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Newton Raphson algorithm for Selective Harmonic Elimination in Asymmetrical CHB Multilevel Inverter using FPGA

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***Abstract*— Asymmetrical structure is used to reduce the number of bridges and gate drive circuits and DC sources. This structure therefore provides the capability to produce higher voltages at higher speeds with low switching frequency which has inherent low switching losses and high converter efficiency.**
Newton Raphson (N-R) algorithm is investigated for the selective harmonic elimination (SHE) to calculate switching angles for a range of variation of the modulation rate 'r' for an asymmetrical cascaded multilevel inverter control.
Based on simulation studies, performance of the proposed algorithm for a nine level asymmetrical cascaded H-bridge inverter, is evaluated and experimentally tested on an prototype using FPGA to implement SHE based on N-R algorithm.

***Keywords*— Symmetrical/Asymmetrical CHB multilevel inverter, Newton Raphson algorithm, SHE, THD, FPGA.**

I. INTRODUCTION

Multilevel inverters are an attractive solutions for the high power applications due to their better performance compared to two-level inverter; there are three types named as diode clamped multilevel inverter, flying capacitor multilevel inverter and cascaded multilevel [1], [2].

Compared to diode clamped and flying capacitor type, cascaded H-bridge (CHB) inverter requires least number of components to achieve same number of voltage levels and optimized circuit layout is possible because each level have same structure and there is no extra clamping diodes or capacitors [3], [4].

The asymmetrical cascaded multilevel inverters generate a higher number of output levels in comparison with the symmetrical cascaded multilevel inverters with the same number of power electronic devices because of the different amplitude of its DC voltage sources. As a result, the installation space and total cost of an asymmetrical cascaded multilevel inverter is lower than that of a symmetrical cascaded multilevel inverter [5], [6], [7].

Several methods are put forth for the harmonic elimination in literature, such as pulse width modulation (PWM), sinusoidal pulse width modulation (SPWM), space vector modulation (SVM), selective harmonic eliminated pulse width modulation (SHEPWM), all of various switching methods produce harmonics and hence, it is interested in selecting the best method to achieve minimum harmonic and total harmonic distortion [8], [9].

In this study, the lower order harmonics can be eliminated by selection of appropriate switching angle. The various optimization algorithms [10], like partical swarm optimization (PSO) [11], symmetrical polynomial and resultant Theory are used to lower the THD and eliminate the Lower order harmonics. In this paper, the N-R based optimization algorithm is used to eliminate the lower order THD for asymmetrical CHB nine level inverter. The proposed method provides better solution for practical application [12].

FPGAs are digital hardware-based devices and they have become an increasingly popular technology in digital prototyping for multilevel inverters due to their speed and flexibility [13].

In this paper, SHE is suggested for a 9-level asymmetrical H-Bridge inverter. The Newton-Raphson method is used to calculate switching angles with the capability to eliminate the lowest order harmonics (5^{th}, 7^{th}, 11^{th}), while maintaining the fundamental component, in order to generate an optimum stepped output waveform. The analytical results are validated through both simulation and experimental results [14].

This paper is organized as follows. Section 2 describes both power topology of asymmetrical cascaded multilevel inverter and asymmetrical CHB inverter. Harmonic elimination based on N-R optimization is explained in section 3. Simulation and experimental results are presented in section 4 and 5 respectively. Finally, the concluding remarks are drawn in section 6.

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II. POWER TOPOLOGY OF ASYMMETRIC AND SYMMETRICAL CHB MULTILEVEL INVERTER

As shown in figure 1, the cascaded multilevel inverter is one of several multilevel configurations. It is formed by connecting single-phase H-bridges inverters in series [15], [3].

In symmetrical cascaded multilevel inverter, where the DC-link voltages of HBs are identical. The number of output levels is normalized by:

$$N = 2h + 1, \quad \text{h: number of H-Bridge} \qquad (1)$$

When the number of HB=4, as shown in figure1, therefore the single phase symmetrical inverter output voltage V_{AO} gives a nine level output voltage: $N = 2 \times 4 + 1 = 9$ for Vdc1=Vdc2=Vdc3=Vdc4=E.

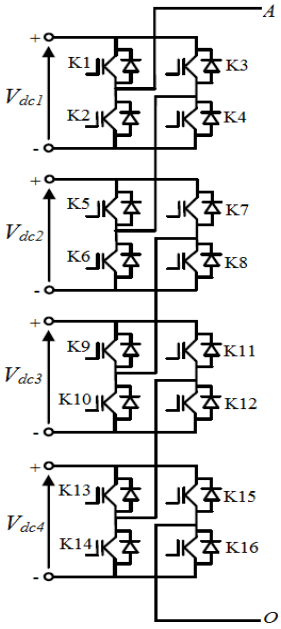

Fig.1 Topology of single phase symmetrical Cascaded 9-level inverter

Unlike symmetrical multilevel CHB inverters which is characterized by partial cells supplied with DC voltages having the same values, asymmetrical converters which are the subject of this study consist of partial cells supplied by different DC voltages, the number of output levels normalized by:

$$N = 2(\sum_{j=1}^{h} \lambda j) + 1\,, \quad \text{where } \lambda j = \frac{Vdcj}{Vdc1} \qquad (2)$$

Asymmetrical nine levels HB inverter, is obtained for Vdc3= 2Vdc1=2Vdc2=2E.

$$N = 2(1 + 1 + 2) + 1 = 9.$$

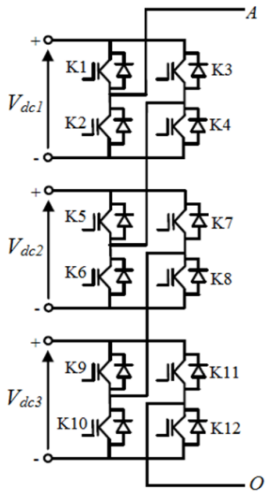

Fig.2 Topology of single phase asymmetrical Cascaded 9-level inverter

The studied asymmetrical 9-level inverter configuration is shown in table 1.

TABLE I
SWITCHING STATES FOR CASCADED SYMMETRICAL 9-LEVEL INVERTER

$V_{dc3}=2V_{dc2}$, $V_{dc1}=V_{dc2}$				
State	**V_{dc1}**	**V_{dc2}**	**V_{dc3}**	**V_{AO}**
1	E	E	2E	4E
2	0	E	2E	3E
3	0	0	2E	2E
4	E	0	0	E
5	0	0	0	0
6	-E	0	0	-E
7	0	0	-2E	-2E
8	0	-E	-2E	-3E
9	-E	-E	-2E	-4E

TABLE II
DC-VOLTAGE SOURCES AND SWITCHES COMPARISON FOR DIFFERENT TOPOLOGIES

Topology	No. of voltage sources	No. of switches	No. of output level
Symmetrical 9-level CHB inverter	4	16	9
Symmetrical 7-level CHB inverter	3	12	7
Asymmetrical 9-level CHB inverter	3	12	9

As shown in table 2, for the same number of bridges, the asymmetrical structure compared to a symmetrical H-bridge topology, can produces a higher number of levels, consequently a better voltage quality, which make the asymmetrical inverter to be a perfect candidate for selective harmonic elimination.

III. HARMONIC ELIMINATION BASED ON NEWTON RAPHSON ALGORITHM

In this section staircase voltage waveform as shown in figure 3 is chosen for the selective harmonic elimination (SHE) technique in multilevel inverters [16], [6]. The problem under consideration is to find appropriate switching angles namely θ_1, θ_2, θ_3 … θ_p so that the p-1 non-triplen odd harmonics can be eliminated and control of the fundamental is also achieved.

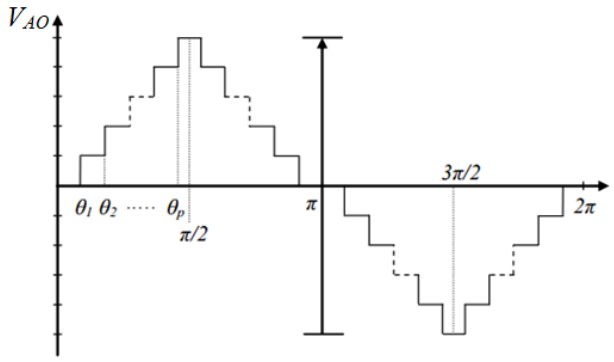

Fig.3 Typical output voltage waveform of a multilevel inverter

Because of the quarter-wave symmetry, the Fourier series expansion of the output voltage V_{AO}, as shown in Figure 3, can be written as:

$$f(\omega t) = 4/\pi \int_0^{\pi/2} V_{AO}(\omega t)\, d\omega t \text{ , for odd n} \quad (3)$$

$$A_n = 0 \text{ , for even n} \quad (4)$$

$$B_n = 0 \text{ , for all n} \quad (5)$$

$$V_{AO}(\omega t) = \sum_{n=1}^{+\infty} A_n sin(n\omega t) \quad (6)$$

p and A_n are the number of switching angles and the magnitude of the n^{th} harmonic order respectively, such as:

$$A_n = \frac{4E}{n\pi} \sum_{i=1}^{p} cos(n\theta_i) \quad (7)$$

For N-level, in the staircase output voltage waveform, the number of the switching angles p to be calculated is given by:

$$p = \frac{N-1}{2} \quad (8)$$

For a nine level inverter output voltage (N=9), the number of harmonics to be eliminated is equal to (p-1) =3.

The maximum fundamental voltage is obtained when all the switching angles are zero.

In this case:

$$A_{1max} = \frac{4p}{\pi} V_{dc1} = \frac{16}{\pi} E \quad (9)$$

It is desirable to control the fundamental component of the output voltage at a certain value and eliminate the low-order harmonics as much as possible. In a three-phase and three-wire system the triplen harmonics will be automatically eliminated. In fact, p switching angles are determined by imposing the amplitude of the fundamental component and eliminate the (p-1) harmonics.

In our case, the four switching angles (θ1, θ2, θ3 and θ4) must be determined to eliminate the first three odd harmonic components (5^{th}, 7^{th} and 11^{th} order). One solution approach for sets of nonlinear transcendental equations (10) is by applying an iterative method based one Newton Raphson algorithm [17], [18], [19].

$$\begin{cases} cos(\theta_1) + \cos(\theta_2) + \cos(\theta_3) + \cos(\theta_4) = r\pi \\ cos(5\theta_1) + \cos(5\theta_2) + \cos(5\theta_3) + \cos(5\theta_4) = 0 \\ cos(7\theta_1) + \cos(7\theta_2) + \cos(7\theta_3) + \cos(7\theta_4) = 0 \\ cos(11\theta_1) + \cos(11\theta_2) + \cos(11\theta_3) + \cos(11\theta_4) = 0 \end{cases} \quad (10)$$

Modulation rate **r** is given as follow:

$$r = \frac{A_1}{pV_{dc1}} = \frac{A_1}{pE} \text{ : Modulation rate} \quad (11)$$

The Newton_Raphson (N-R) method is one of the fastest iterative methods. Here, the N-R is used in Matlab to solve the set of transcendental equations in (10), and the following matrices are implemented;

The switching angle matrix,

$$\theta^j = \begin{bmatrix} \theta_1^j \\ \theta_2^j \\ \theta_3^j \\ \theta_4^j \end{bmatrix} \quad (12)$$

The nonlinear system matrix,

$$F(\theta) = \begin{bmatrix} cos(\theta_1) & cos(\theta_2) & cos(\theta_3) & cos(\theta_4) \\ cos(5\theta_1) & cos(5\theta_2) & cos(5\theta_3) & cos(5\theta_4) \\ cos(7\theta_1) & cos(7\theta_2) & cos(7\theta_3) & cos(7\theta_4) \\ cos(11\theta_1) & cos(11\theta_2) & cos(11\theta_3) & cos(11\theta_4) \end{bmatrix} \quad (13)$$

And,

$$\left[\frac{\partial F}{\partial \theta}\right]^j = -\begin{bmatrix} sin(\theta_1^j) & sin(\theta_2^j) & sin(\theta_3^j) & sin(\theta_4^j) \\ 5sin(5\theta_1^j) & 5sin(5\theta_2^j) & 5sin(5\theta_3^j) & 5sin(5\theta_4^j) \\ 7sin(7\theta_1^j) & 7sin(7\theta_2^j) & 7sin(7\theta_3^j) & 7sin(7\theta_4^j) \\ 11sin(11\theta_1^j) & 11sin(11\theta_1^j) & 11sin(11\theta_3^j) & 11sin(11\theta_3^j) \end{bmatrix} \quad (14)$$

The corresponding harmonic amplitude matrix,

$$T = \begin{bmatrix} r\pi \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (15)$$

Generally, equation (7) can be written:

$$F(\theta) = T \quad (16)$$

By using matrices (11) to (16) and the Newton_Raphson method, the statement of algorithm is shown as follows:

- Guess a set of initial values for θ^j with j = 0

Assume, $$\theta^0 = \begin{bmatrix} \theta_1^0 \\ \theta_2^0 \\ \theta_3^0 \\ \theta_4^0 \end{bmatrix} \quad (17)$$

- Calculate the value of

$$F(\theta^0) = F^0 \quad (18)$$

- Linearize equation (10) about θ^0

$$F^0 + \left[\frac{\partial F}{\partial \theta}\right]^0 d\theta^0 = T \quad (19)$$

And,

$$d\theta^0 = \begin{bmatrix} d\theta_1^0 \\ d\theta_2^0 \\ d\theta_3^0 \\ d\theta_4^0 \end{bmatrix} \quad (20)$$

- Solve $d\theta^0$ from equation (19),

$$d\theta^0 = INV\left[\frac{\partial F}{\partial \theta}\right]^0 (T - F^0) \quad (21)$$

Where $INV\left[\frac{\partial F}{\partial \theta}\right]^0$ is the inverse matrix of $\left[\frac{\partial F}{\partial \theta}\right]^0$

- As updated the initial values,

$$\theta^{j+1} = \theta^j + d\theta^j \quad (22)$$

Repeat the process for equations (15) to (19), until $d\theta^j$ is satisfied to the desired degree of accuracy, and the solutions must satisfy the condition:

$$\theta_1 < \theta_2 < \theta_3 < \theta_4 < \frac{\pi}{2} \quad (23)$$

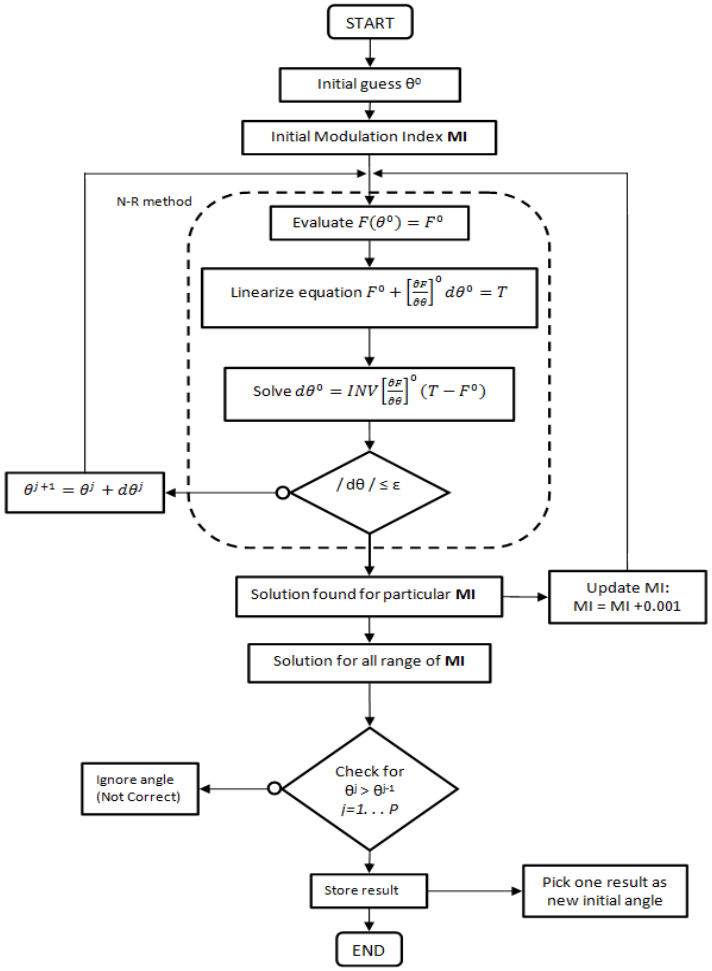

Fig.4 Flowchart of Newton Raphson algorithm

IV. SIMULATION RESULTS

By using MATLAB program, N-R technique returns all the possible combinations of the switching angles for different values of **r**. The result is represented by figure 5, where one can see the presence of unique solutions of angles for 0.826 ≤ r ≤ 0.9 and for 0.925 ≤ r ≤ 1.0. On the other side, the system does not accept any solution.

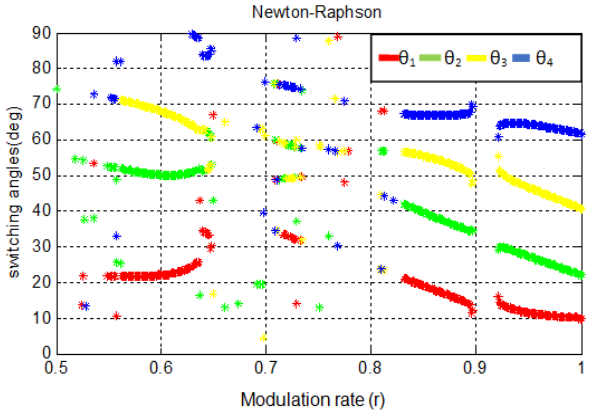

Fig.5 Switching angles versus modulation rate based on N-R algorithm

The residual THD through the 41st harmonic is shown for these solution sets in Figure 6.The THD is defined by:

$$THD(\%) = 100 \times \frac{1}{A_1}\sqrt{\sum_{n=3,5,7\ldots}^{\infty} A_n^2} \qquad (24)$$

The best angle values are therefore the ones leading to the lowest THD. The THD is a quantifiable expression for determining how much the signal has been distorted. The greater are the amplitudes of the harmonics, the greater are the distortions.

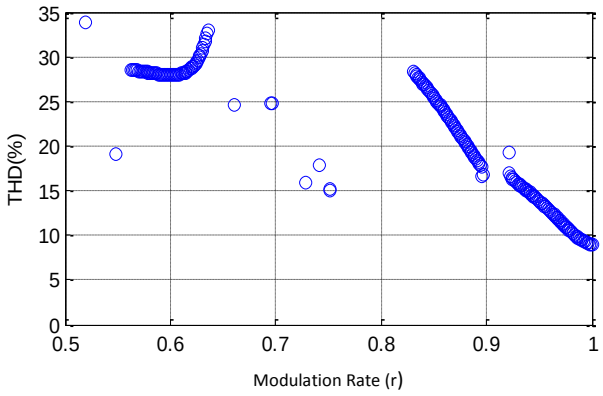

Fig.6 THD vs Modulation Rate besed on N-R algorithm

Single phase asymmetrical cascaded nine level inverter is used to drive R-L load (R =220Ω, L= 0.5mH) such as the first HB inverter unit (HB1), second HB inverter unit (HB2) and HB3 DC sources voltages are Vdc1= Vdc2=E=50V and Vdc3=2E=100V, respectively. The modulation rate is chosen to be equal to 1, (r=1) and output voltage frequency: f=50 Hz.

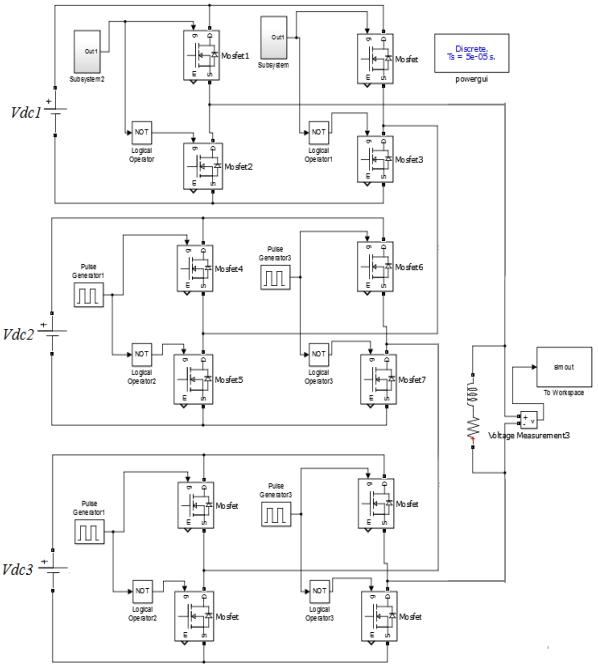

Fig.7 MATLA/Simulink model of single phase asymmetrical nine level inverter

Using Matlab-Simulink, asymmetrical nine level CHB inverter simulation output voltage and its FFT analysis based equal calculated switching angles (ECSA) technique and N-R algorithm, are depicted in Figures 8, 9, and figures 10 and 11, respectively.

The four switching angles obtained by N-R algorithm such, θ1 = 10.01°, θ2 = 22.14°, θ3 = 40.75°, θ4 = 61.75°.

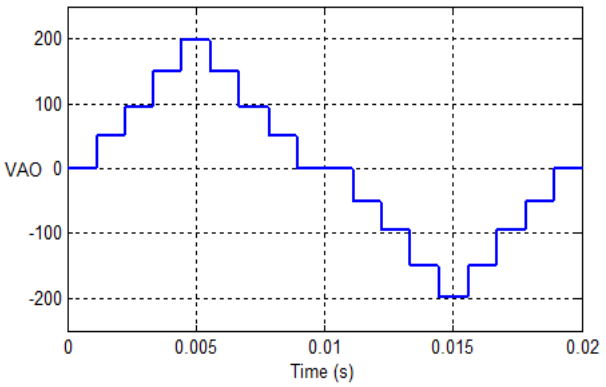

Fig.8 Single phase nine level inverter output voltage waveform (V_{AO}) based on ECSA technique

For, θ1 = θ2 = θ3 = θ4 = 20°

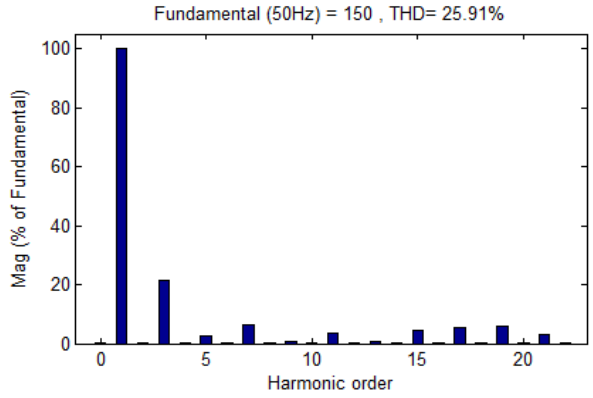

Fig.9 FFT analysis of the nine level inverter output voltage waveform (V_{AO}) based on ECSA

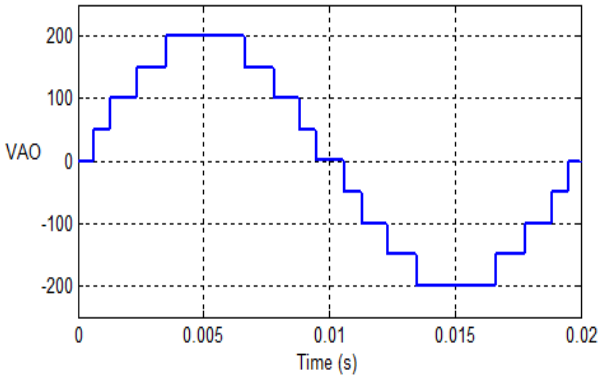

Fig.10 Single phase nine level inverter output voltage wave form (V_{AO}) based on N-R Algorithm

$\theta 1 = 10.01°$, $\theta 2 = 22.14°$, $\theta 3 = 40.75°$, $\theta 4 = 61.75°$

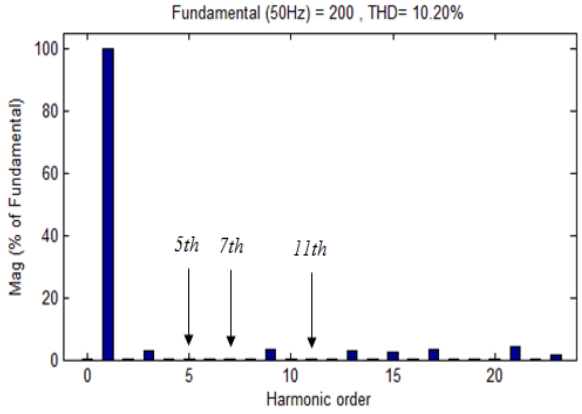

Fig.11 FFT analysis of the nine level output voltage waveform (V_{AO}) based on N-R algorithm for **r**=1

From the spectrum analysis, it is inferred that the THD Newton Raphson based is 10.20% and that for ECSA technique is 25.91%.

In Figure 8 and figure 9 represent the nine level inverter output voltage and its FFT analysis respectively, based on ECSA technique. Figure 9 reveals harmonics 5, 7 and 11 in entirety, reason why the THD is higher than that obtained based on N-R, hence an output voltage waveform represent a poor quality signal. In fact the higher harmonic range in ECSA technique is explained by the absence of the optimization technique in order to eliminate, 5^{th}, 7^{th} and 11^{th} harmonics. However, figure 11, when N-R algorithm is applied, it is clearly identified that the 5^{th}, 7^{th} and the 11^{th} harmonics are completely eliminated, which explains the significant improvement in harmonic profile.

V. EXPREMENTAL RESULTS

The SPARTAN 6 VHDL program is verified and simulated using Xilinx-ISE 13.1 software. Once the program is dumped on the FPGA kit, it acts as a controller and generates gating pulses given in figure 7.

The output of the gating signals can be observed in digital storage oscilloscope (DSO) as given in figure 8, where gating signals are generated based on NR algorithm.

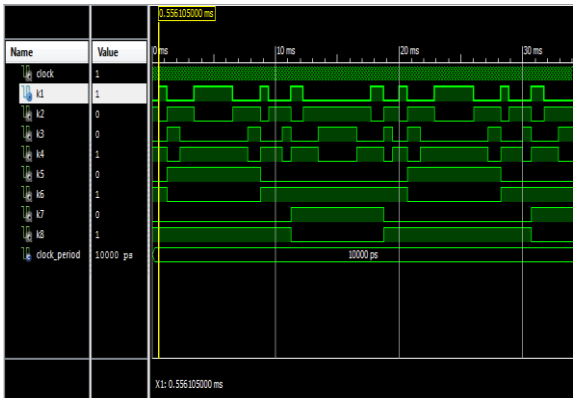

Fig.12 VHDL test bench simulation of the nine levels inverter power switches control signals (K1- K8)

Fig.13 Photograph of the DSO display the control signals based on NR and generated from FPAG- XILINX

VI. CONCLUSION

This paper has presented a modular method for implementing SHE in an FPGA for a single-phase 9-level asymmetrical CHB multilevel inverter. With the asymmetrical topology, the output quality can be improved with a lower number of switches. Using Newton-Raphson's method, the objectives are achieved by eliminating the 5^{th}, 7^{th} and 11^{th} harmonics of the output voltage.

Simulation results prove the precision and efficiency of the N-R algorithm compared to ECSA.

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