

Complex Multiple Support Vector Machine Regression for Frequency Selective MIMO-OFDM System

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Abstract—This paper proposes a new scheme to track the frequency selective time variant channel induced by multipath fading wireless Multiple-Input Multiple-Output Orthogonal Frequency Division Multiplexing (MIMO-OFDM) system under high mobility conditions in the presence of Gaussian and non-Gaussian impulsive noise interfering with reference signals. The estimation of the channel is performed by using a nonlinear channel estimator based on a complex Multiple Support Vector Machine Regression (M-SVR) which is developed and applied to MIMO Long Term Evolution (LTE) Downlink with Vertical-Bell Labs Advanced Space Time (V-BLAST) detection algorithm. The obtained results confirm the effectiveness of the proposed technique to track the fading channel under high mobility conditions (350 Km/h) in the presence of different nonlinearities.

Index Terms—Complex M-SVR, MIMO-OFDM, V-BLAST, impulsive noise, LTE.

I. INTRODUCTION

Multiple-Input Multiple-Output (MIMO) systems have attracted the interest of many researchers because of which have been proposed for increasing reliability of the wireless systems as well as communication capacity. The exploitation of the spatial dimension by using the space division multiplexing (SDM) technique is a promising solution for significant increase of bandwidth efficiency and performance under fading channels. Fundamentally, the SDM method transmits different data streams on different transmit antennas simultaneously, which increase the signal to noise ratio and capacity. By using multiple antennas at the receiver side, the different mixed data streams can be recovered by SDM approaches like Vertical-Bell Labs Advanced Space Time (V-BLAST) algorithm detailed in [1].

Orthogonal Frequency Division Multiplexing (OFDM) technology avoids channel multipath effect by converting the wide-band frequency selective channel into a set of narrow band flat subcarrier. The modulated symbol rate on each subcarrier is lower in comparison to the channel delay spread, thus the intersymbol interference (ISI) can be prevented. Therefore, the combination of MIMO and OFDM approaches (MIMO-OFDM)

is an attractive technique for the wireless cellular systems especially over a fading channel.

In MIMO-OFDM systems, channel estimation task is very important to the coherent detection especially in the presence of non-Gaussian impulsive noise. In fact, impulsive noise can be present in a practical environment, so the channel becomes nonlinear. In this context, impulsive noise can significantly influence the performance of the MIMO-OFDM system since the time of the arrival of an impulse is unpredictable and shapes of the impulses are not known and they vary considerably. Additionally, impulses usually have very high energy which can be much greater than the energy of the useful signal.

There are many channel estimation techniques proposed for MIMO-OFDM systems. The channel estimation technique used in this paper is based on the M-SVR (Multiple Support Vector Machine Regression) which training sequences are placed in each OFDM symbol to obtain the transmission environment parameters in the presence of impulsive noise under high mobility conditions. Indeed, the principle of the proposed nonlinear complex M-SVR algorithm is to exploit the information provided by the pilot signals to estimate all subchannel frequency responses. Thus, the proposed algorithm is developed in terms of the RBF (Radial Basis Function) kernel and applied to LTE downlink system.

This paper is organized as follows. Section II introduces the MIMO-OFDM system. In section III, the M-SVR channel estimator is provided. In section IV, we make some simulation results. Finally, section V concludes the paper.

II. MIMO-OFDM SYSTEM

In a MIMO-OFDM system, the output signal at each receive antenna Rx is a mixed signal consisting of the data streams coming from each transmit antenna Tx. Assuming that the cyclic prefix is longer than the channel response length, the receive signal at the j^{th} Rx antenna can be presented in the

frequency domain as follows:

$$R_j[l, k] = \sum_{i=1}^{N_t} H_{ij}[l, k] X_i[l, k] + W_j[l, k] + I_j[l, k],$$

$$j = 1, \dots, N_r, \quad 0 \leq k \leq N - 1, \quad (1)$$

where $H_{ij}[l, k]$ represents the channel frequency response corresponding to the l^{th} OFDM symbol and the k^{th} subcarrier transmitted between the i^{th} Tx antenna and the j^{th} Rx antenna. Let N , N_t and N_r denote the number of subcarriers, the number of Tx antennas and the number of Rx antennas, respectively. $X_i[l, k]$ denotes the data transmitted from the i^{th} Tx antenna at the l^{th} OFDM symbol on the k^{th} subcarrier. $W_j[l, k]$ is the additive white Gaussian noise (AWGN) at the j^{th} receiver antenna, with zero mean and variance σ_w^2 , and is assumed to be uncorrelated for different j 's, k 's or l 's.

$I_j[l, k]$ is the impulsive noise in the frequency domain which is modeled as a Bernoulli-Gaussian process and it was generated in time domain with the Bernoulli-Gaussian process function $i(n) = \mu(n)\lambda(n)$ where $\mu(n)$ is a random process with Gaussian distribution and power σ_{BG}^2 , and $\lambda(n)$ is a random process with probability [2]

$$P_r(\lambda(n)) = \begin{cases} p, & \lambda = 1 \\ 1 - p, & \lambda = 0 \end{cases} \quad (2)$$

Equation (1) can be expressed as

$$R_j[l, k] = \sum_{i=1}^{N_t} H_{ij}[l, k] X_i[l, k] + e_j[l, k], \quad (3)$$

where $e_j[l, k]$ is the residual noise which represents the sum of the AWGN noise $W_j[l, k]$ and impulsive noise $I_j[l, k]$ in the frequency domain.

We consider the following channel impulse response of the mobile wireless frequency-selective multipath fading channel:

$$h(\tau, t) = \sum_{q=0}^{L-1} h_q(t) \delta(t - \tau_q), \quad (4)$$

where $h_q(t)$ denotes the impulse response representing the complex gain of the q^{th} path, τ_q represents the random delay of the q^{th} path and L is the number of multipaths in the channel.

Since the mobile wireless channel is frequency selective time variant, it is necessary to track the channel response continuously. Therefore, the learning and estimation phases for the MIMO-OFDM system under consideration are repeated for each OFDM symbol in order to track the channel variations.

III. NONLINEAR COMPLEX M-SVR ESTIMATOR

We note first that the indices i and j throughout this section denotes the i^{th} and j^{th} antenna at the transmitter and receiver side respectively of the considered MIMO system.

Let the OFDM frame contains Nl OFDM symbols which every

symbol includes N subcarriers. The transmitting pilot symbols are $\mathbf{X}_i^P = \text{diag}(X_i(l, m\Delta P))$, $m = 0, 1, \dots, N_P - 1$, where l and m are labels in time domain and frequency domain respectively, and ΔP is the pilot interval in frequency domain.

The proposed channel estimation approach is based on nonlinear complex M-SVR algorithm which has two separate phases: learning phase and estimation phase. In learning phase, we estimate first the subchannels pilot symbols according to LS criterion to strike $\min [(Y_j^P - \mathbf{X}_i^P \mathbf{F} h_{i,j})(Y_j^P - \mathbf{X}_i^P \mathbf{F} h_{i,j})^H]$ [3], as

$$\hat{H}_{i,j}^P = \mathbf{X}_i^{P-1} Y_j^P, \quad (5)$$

where $Y_j^P = Y_j(l, m\Delta P)$ and $\hat{H}_{i,j}^P = \hat{H}_{i,j}(l, m\Delta P)$ are the received pilot symbols and the estimated frequency responses for the l^{th} OFDM symbol at pilot positions $m\Delta P$, respectively.

Then, in the estimation phase and by the interpolation mechanism, frequency responses of data subchannels can be determined. Therefore, frequency responses of all the OFDM subcarriers are

$$\hat{H}_{i,j}(l, k) = f_{i,j}(\hat{H}_{i,j}^P(l, m\Delta P)), \quad (6)$$

where $k = 0, \dots, N - 1$, and $f_{i,j}(\cdot)$ is the interpolating function, which is determined by the nonlinear complex M-SVR approach.

We used the following regression function:

$$\hat{H}_{i,j}(m\Delta P) = \mathbf{w}_{i,j}^T \boldsymbol{\varphi}_{i,j}(m\Delta P) + b_{i,j} + e_{i,j}^m, \quad (7)$$

for $m = 0, \dots, N_P - 1$ and $\mathbf{w}_{i,j}$ is the weight vector, $b_{i,j}$ is the bias term and residuals $\{e_{i,j}^m\}$ account for the effect of both approximation errors and noise.

To improve the performance of the estimation algorithm, a robust cost function is introduced which is ε -Huber robust cost function given by [4]

$$\mathcal{L}^\varepsilon(e_{i,j}^m) = \begin{cases} 0, & |e_{i,j}^m| \leq \varepsilon \\ \frac{1}{2\gamma} (|e_{i,j}^m| - \varepsilon)^2, & \varepsilon \leq |e_{i,j}^m| \leq e_C \\ C(|e_{i,j}^m| - \varepsilon) - \frac{1}{2}\gamma C^2, & e_C \leq |e_{i,j}^m|, \end{cases} \quad (8)$$

where $e_C = \varepsilon + \gamma C$, ε is the insensitive parameter which is a positive scalar that represents the insensitivity to a low noise level, whereas parameters γ and C control essentially the trade-off between regularization and losses.

Let $\mathcal{L}^\varepsilon(e_{i,j}^m) = \mathcal{L}^\varepsilon(\Re(e_{i,j}^m)) + \mathcal{L}^\varepsilon(\Im(e_{i,j}^m))$ since $\{e_{i,j}^m\}$ are complex, where $\Re(\cdot)$ and $\Im(\cdot)$ represent real and imaginary parts, respectively.

Now, we can state the primal problem as

$$\begin{aligned}
\min \quad & \frac{1}{2} \|\mathbf{w}_{i,j}\|^2 + \frac{1}{2\gamma} \sum_{m \in I_1} (\xi_{i,j}^m + \xi_{i,j}^{m*})^2 \\
& + C \sum_{m \in I_2} (\xi_{i,j}^m + \xi_{i,j}^{m*}) + \frac{1}{2\gamma} \sum_{m \in I_3} (\zeta_{i,j}^m + \zeta_{i,j}^{m*})^2 \\
& + C \sum_{m \in I_4} (\zeta_{i,j}^m + \zeta_{i,j}^{m*}) - \frac{1}{2} \sum_{m \in I_2, I_4} \gamma C^2 \quad (9)
\end{aligned}$$

constrained to

$$\begin{aligned}
\Re(\hat{H}_{i,j}(m\Delta P) - \mathbf{w}_{i,j}^H \boldsymbol{\varphi}_{i,j}(m\Delta P) - b_{i,j}) &\leq \varepsilon + \xi_{i,j}^m \\
\Im(\hat{H}_{i,j}(m\Delta P) - \mathbf{w}_{i,j}^H \boldsymbol{\varphi}_{i,j}(m\Delta P) - b_{i,j}) &\leq \varepsilon + \zeta_{i,j}^m \\
\Re(-\hat{H}_{i,j}(m\Delta P) + \mathbf{w}_{i,j}^H \boldsymbol{\varphi}_{i,j}(m\Delta P) + b_{i,j}) &\leq \varepsilon + \xi_{i,j}^{m*} \\
\Im(-\hat{H}_{i,j}(m\Delta P) + \mathbf{w}_{i,j}^H \boldsymbol{\varphi}_{i,j}(m\Delta P) + b_{i,j}) &\leq \varepsilon + \zeta_{i,j}^{m*} \\
\xi_{i,j}^{m(*)}, \zeta_{i,j}^{m(*)} &\geq 0, \quad (10)
\end{aligned}$$

for $m = 0, \dots, N_P - 1$, where $\xi_{i,j}^m$ and $\xi_{i,j}^{m*}$ are slack variables which stand for positive, and negative errors in the real part, respectively. $\zeta_{i,j}^m$ and $\zeta_{i,j}^{m*}$ are the errors for the imaginary parts. I_1, I_2, I_3 and I_4 are the set of samples for which:
 I_1 : real part of the residuals are in the quadratic zone;
 I_2 : real part of the residuals are in the linear zone;
 I_3 : imaginary part of the residuals are in the quadratic zone;
 I_4 : imaginary part of the residuals are in the linear zone.

To transform the minimization of the primal functional (9) subject to constraints in (10), into the optimization of the dual functional, we must first introduce the constraints into the primal functional to obtain the primal-dual functional. Then, by making zero the primal-dual functional gradient with respect to $\omega_{i,j}$, we obtain an optimal solution for the weights

$$\mathbf{w}_{i,j} = \sum_{m=0}^{N_P-1} \psi_{i,j}^m \boldsymbol{\varphi}_{i,j}(m\Delta P) = \sum_{m=0}^{N_P-1} \psi_{i,j}^m \boldsymbol{\varphi}_{i,j}(P_m), \quad (11)$$

where $\psi_{i,j}^m = (\alpha_{R,m,i,j} - \alpha_{R,m,i,j}^*) + j(\alpha_{I,m,i,j} - \alpha_{I,m,i,j}^*)$ with $\alpha_{R,m,i,j}$, $\alpha_{R,m,i,j}^*$, $\alpha_{I,m,i,j}$, $\alpha_{I,m,i,j}^*$ are the Lagrange multipliers for real and imaginary parts of the residuals and $P_m = (m\Delta P)$, $m = 0, \dots, N_P - 1$ are the pilot positions. Let the Gram matrix defined by

$$\mathbf{G}_{i,j}(u, v) = \langle \boldsymbol{\varphi}_{i,j}(P_u), \boldsymbol{\varphi}_{i,j}(P_v) \rangle = K_{i,j}(P_u, P_v), \quad (12)$$

where $K_{i,j}(P_u, P_v)$ is a Mercer's kernel which represents the Radial Basis Function (RBF) kernel matrix [5] which allows obviating the explicit knowledge of the nonlinear mapping $\boldsymbol{\varphi}(\cdot)$. A compact form of the functional problem can be stated in matrix format by placing optimal solution $\mathbf{w}_{i,j}$ into the primal dual functional and grouping terms. Therefore, the dual problem

consists of

$$\begin{aligned}
\max \quad & - \frac{1}{2} \boldsymbol{\psi}_{i,j}^H (\mathbf{G}_{i,j} + \gamma \mathbf{I}) \boldsymbol{\psi}_{i,j} + \Re(\boldsymbol{\psi}_{i,j}^H \mathbf{Y}_{i,j}^P) \\
& - (\boldsymbol{\alpha}_{R,i,j} + \boldsymbol{\alpha}_{R,i,j}^* + \boldsymbol{\alpha}_{I,i,j} + \boldsymbol{\alpha}_{I,i,j}^*) \mathbf{1} \varepsilon \quad (13)
\end{aligned}$$

constrained to

$$0 \leq \alpha_{R,m,i,j}, \alpha_{R,m,i,j}^*, \alpha_{I,m,i,j}, \alpha_{I,m,i,j}^* \leq C, \quad (14)$$

where $\boldsymbol{\psi}_{i,j} = [\psi_{i,j}^0, \dots, \psi_{i,j}^{N_P-1}]^T$; \mathbf{I} and $\mathbf{1}$ are the identity matrix and the all-ones column vector, respectively; $\boldsymbol{\alpha}_{R,i,j}$ is the vector which contains the corresponding dual variables, with the other subsets being similarly represented. The weight vector can be obtained by optimizing (13) with respect to $\alpha_{R,m,i,j}$, $\alpha_{R,m,i,j}^*$, $\alpha_{I,m,i,j}$, $\alpha_{I,m,i,j}^*$ and then substituting into (11). Therefore, and after learning phase, frequency responses at all subcarriers in each OFDM symbol can be obtained by SVM interpolation

$$\hat{H}_{i,j}(k) = \sum_{m=0}^{N_p-1} \psi_{i,j}^m K_{i,j}(P_m, k) + b_{i,j}, \quad (15)$$

for $k = 1, \dots, N$. Note that, the obtained subset of dual multipliers which are nonzero will provide with a sparse solution. As usual in the SVM framework, the free parameter of the kernel and the free parameters of the cost function have to be fixed by some a priori knowledge of the problem, or by using some validation set of observations [2].

IV. SIMULATION RESULTS

The specification parameters of an extended vehicular A model (EVA) for downlink LTE system with the excess tap delay and the relative power for each path of the channel are presented in table 1. These parameters are defined by 3GPP standard [6]. In order to demonstrate the effectiveness

TABLE I
EXTENDED VEHICULAR A MODEL (EVA) [6].

Excess tap delay [ns]	Relative power [dB]
0	0.0
30	-1.5
150	-1.4
310	-3.6
370	-0.6
710	-9.1
1090	-7.0
1730	-12.0
2510	-16.9

of our proposed technique and evaluate the performance in the presence of impulsive noise under high mobility conditions, we used a varied range of signal-to-impulse ratio (SIR) which it ranged from -20 to 20 dB. The SIR is given by [2]

$$\begin{aligned}
SIR_{dB} &= 10 \log_{10}(E|IDFT(R_j[l, k]) - IDFT(W_j[l, k]) \\
&- IDFT(I_j[l, k])|^2 / \sigma_{BG}^2). \quad (16)
\end{aligned}$$

We consider a scenario for MIMO-OFDM downlink LTE system with V-BLAST detection algorithm for a mobile speed equal to 350 Km/h. The simulated system parameters are according to 3GPP specifications presented in [7], [8] and [9]. Two Tx and four Rx antennas are used for the MIMO-OFDM system. The channel length L is assumed to be 9. There are a total of 512 subcarriers so that the FFT/IFFT size is 512. The OFDM symbol period is $72 \mu s$. The channel bandwidth consists of $B = 5 MHz$ and the spacing between subcarriers is 15 KHz. Modulation in subcarriers is 16-QAM and the carrier frequency is 2.15 GHz.

The nonlinear complex M-SVR estimate a number of OFDM symbols in the range of 1400 symbols per receive antenna, corresponding to ten radio frame LTE. Note that, the LTE radio frame duration is 10 ms [7], which is divided into 10 subframes. Each subframe is further divided into two slots, each of 0.5 ms duration. We Notice that, in the LTE system, when two or more transmitter antennas are applied, the pilot symbols are transmitted orthogonally in space. Indeed, these orthogonality in space is obtained by letting all other antennas be silent in the resource element in which one antenna transmits a pilot symbol.

For comparison purposes, Least squares (LS), Minimum Mean Squares Error (MMSE) and Decision Feedback channel estimates are simultaneously obtained in the frequency domain in all cases. Fig. 1 represents the Bit Error Rate (BER) improvements that can be attained with nonlinear complex M-SVR and other conventional algorithms. All techniques are simulated in the presence of non-Gaussian impulsive noise with $p = .1$. This figure confirms that nonlinear complex M-SVR algorithm outperforms other algorithms especially for low SIR values where nonlinearities increase. The complex M-SVR algorithm parameter values are set as: C ranging from 10 to 1000, γ from 10^{-2} and 10^{-5} , and ε within a range from .001 and 0.1.

Accordingly, we take into account in Fig. 2 the impulsive noise with different values ($p = .05$ and $p = .1$) for different receive antennas ($N_r = 2$, $N_r = 3$ and $N_r = 4$) with $N_t = 2$. It is clear that the behavior of the nonlinear complex M-SVR performs better for low nonlinearity (low value of p) and for high number of receive antennas.

Regarding the complexity of the used estimators, LS is the least complex estimator because it contains only one matrix inversion operation. However, the Decision Feedback estimator contains two operations of matrix inversion and two operations of matrix multiplication, while the MMSE estimator suffers from high complexity since a matrix inversion is needed each time the data change. On the other hand, the M-SVR estimator uses quadratic programming (*quadprog* function in Optimization MATLAB Toolbox) with the functions *Buffer* and *kron* for fast computation of kernel matrix using the Kronecker product, and thus the algorithm becomes faster.

V. CONCLUSION

This paper describes a new semi-blind MIMO-OFDM channel estimation algorithm based on the M-SVR method to compensate and estimate channel effects for a MIMO-OFDM wireless communication system. Indeed, this paper adopts a nonlinear complex M-SVR based channel estimator for LTE downlink system with V-BLAST detection algorithm in the presence of impulsive noise interfering with OFDM pilot symbols under high mobility conditions (350 Km/h). Our formulation is based on nonlinear complex M-SVR specifically developed for comb type pilot arrangement-based MIMO-OFDM system. The proposed method is based on learning process that uses training sequence to estimate the channel variations. Therefore, pilot symbols are inserted into different subcarriers at different antennas in order to increase the convergence rate and the estimation accuracy. Through experimentation, results have confirmed the capabilities of the proposed nonlinear complex MIMO-OFDM M-SVR estimator in the presence of Gaussian and non-Gaussian impulsive noise interfering with the pilot symbols for a high mobile speed when compared to some conventional techniques.

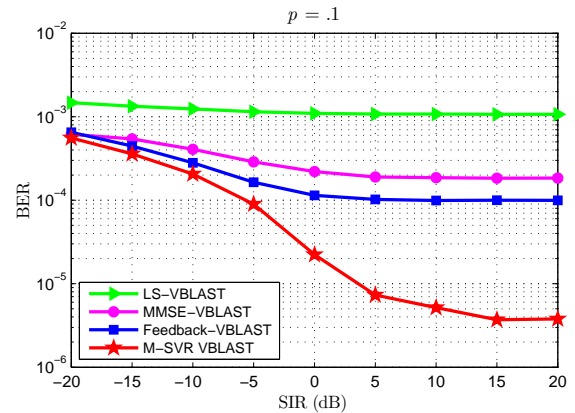


Fig. 1. BER performance as a function of SIR for (2×4) MIMO-VBLAST system for a mobile speed at $350 Km/h$ with $p = .1$.

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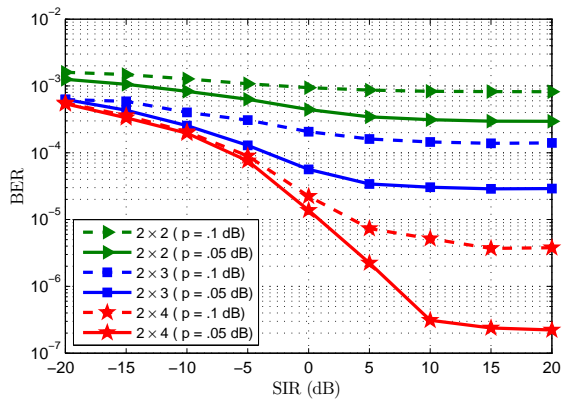


Fig. 2. BER performance for MIMO-VBLAST system with various receive antennas at mobile speed = 350Km/h with $p = .05$ and $p = .1$.

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