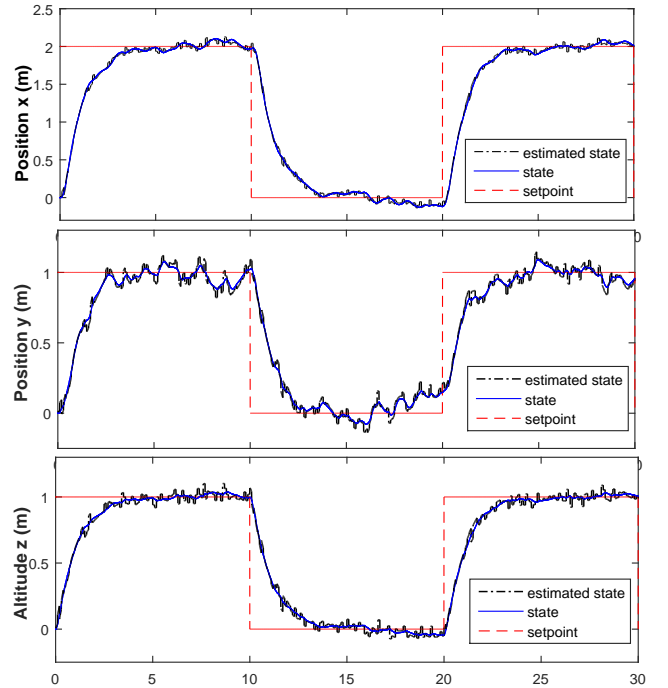


Fig. 2: LQG control based position response of the Quadrotor.

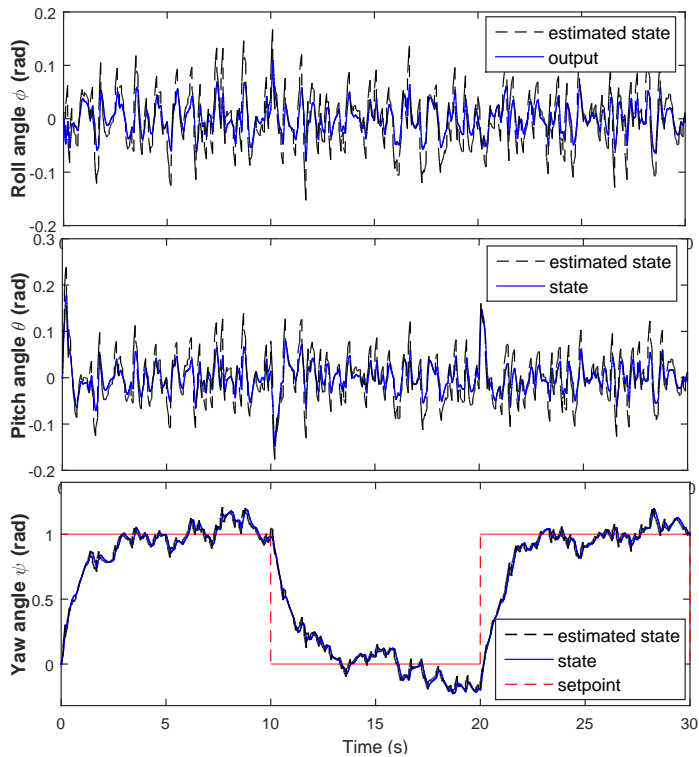


Fig. 3: LQG control based attitude response of the Quadrotor.

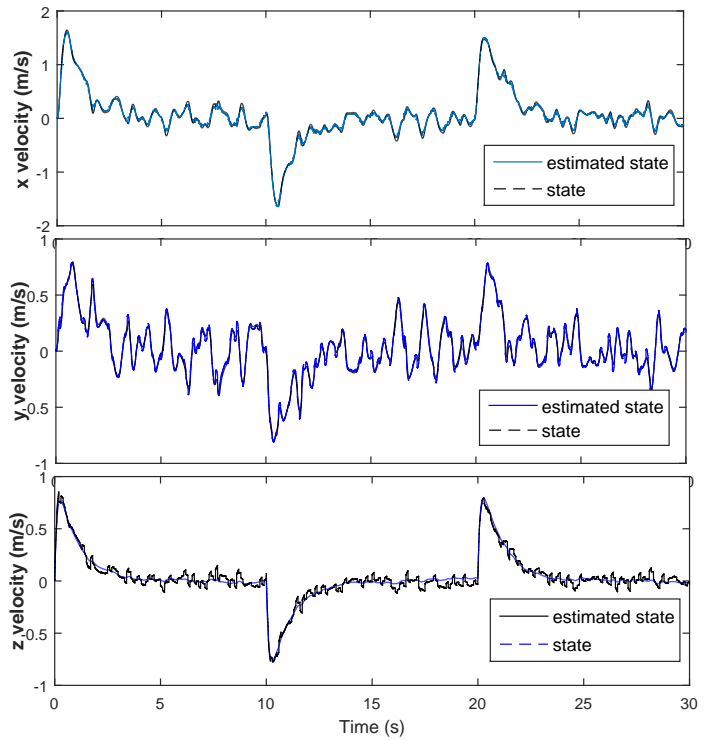


Fig. 5: LQG control based linear velocity response of the Quadrotor.

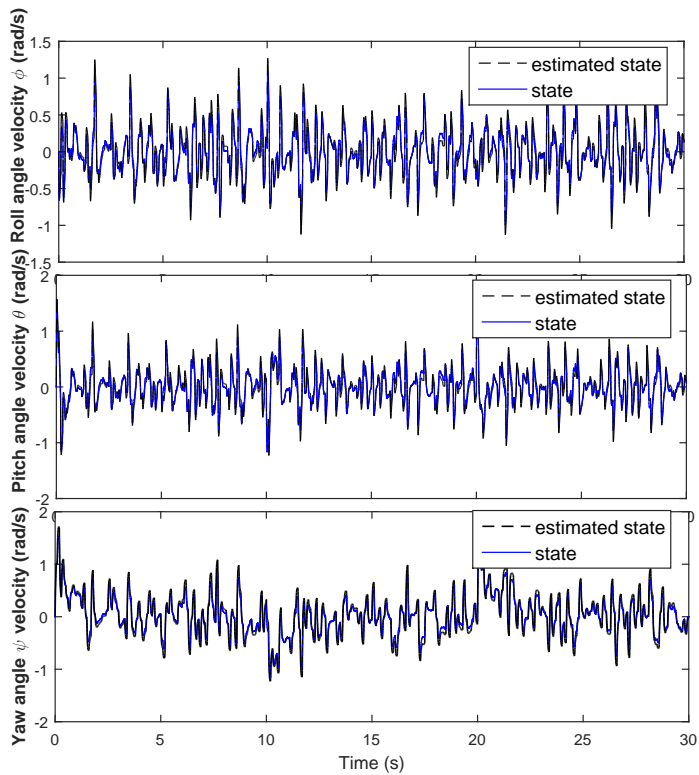


Fig. 4: LQG control based angular velocity response of the Quadrotor.

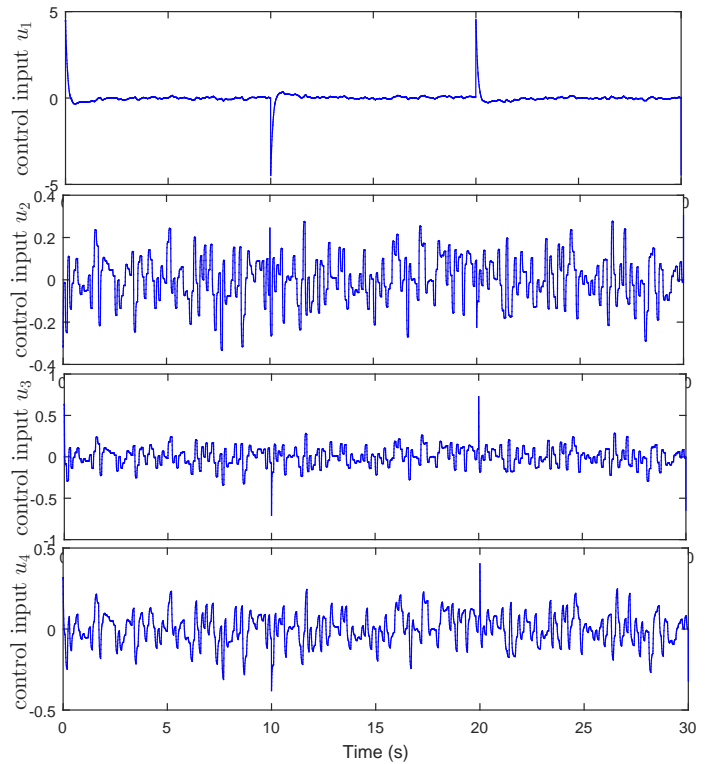


Fig. 6: LQG control law signals applied to the Quadrotor.

It can be observed from these simulation results that the estimated and real system states are close and similar for the position and the yaw dynamics. The estimation errors are negligible and a good reconstitution of the system state is obtained. On the other hands, the stabilization objectives of the LQG controller are made with a satisfied tracking performance. The effectiveness of the proposed LQG control approach is guaranteed.

V. CONCLUSION

In this paper, we established a nonlinear dynamical model of a Quadrotor UAV using the Newton-Euler formalism, extensively adopted in the literature. All aerodynamic forces and moments of the studied Quadrotor UAV are described within an inertial frame. Such an established dynamical model is then used to design a LQG controller for the stabilization of the altitude and attitude of the rotorcraft. Parameters design of the proposed LQG control approach, i.e., the weighting matrix R and Q are obtained thanks to several trials-errors procedures. Finally, some demonstrative simulation results are obtained under the MATLAB/Simulink environment in order to show the effectiveness of the proposed flight stabilization approach. Forthcoming works deal with the optimization of all LQG control parameters based on metaheuristics-based techniques and within the discrete-time framework.

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