

# Rigorous Study of a microstrip line excited with perpendicular coaxial source using MGEC MoM based Method

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**Abstract**— This paper describes a new approach based on the MGEC method [1] which should enable analyzing circuits in different plans. In this study, we consider an excitation source of type coaxial cable localized in a perpendicular plan to the circuit. This type of excitation requires manipulating MGEC method in full 3D context. The originality of this work is the introduction of new operators expressing transformations from one plan to another.

**Keywords**— MoM, 3D structures, coaxial source, discontinuity, MGEC

## I. INTRODUCTION

In the literature, various techniques have been developed for the characterization of discontinuities in planar circuits [2, 3, 4, 5, 6, 7, 8, 9, 10 and 11]. In these methods, several types of excitation sources were used in order to guarantee an acceptable coherence and efficiency as to better describe the theoretical formulation. This problem was first raised in 1985 by Janson [12]. He explains the difficulty of finding a natural formulation for the term excitation [13]. Indeed, the introduction of a source makes the boundaries conditions verification more difficult in its vicinity.

Because of its simplicity, the method of moments (MoM) [2] is often used for solving such problems (MGEC being one of its applications). However, MoM takes only into account the planar excitation sources incorporated in the same plan as the circuit. Actually, sources are either perpendicular to the circuit plan or are put in the ground plan level. In both cases, excitation should be modeled in a different plan than the circuit. Here where the 2.5D MoM method finds its limits.

To simplify this problem most studies model the excitation source as a localized planar element that is integrated in the same plan as the circuit.

Hence the importance of our study as we propose a more accurate modelling by taking into account the excitation source plan. In fact, our new approach allows modelling different planar structures irrespective of the source localization plan. Real coaxial cable excitations located in a perpendicular plan, from the one hand, or the line coaxial excitations located at the ground plan, on the other hand, are

to be better expressed and modeled. In this paper, we consider excitation by coaxial cable.

Our approach is based on the definition of new admittance operators for connecting the excitation source to various electrical quantities. These new operators are used to model the electric field and current density at the circuit plan by means of three-dimensional transformations describing the passage from one plan to another.

Next paragraph is an introduction to the new formulation and calculation of the different operators in the case of a coaxial source located at the perpendicular plan to the circuit. Then, we will present various simulation results with our new formulation. Finally, we validate results by means of comparison with commercial software HFSS.

## II. NEW FORMULATION

Coaxial cable is widely used as excitation source. We will take this case as a concrete example of source located in a perpendicular plan to the circuit. To avoid discontinuity between the source and the circuit, we take as an expression of the source, the fundamental mode of coaxial cable (quasi-TEM) [14, 15].

This source is defined by a unitary function  $|e_1(x, y)\rangle$ , where:  $\langle e_1 | j_1 \rangle = 1$  and  $|j_1\rangle$  is the current density relative to  $|e_1(x, y)\rangle$  on the plan (xoy).

To validate our numerical approach, we consider a microstrip line short-circuited at a distance  $l$  from the source as shown in Fig. 1.

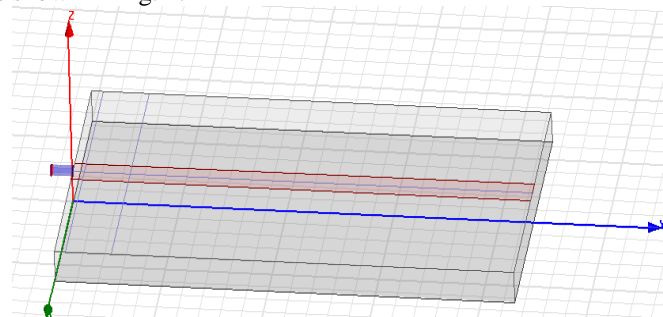


Fig. 1 Microstrip short-circuited line

Dimensions are detailed in the Table 1:

TABLE I  
STUDY STRUCTURE DIMENSIONS

	length	width	height
<b>Line</b>	$l=\lambda/4$	$w=2\text{mm}=0.023\lambda$	-
<b>dielectric</b>	$l=\lambda/4$	$a=12.7\text{mm}=0.15\lambda$	$h_0=11.43\text{mm}=0.133\lambda$
<b>Box</b>	$l=\lambda/4$	$a=12.7\text{mm}=0.15\lambda$	$h=1.27\text{mm}=0.15\lambda$

With: freq=3.5 GHz,  $\epsilon_r=1$

According to the method of generalized equivalent circuit MGEC associated to the MoM in order to establish relationship between the various electrical quantities of structure (E, J). We use circuit at Fig. 2 which depicts the equivalent circuit relative to the structure of Fig.1.

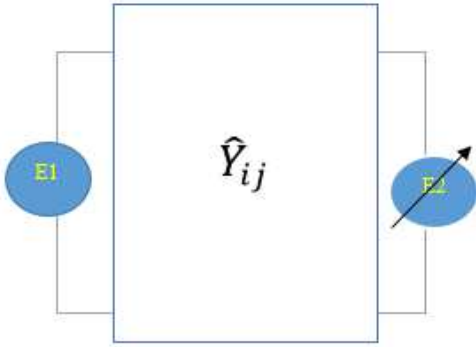


Fig. 2 generalize equivalent circuit of Microstrip short-circuited line

We consider here admittance operator. The electromagnetic quantities in the source plan and circuit plan can be written using the following relations:

$$\begin{cases} |J_1\rangle = |\hat{Y}_{11} E_1\rangle + |\hat{Y}_{12} E_2\rangle \\ |J_2\rangle = |\hat{Y}_{21} E_1\rangle + |\hat{Y}_{22} E_2\rangle \end{cases} \quad (1)$$

Where:

$J_1$ : current density of source

$E_1$ : electric field of source

$E_2$  and  $J_2$  are the electric field and the current density defined in the plan of the circuit to study

The admittances operators  $\hat{Y}_{ij}$  define the relationship between the electric fields of the TE and TM modes of  $j^{\text{th}}$  plan and the current density generated by these modes on the  $i^{\text{th}}$  plan. These  $\hat{Y}_{ij}$  can be determined by applying the superposition theorem to the previous system and decomposing these operators on a homogeneous basis (TE and TM) associated with each one.

If  $f_{mn}$  and  $g_{pq}$  are the orthonormal basis of electric field and current density defined on the source and the circuit plans then we can write for  $i=1,2$  and  $j=1,2$

$$\hat{Y}_{ij} = \sum_{mn} \sum_{pq} |g_{pq}^{(i)}\rangle^{TE, TM} y_{mn, pq}^{TE, TM} \langle f_{mn}^{(j)} |^{TE, TM} \quad (2)$$

If  $i = j$ ,  $\hat{Y}_{ij}$  coincides with the admittance operator defined on the  $i^{\text{th}}$  plan ( $i=1$ : vertical plan,  $i=2$ : circuit plan). In this case, we have  $g_{pq} = f_{mn}$  and the equation (2) becomes:

$$\hat{Y}_{ii} = \sum_{mn} |f_{mn}\rangle^{TE, TM} y_{mn}^{TE, TM} \langle f_{mn} |^{TE, TM} \quad (3)$$

To determine the different electromagnetic quantities, we use a variant of moment's method that is the method of Galerkin [16]. The advantage of this method is the rapid convergence of his algorithm, because the computational effort is limited to the resolution of an integral equation expressing the boundary conditions of the electromagnetic fields on a section of the circuit.

We choose the test functions of electric field type  $\varphi_i(x, z)$  that describes the electric field E. Its functions are assumed to be virtual sources defined in the circuit plan and must meet the boundary conditions of the circuit plan (xoz). They are zero on the metal and non-zero on the dielectric. The current density J is zero on the dielectric and non-zero on the metal.

In this basis, the electric field is expressed:

$$E = \sum_{k=1}^K x_k \varphi_k(x, z) \quad (4)$$

With:

$x_k (k = 1 \dots K)$ : Projection of E on this basis

K: the number of test functions necessary for the convergence of electric field E

The application of Galerkin to the system (1), consists of projecting the first equation on the function of unitary source  $|e_{01}\rangle$ .

$$\langle e_{01} | J_1 \rangle = \langle e_{01} | \hat{Y}_{11} E_1 \rangle + \langle e_{01} | \hat{Y}_{12} E_2 \rangle \quad (5)$$

Also, we project the second equation of the system (1) on the different test functions.

$$\langle \varphi_i | J_2 \rangle = \langle \varphi_i | \hat{Y}_{21} E_1 \rangle + \langle \varphi_i | \hat{Y}_{22} E_2 \rangle, \text{ for } i=1 \dots K \quad (6)$$

We assume that:  $E_{01} = V_{01} e_{01}$  and  $J_{01} = I_{01} j_{01}$

With  $\langle e_{0i} | j_{0i} \rangle = 1$

Considering the relationship between the current density and the test functions, the resolution of equations (5) and (6) allows us to establish the expression of the  $y_{in}$  input impedance:

$$y_{in} = \frac{I_1}{J_1} = Y_{11} - C_1 A^{-1} B_1 \quad (7)$$

With:

$$Y_{11} = \langle e_{01} | \hat{Y}_{11} e_{01} \rangle \quad (8)$$

$$A(i, k) = \langle \varphi_i | \hat{Y}_{22} \varphi_k \rangle, i = 1 \dots K \quad (9)$$

$$B(j) = \langle \varphi_j | \hat{Y}_{21} e_{01} \rangle, j = 1 \dots K \quad (10)$$

$$C(i) = \langle e_{01} | \hat{Y}_{12} \varphi_i \rangle, i=1 \dots K \quad (11)$$

To model the electric field, it is sufficient to determine the coefficients  $x_k$  of the test functions. The vector  $X = (x_1, \dots, x_k, \dots, x_K)^T$  is written in the following form (12):

$$X = -\sum_i V_{oi} A^{-1} B_i \quad (12)$$

Using this new formulation of admittance operators  $\hat{Y}_{ij}$  describing the passage of the source plan to the circuit plan (and vice versa) we have been able to evaluate input admittance  $y_{in}$  expression. The next paragraph is dedicated to validation of the presented formulation by deducing electric field and current density on the circuit plan.

### III. NUMERICAL RESULTS

Several admittances operators are used to perform the passage from one plan to another requiring several large-sized matrices manipulation and cpu-consuming integral calculations. Using generic tools such as MATLAB, lacks of fast hybrid symbolic/numeric calculation and has no built-in cache support neither save-points (to stop and resume calculation on need) concept. For these reasons, we considered using a TMWLib library (for Tiny MicroWave Library) developed by Dr Taha Ben Salah during his research work [17]. The library is based on Java/Scala programming languages, is fully modular, feature rich and scalable.

To validate the presented approach, we evaluate the input impedance  $y_{in}$  and compare it to the theoretical input impedance of a microstrip short circuited line. We also deduce some electromagnetic characteristics (current density  $J$  and electric field  $E$ ) to verify boundary conditions.

#### A. The input admittance $y_{in}$

Figure 3 illustrates the simulation result of the input impedance  $y_{in}$  given by equation 7 and that of the theoretical input impedance of a line short circuited at a distance  $l$  from the source given by the equation 13.

$$y_{in}^{th} = -j \cotan(\beta l) \quad (13)$$

With  $\beta$  is the propagation constant of the fundamental mode.

The  $y_{in}$  curve is evaluated at convergence with 5 extended test functions (trigonometric type), 80000 TE and TM mode functions  $f_{mn}$  (source plan) and 54000 TE and TM modes functions (circuit plan)  $g_{pq}$ .

We notice that the two curves are quite similar confirming the validity of the presented formulation with a relative error of at most 1% with a slight difference between the two curves at the poles.

We also note that this result is consistent with theoretical expectations since we have used the fundamental mode of a coaxial cable as an excitation source which explains this perfect matching between source and dipole.

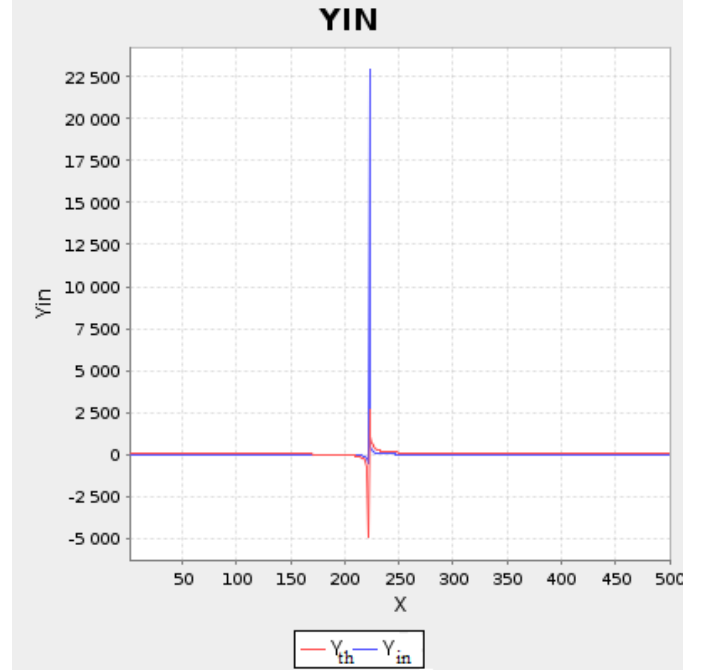


Fig. 3 The input admittance  $y_{in}$  comparison

#### B. Electromagnetic characteristics

We consider now the electromagnetic field projected on the TE and TM mode functions (equation 14) and the current density (equation 15) in three cases: MGEC'3D (our new approach), traditional MGEC (with a planar source) and HFSS (source modal).

$$E = \sum_{p,q} \sum_{i=1}^{N_g} x_i \langle g_{pq}^{TE, TM} | \varphi_i \rangle g_{pq}^{TE, TM} \quad (14)$$

$$J = \sum_{p,q} \sum_{i=1}^{N_g} x_i \langle g_{pq}^{TE, TM} | \varphi_i \rangle y_{pq}^{TE, TM} g_{pq}^{TE, TM} \quad (15)$$

With:

$g_{pq}^{TE, TM}$  : TE and TM mode functions in the circuit plan (xoz)

$y_{pq}^{TE, TM}$  : TE and TM mode admittance at the short-circuit ( $y=-h$ ) and ( $y=h$ )

$$y_{pq}^{TE} = \frac{\gamma_{pq}}{j \omega \mu} \cotan h(\gamma_{pq} h_i) \quad (16)$$

$$y_{pq}^{TM} = \frac{j \omega \epsilon_i}{\gamma_{pq}} \cotan h(\gamma_{pq} h_i) \quad (17)$$

$\gamma_{pq}$  : The propagation constant of a mode  $g_{pq}(x, z)$ , such as :

$$\gamma_{mn}^2 = \left(\frac{m \pi}{a}\right)^2 + \left(\frac{n \pi}{h_i}\right)^2 - \omega^2 \epsilon_i \mu_0 \quad (18)$$

Where:

$$\begin{cases} \epsilon_i = \epsilon_0 \epsilon_r; h_i = -h; \text{ for } -h \leq y \leq 0 \\ \epsilon_i = \epsilon_0; h_i = h_0; \text{ for } 0 < y \leq h_0 \end{cases}$$

### 1) Electromagnetic field E:

Figures 4 and 5 provide a comparison of the results of the normalized electric field  $E_y$  (field in the direction of propagation) at the microstrip short-circuited line between the new approach and results gathered using HFSS commercial simulation software.

Figure 4 illustrates a section along the axis (oy). We observe that the two curves have the same envelop shape (regardless of Gibbs effect fluctuations), a maximum at the source then it follows a rapid attenuation moving away from the source.

Figure 5 presents a section along the axis (ox). We observe that the new technique results still coincide with Commercial Software ones while defining a smaller bandwidth which can be considered as more accurate for such a structure.

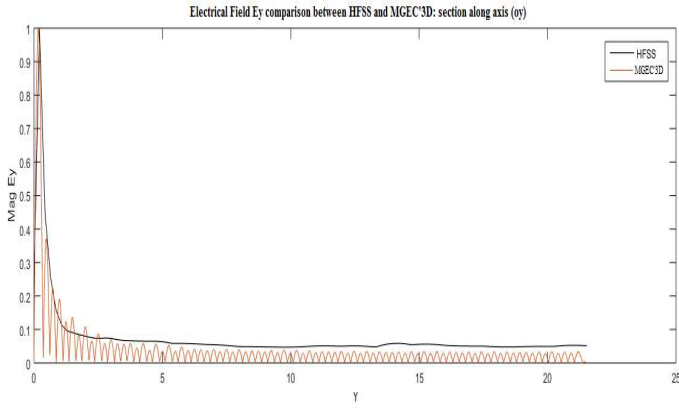


Fig. 4 Electrical field  $E_y$  comparison between HFSS and MGEC'3D: section along axis (oy)

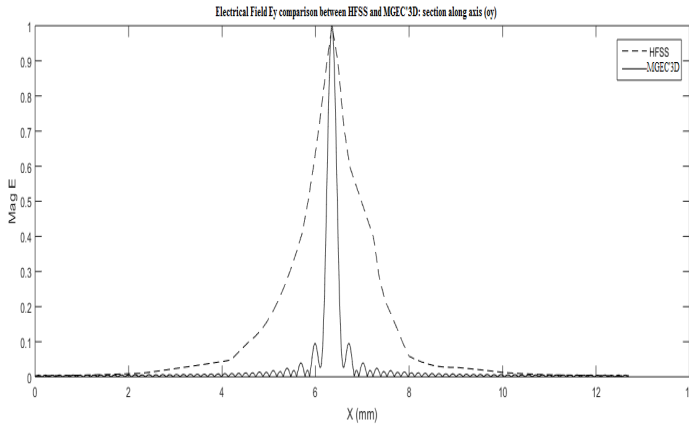


Fig. 5 Electrical field  $E_y$  comparison between HFSS and MGEC'3D: section along axis (ox)

Figure 6 illustrates that the electric field  $E_x$  is maximum at the level of discontinuity and is zero elsewhere with concordance to boundary conditions.



Fig. 6 Electrical field  $E_x$  with MGEC'3D

### 2) Current density

Figures 7 and 8 illustrate shapes of the current density (MGEC'3D and in HFSS). New technique behaves more smoothly and more symmetric due to chosen rooftop x-only test functions.

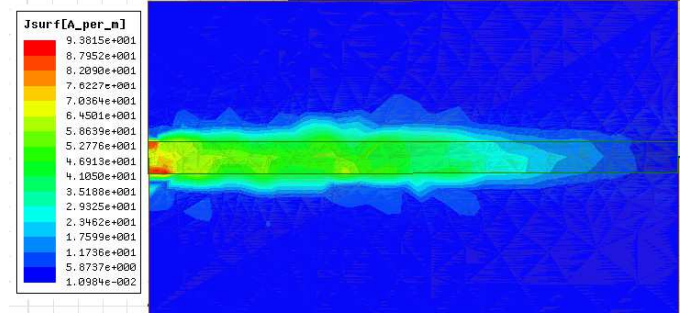


Fig. 7 Current density with HFSS

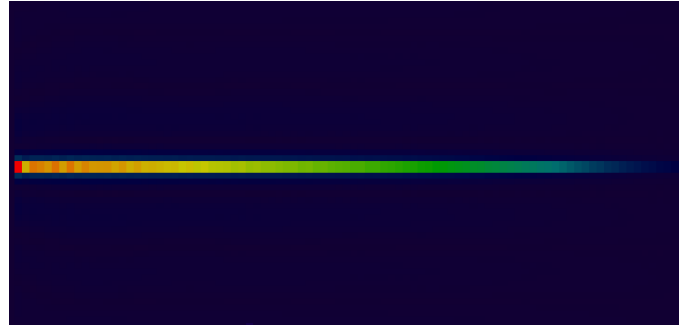


Fig. 8 Current density with MGEC'3D (coaxial source)

## IV. CONCLUSIONS

We proposed in this paper a new formulation of the MGEC method to help studying planar structures excited by coaxial sources located at a perpendicular plan. We introduced a new formulation based on new admittance operators and three-dimensional transformations to depict the transition from one plan to another. We applied the approach to a microstrip short-circuited line for validation and compared input impedance results to theoretical input impedance of the same structure to get a 1% relative error.

We also determined the current density and the electric field and compared them with ones obtained with the commercial software HFSS.

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