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# Modeling and LQG Controller Design for a Quad Tilt-Wing UAV

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Abstract—In this paper, a nonlinear dynamical model of a particular class of convertible Unmanned Aerial Vehicle (UAV), called Quad Tilt-Wing (QTW), is established and used for the Linear Quadratic Gaussian (LQG) control design of such a rotorcraft. With four rotors/wings and a tilting mechanism for each pair, this Vertical Take-Off and Landing (VTOL) vehicle can take-off and landing vertically, such as helicopters, and fly horizontally like a fixed-wing aircraft. All aerodynamic forces and moments of the studied QTW are described within an inertial frame. Then, a dynamical model, relating to the vertical flight mode of the QTW, is derived using the Newton-Euler formalism. A LQG based control approach is investigated to stabilize the attitude and altitude of the QTW drone. Several numerical results are carried out in order to show the ability and effectiveness of the proposed control approach despite the addition of aerodynamic disturbances that represent small airstreams and winds.

Keywords—convertible UAV, Quad Tilt-Wing, modeling, vertical flight dynamics, LQG control, altitude and attitude stabilization, MIMO systems.

#### I. Introduction

In recent decades, the use of the UAVs (Unmanned Aerial Vehicles) experienced a real prosperity in the fields of reconnaissance and military surveillance as well as in various civil applications [1]. UAV is divided in two major categories: the rotary-wing systems which lift is provided by the rotation of the propeller blades and the fixed-wing ones which lift is provided by the airflow over the wings induced by the own movement of the vehicle [2].

The rotary wing UAV is able to take off and landing vertically and perform the hover flight, which is very useful for many applications (inspection, surveillance, take-off from a restricted zone, etc.). However, they cannot fly forward at high speed carrying large payloads. Contrariwise, the fixedwing UAV can fly forward at high speed. But, they need always of a landing strip because the inability to vertical flight. For that, a new family of aerial vehicles is recently appeared and called convertible aircrafts to be currently used at the forefront of aerial robotic researches [3]. This kind of UAV has a hybrid structure which combines the advantages of rotary-wing and fixed-wing aircrafts allowed to experience the best effects of aerodynamic lift and minimize the energy consumed in forward flight. These vehicles can toggle between the vertical and horizontal flights. The transition between the two phases of flight can be achieved by tilting the full vehicle body when the latter takes off and lands on its tail, then tilts horizontally for forward flight (tail-sitter or tilt-body) or by tilting only its rotors using a dedicated mechanism (tilt-rotor or tilt-wing).

The most famous model of convertible tilt-rotor is the Bell Eagle Eye [4] which is based on two rotors with a tilt mechanism and a single wing. There exist some aerial vehicles with a two tilt-rotor technology similar as the Boeing-V22 Osprey [5] and the BIROTAN [6]. In [7] and [8], the authors developed a new design of tilt-rotor vehicle called Quadplan which can take off and landing vertically. However, the disadvantage of using two rotors is that it is impossible to fly when one of these rotors fails. Therefore, it is desirable to use four rotors. In addition, the tilting of the wings gives more efficiency of the slipstream effect. A new convertible UAV design that is capable of solving the above mentioned problems is developed. This vehicle, called Quad Tilt-Wing (OWT), has four rotors and wings with a tilt mechanism for each pair. Several experimental platforms were carried with this body structure [9], [11].

Among many control approaches developed in the literature for stabilizing the attitude and altitude dynamics of the QTW, some ones are around the Linear Quadratic (LQ) and sliding mode with recursive nature controllers [12], adaptive hierarchical control [13], hybrid model predictive control [14] and dynamics inversion method [15]. In this paper, a LQG based control approach is proposed to stabilize both the altitude and the attitude of the studied QTW drone. This control strategy combines a LQ controller and a Kalman estimator to improve the reconstruction of the non-observed plant states and the disturbances rejection.

The remainder of this paper is organized as follows. In Section II, the principle and the different modes of flight for the QTW UAV are presented and a dynamical model is derived for the vertical flight dynamics using the Newton-Euler formalism. Section III presents the designed LQG control approach for the altitude and attitude stabilization of the QTW VTOL aircraft. All numerical simulation results, obtained with the proposed stabilization approach, are shown and discussed in Section IV. Conclusion and future works are finally presented in Section V.

# II. MODELING OF THE QTW UAV

In this section, we describe the mathematical model of the studied QTW in the vertical flight dynamics. The different operation modes and the related aerodynamic forces and torques are firstly presented.

## A. Description of the flight modes

As shown in Fig. 1, the convertible aircraft has two fundamental motion modes: the vertical and the horizontal flights. A transition operation mode, that interposes these two flights, is also noted. The switching from one mode to another is affected using the well known tilting mechanism. During vertical takeoff and landing, the QWT is based only on its rotors and behaves like a Quadrotor with H-type structure. The tilt angles of the wings are nearly equal to 90° with the horizontal plane. At the horizontal flight mode, the aircraft behaves like a conventional plane with a tilt angle of wings almost equal to zero degree.

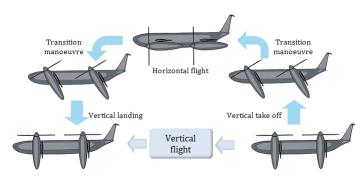


Fig. 1. The Quad Tilt-Wing's operation flight modes.

#### B. Dynamical model in the vertical flight

The modeling of flying robots is a delicate task since the dynamics of the system is strongly nonlinear and fully coupled. In this section, the Newton-Euler formalism is used to develop a nonlinear mathematical model of the studied QTW. To simplify this study, different assumptions will be made for the development of a dynamical model to be used for the observation and the control stages. So, we suppose that [16]:

- the QTW aerial vehicle is a 6 Degree-Of-Freedom (DOF) rigid body;
- the flexibility of the aircraft wings and fuselage are neglected;
- the aerodynamic center and center of gravity are coincident:
- the aircraft weight is considered constant.

To establish such a model, we use an earth fixed inertial reference frame, denoted as  $R^i = \{O_i, x_i, y_i, z_i\}$ , and the body fixed reference frame, given by  $R^b = \{O_i, x_b, y_b, z_b\}$ , as shown in Fig. 2.

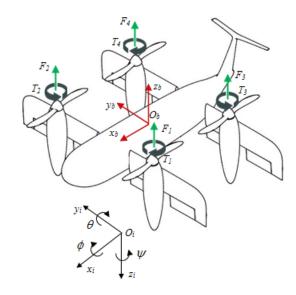


Fig. 2. Coordinate systems external forces acting on the QTW aircraft.

The orientation of the body frame  $R^b$  with respect to the inertial one  $R^i$  is expressed by the following transformation matrix:

$$R_{ib} = \begin{pmatrix} c\psi c\theta & s\phi s\theta c\psi - s\psi c\phi & c\phi s\theta c\psi + s\psi \sin \phi \\ s\psi c\theta & s\phi s\theta s\psi + c\psi c\theta & c\phi s\theta s\psi - s\phi c\psi \\ -s\theta & s\phi c\theta & c\phi c\theta \end{pmatrix}$$
(1)

where,  $s(.) = \sin(.)$  and  $c(.) = \cos(.)$ ,  $\phi$ ,  $\theta$  and  $\psi$  are the roll, pitch and yaw Euler angles, respectively. These angles represent the rotation motions of the QTW body around the axis  $x_b$ ,  $y_b$  and  $z_b$ , respectively.

The position and linear velocity of the QTW gravity center G are given in the inertial frame as follows:

$$P_i = \left(\begin{array}{ccc} x & y & z \end{array}\right)^T \tag{2}$$

$$V_i = \dot{P}_i = \begin{pmatrix} \dot{x} & \dot{y} & \dot{z} \end{pmatrix}^T \tag{3}$$

The attitude and the angular velocities in the inertial frame are respectively described as:

$$\alpha_i = \left(\begin{array}{ccc} \phi & \theta & \psi \end{array}\right)^T \tag{4}$$

$$\Omega_i = \dot{\alpha}_i = \left( \begin{array}{ccc} \dot{\phi} & \dot{\theta} & \dot{\psi} \end{array} \right)^T \tag{5}$$

The relations for the transformation between the linear and angular velocities, presented in the inertial and body frames, are given respectively by Eq. (6) and Eq. (7):

$$V_b = \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix} = R_{ib}^T(\phi, \theta, \psi) \times V_i = R_{bi}(\phi, \theta, \psi) \times V_i \quad (6)$$

$$\Omega_b = \begin{pmatrix} p \\ q \\ r \end{pmatrix} = \mathbf{E}(\theta, \phi) \times \Omega_i = \mathbf{E}(\theta, \phi) \times \begin{pmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{pmatrix}$$
(7)

where:

$$\mathbf{E}(\theta, \phi) = \begin{bmatrix} 1 & 0 & -s\theta \\ 0 & c\phi & s\phi c\theta \\ 0 & -s\phi & c\phi c\theta \end{bmatrix}$$
(8)

During the hover flight, the QTW UAV behaves like a Quadrotor, i.e., it is based only on its rotors. In this case, the basic movements of the QWT are achieved by varying the speed of each rotor there by changing the thrust produced. The rotations of propellers create torques in the clockwise direction (T1, T4) and counter-clockwise direction (T2, T3) for propellers (1, 4) and (2, 3), respectively. In this way, the altitude stabilization is preserved if the rotations of rotors are equals. The QWT inclines toward the direction of the slower rotor, which takes into account a translational movement along this rotor axis. For example, the vehicle will move along y axis if the roll angle is changed either by increasing the speeds rotations of the rotors 1 and 2 or increasing the speeds rotations of the rotors 3 and 4.

The translational and rotational dynamics of the QTW are expressed in the vehicle body frame. Using the Newton-Euler formalism, the dynamical plant model can be described by Eq. (9) and Eq. (10)

$$m\dot{V}_i = F_t \tag{9}$$

$$I_b \dot{\Omega}_b = -M_t + \Omega_b \wedge (I_b \times \Omega_b) \tag{10}$$

where m denotes the mass of the QTW,  $F_t$  and  $M_t$  represent the sum of external forces and moments acting on the vehicle center of gravity, respectively,  $I_b = diag\left(I_{xx}, I_{yy}, I_{zz}\right)$  denotes the inertia matrix of the QTW,  $I_{xx}$ ,  $I_{yy}$  and  $I_{zz}$  are the inertias in the body reference frame.

The total external force  $F_t$  acting on the convertible QTW center of gravity consists of the total thrust forces F of the four rotors, the gravity force  $F_g$  and the external disturbances  $F_d$  like the wind gusts. Since these forces are expressed in the body frame, they must be transformed by the rotation matrix  $R_{ib}$  in the inertial one to obtain the following expression:

$$F_t = R_{ib}(F + F_a + F_d) \tag{11}$$

where

where: 
$$F = \left[0, 0, -\sum_{i=1}^{4} F_i\right]^T \text{ and } F_g = mg \begin{bmatrix} -s\theta \\ s\phi c\theta \\ c\phi c\theta \end{bmatrix}$$

Note that  $F_i = k\omega_i^2$  where k > 0 is the lift coefficient,  $\omega_i$  the angular speed of the  $i^{th}$  motor (i= 1, 2, 3 and 4) and g is the acceleration due to gravity.

The total external torque  $M_t$  acting on the convertible QTW center of gravity consists of the torques M induced by the rotors thrust forces, the gyroscopic effects torques  $M^h_{gyro}$  and  $M^b_{gyro}$  due to the rotation of the propellers and the movement of the aircraft, respectively, and the torque  $M_d$  due to the external disturbances. It is given by:

$$M_t = M + M_{gyro}^h + M_{gyro}^b + M_d (12)$$

All these defined torques are expressed respectively as follows:

$$M = \begin{bmatrix} M_{\phi} \\ M_{\theta} \\ M_{\psi} \end{bmatrix} = \begin{bmatrix} l_s \left[ (F_1 + F_3) - (F_2 + F_4) \right] \\ l_l \left[ (F_1 + F_2) - (F_3 + F_4) \right] \\ \lambda \left( F_1 - F_2 - F_3 + F_3 \right) \end{bmatrix}$$
(13)

$$M_{gyro}^{h} = \sum_{i=1}^{4} J_r \left( \eta_i \Omega_b \wedge \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} \omega_i \right)$$
 (14)

$$M_{qyro}^b = \Omega_b \wedge I_b \Omega_b \tag{15}$$

where  $l_l$  and  $l_s$  denote the distances between the rotors and the vehicle's center of gravity in x and y directions, respectively,  $\eta_{(1,2,3,4)}=(1,-1,-1,1)$  and  $J_r$  denotes the z-axis inertia of the rotors propellers.

The torque  $T_i = \lambda_i F_i$  created by the rotors is proportional to the thrust forces with a torque/force constant that depends on the geometry of the propellers. For the clockwise propellers, we have  $\lambda_{2,3} = \lambda$ , while for the counter-clockwise ones, we obtain  $\lambda_{1,4} = -\lambda$ .

Substituting in Eq. (9) each term by its expression, previously established, we obtain the following differential equations which define the translation dynamics of the QTW:

$$\begin{cases}
\ddot{x} = \frac{1}{m} (-c\phi s\theta c\psi - s\phi s\psi) u_1 \\
\ddot{y} = \frac{1}{m} (-c\phi s\theta s\psi + s\phi c\psi) u_1 \\
\ddot{z} = \frac{-c\phi c\theta}{m} u_1 + g
\end{cases}$$
(16)

Substituting in Eq. (10) each term by its expression, previously established, we obtain the following differential equations which define the rotational dynamics of the OTW:

$$\begin{cases} \dot{p} = \frac{u_2}{I_{xx}} + \frac{I_{yy} - I_{zz}}{I_{xx}} qr - \frac{J_r}{I_{xx}} q\omega_p \\ \dot{q} = \frac{u_3}{I_{yy}} + \frac{I_{zz} - I_{xx}}{I_{yy}} pr + \frac{J_r}{I_{yy}} p\omega_p \\ \dot{r} = \frac{u_4}{I_{zz}} + \frac{I_{xx} - I_{yy}}{I_{zz}} pq \end{cases}$$
(17)

where  $\omega_p = \omega_1 - \omega_2 - \omega_3 + \omega_4$  is the overall residual rotor angular velocity.

Finally, the control inputs of the QTW aircraft are defined as follows:

$$U = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} = \begin{bmatrix} k & k & k & k \\ kl_s & -kl_s & kl_s & -kl_s \\ kl_l & kl_l & -kl_l & -kl_l \\ k\lambda & -k\lambda & -k\lambda & k\lambda \end{bmatrix} \begin{bmatrix} \omega_1^2 \\ \omega_2^2 \\ \omega_3^2 \\ \omega_4^2 \end{bmatrix}$$
(18)

From Eq. (18), it can be observed that the input  $u_1$  is related to the altitude and the inputs  $u_2$ ,  $u_3$  and  $u_4$  are related to the attitude of the QTW.

## III. LQG CONTROLLER DESIGN

# A. LQG control formulation

As mentioned by M. Green and D. Limebeer in [17], unlike the Linear Quadratic (LQ) control, the LQG control approach has the advantage to be applied to systems whose state is not measured or the measurements are affected by noises. Developed at the beginning at the second half of the  $20^{th}$ century, the LQG technique witnessed a great success and evolution where it was applied in many aerospace systems like the Apollo space program for the stabilization of launchers.

In the LQG control, it is considered that the plant dynamics is linear and the measurement noise and disturbance signals are stochastic. Hence, we have a plant model with the following form:

$$\begin{cases} \dot{x} = \mathbf{A}x + \mathbf{B}u + w \\ y = \mathbf{C}x + \mathbf{D}u + v \end{cases}$$
 (19)

where w and v are the disturbance process and measurement noise inputs, respectively. They are usually assumed to be Gaussian stochastic processes with constant covariance matrices V and W given as follows:

$$E\left\{vv^{T}\right\} = V, E\left\{ww^{T}\right\} = W$$

The LQG control problem is to find the optimal control law which minimizes the following criterion:

$$J = \lim_{t \to \infty} E \left\{ \frac{1}{tf} \int_{0}^{tf} (x^{T} \mathbf{Q} x + u^{T} \mathbf{R} u) dt \right\}$$
 (20)

where  $\boldsymbol{Q}$  and  $\boldsymbol{R}$  are two weighting matrices with  $\boldsymbol{Q} = \boldsymbol{Q}^T \geqslant 0$ and  $\mathbf{R} = \mathbf{R}^T > 0$  and  $E\{.\}$  is the expectation operator.

The solution of this problem is based on the Separation Theorem [18], as illustrated in Fig. 3. This theorem states that the solution of this optimal control problem for stochastic process consists of the two following steps:

• Determining an optimal estimate  $\hat{x}$  of the state x by minimizing the variance of the error  $E\left\{\left(x-\hat{x}\right)^T\left(x-\hat{x}\right)\right\}$  using a Kalman filter based method. This estimated optimal state is generated by the following equation:

$$\dot{\hat{x}} = A\hat{x} + Bu + L(y - C\hat{x} - Du) \tag{21}$$

where  $L = P_f C^T V^{-1}$  denotes the gain of the Kalman estimator. The matrix  $P_f = P_f^T > 0$  is the unique positive-semidefinite solution of the following Riccati algebraic equation:

$$P_f A^T + A P_f - P_f C^T V^{-1} C P_f + W = 0$$
 (22)

· Design an optimal state feedback controller applying to  $\hat{x}$  as an exact measure of the state vector. Such a control law is given by:

$$u(t) = -\mathbf{K}\hat{x}(t) \tag{23}$$

where K is the calculated optimal gain based on the LQ control problem [19].

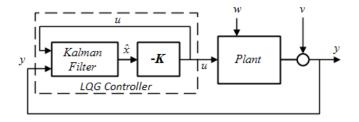


Fig. 3. Separation Theorem Principle.

From the plant model given by Eq. (19), the optimal estimated state and the optimal feedback control law expressions are summarized in Eq. (24), respectively:

$$\begin{cases} \dot{\hat{x}} = (\mathbf{A} - \mathbf{L}\mathbf{C} - \mathbf{B}\mathbf{K})\hat{x} + \mathbf{L}y \\ u = -\mathbf{K}\hat{x} \end{cases}$$
 (24)

### B. LQG Hovering Control of the QT

To design a position controller for the guidance/control of the QTW in hovering, Eq. (16) and Eq. (17) are putted into a nonlinear  $12^{th}$  order model with states corresponding to the position  $P_i$ , the attitude angles  $\alpha_i$ , the linear and angular velocities  $V_i$  and  $\Omega_b$ , respectively. The system is equivalent to a nonlinear dynamical model of the form:

$$\dot{X} = f(X, U) \tag{25}$$

where  $X = \left(\phi,\dot{\phi},\theta,\dot{\theta},\psi,\dot{\psi},z,\dot{z},x,\dot{x},y,\dot{y}\right)^T$  is the state vector, and  $U = (u_1, u_2, u_3, u_4)^T$  denotes the control input.

In this study, the QTW model is linearized around an equilibrium operating point, e.g. in hover flight condition with constant altitude. The LQG control structure, retained for the convertible QWT, is given in Fig. 4. The linear model is characterized by the matrices A and B, which are computed as follows:

$$\boldsymbol{A} = \begin{bmatrix} \frac{\partial f_{1}(X,u)}{\partial X_{1}|_{x=x^{*}}} & \cdots & \frac{\partial f_{1}(X,u)}{\partial X_{12}|_{x=x^{*}}} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_{12}(X,u)}{\partial X_{1}|_{x=x^{*}}} & \cdots & \frac{\partial f_{12}(X,u)}{\partial X_{12}|_{x=x^{*}}} \end{bmatrix}$$

$$\boldsymbol{B} = \begin{bmatrix} \frac{\partial f_{1}(X,u)}{\partial u_{1}|_{u=u^{*}}} & \cdots & \frac{\partial f_{1}(X,u)}{\partial u_{4}|_{u=u^{*}}} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_{12}(X,u)}{\partial u_{1}|_{u=u^{*}}} & \cdots & \frac{\partial f_{12}(X,u)}{\partial u_{4}|_{u=u^{*}}} \end{bmatrix}$$

$$(26)$$

$$\boldsymbol{B} = \begin{bmatrix} \frac{\partial f_1(X,u)}{\partial u_1|_{u=u^*}} & \cdots & \frac{\partial f_1(X,u)}{\partial u_4|_{u=u^*}} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_{12}(X,u)}{\partial u_1|_{u=u^*}} & \cdots & \frac{\partial f_{12}(X,u)}{\partial u_4|_{u=u^*}} \end{bmatrix}$$
(27)

Through a trial-error process, we choose the weighting matrices Q and R of the LQ controller as follows:

$$Q = 10^{-1} I_{12} \tag{28}$$

$$\mathbf{R} = \begin{bmatrix} 10^{-2} & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 \\ 0 & 0 & 10 & 0 \\ 0 & 0 & 0 & 10 \end{bmatrix}$$
 (29)

where  $I_{12}$  is the  $12 \times 12$  identity matrix.

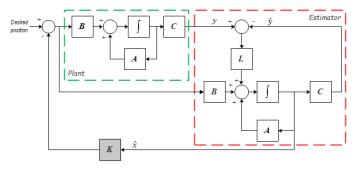


Fig. 4. Proposed LQG structure for the QTW position control.

After that the noise covariance matrices are determined, we solved this formulated problem in MATLAB environment to find the state feedback gain matrix and the optimal Kalman estimator gains K and L, given by:

$$\boldsymbol{K} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & -3.16 & -5.94 & 0 & 0 & 0 & 0 \\ 1.72 & 0.88 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.31 & -0.73 \\ 0 & 0 & 1.54 & 0.71 & 0 & 0 & 0 & 0 & 0.31 & 0.70 & 0 & 0 \\ 0 & 0 & 0 & 0.31 & 0.43 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$
(30)

# IV. SIMULATION RESULTS AND DISCUSSION

In order to observe the performances of the proposed control structure, we simulate the model of the studied QTW UAV using the MATLAB/Simulink environment. From the initial position and attitude  $\alpha_i(0)=P_i(0)=0$  and while retaining the reference position and yaw angle, the simulation results of the LQG control implementation in vertical flight mode are illustrated in Fig. 5 to Fig. 8. The disturbance process and measurement noise inputs are modeled with Gaussian random variables of zero mean and variance equal to 0.01.

The variation of the estimated and real system states for the position and attitude control are depicted in Fig. 5 and Fig. 6. As shown in Fig. 7 and Fig. 8, the estimated and real system states are close and similar for the linear and angular velocities. We can observe that the estimation errors for the position, attitude and velocities are negligible, hence the good reconstitution of the system state. So, the proposed LQG control approach gives high performances in the studied QTW position and attitude stabilization.

## V. CONCLUSION

In this paper, a nonlinear model for the vertical flight dynamics of the convertible QTW has been developed. The LQG approach is presented to solve the position QTW stabilization using a linearized model of the vehicle around an

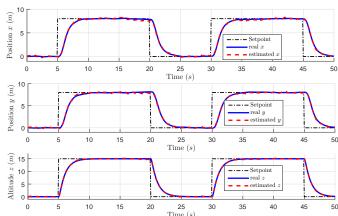


Fig. 5. LQG control based position response of the QTW UAV.

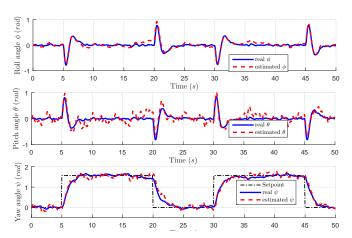


Fig. 6. LQG control based attitude response of the QTW UAV.

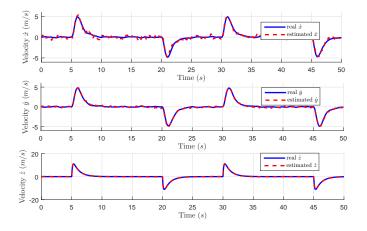


Fig. 7. LQG control based linear velocities response of the QTW UAV.

equilibrium operating point in the hovering condition. The developed LQG controller ensures the position stabilization of the QWT drone with a satisfied tracking performance. The presented simulation results show the effectiveness of the proposed stabilization approach. As future works, we intend to focus on the control in horizontal flight and transition

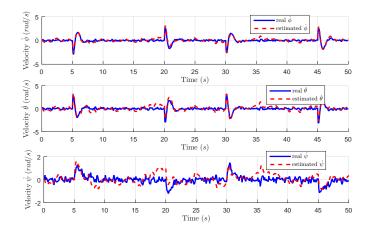


Fig. 8. LQG control based angular velocities response of the QTW UAV.

modes. Furthermore, an advanced Computer Aided Design (CAD) methodology for the Hardware-In-the-Loop (HIL) cosimulation and rapid prototyping is under development to improve the proposed LQG controller effectiveness.

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