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Continuous-time LPV system identification with fractional models

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***Abstract*—This paper deals with the continuous-time identification of linear parameter varying (LPV) systems with fractional models in a noisy output context. Two methods developed to identify the continuous-time systems with rational models are extended to identify the coefficients of fractional models.**
The developed estimators are based on least squares: the fractional-linear parameter varying-ordinary least squares (fLPV-OLS) and the fractional-linear parameter varying-iterative least squares (fLPV-ILS). A fractional state variable filter approach (SVF) is used.
The performances of the developed estimators are analyzed through a numerical example. The influence of the Signal-to Noise Ratio (SNR) is studied using Monte Carlo simulations.

***Index Terms*—system identification, fractional differentiation, linear parameter varying, fractional SVF, least squares.**

I. Introduction

The developed methods for continuous-time system identification with fractional Linear Time Invariant (LTI) models have been treated in several frameworks aiming open loop system identification [1]–[7] and closed loop system identification [8], [9].
Two different classes of identification methods are developed: the first one is based on an equation error and consists in supposing that the fractional orders are known *a priori* and only the linear coefficients are estimated. The second class is based on an output error and consists in estimating both linear coefficients and fractional differentiation.

To identify a continuous-time (CT) model two main approaches are investigated: the direct and the indirect approaches. Both of them use sampled data. The indirect approach consists in identifying a discret-time (DT) model using DT techniques, then convert it into a CT model. The direct approach is based on CT strategies, thus a CT model is identified directly. The direct approach is considered in this paper.

The developed works on fractional systems identification are almost for LTI models. However, in practice, physical behaviours may presents a time varying nature.

A system where its parameters are variables in time is called Linear Parameter Varying (LPV) system. Several methods have been proposed in the past few years to solve the LPV system identification in CT and DT. For an overview of the developed methods, refer to [10]–[18].

Our main contribution is to extend a CT system identification methods with LPV rational models to the fractional case. The developed methods are based on the Least Squares (LS) techniques and called fractional-linear parameters varying-ordinary least squares (*fLPV-OLS*) and fractional-linear parameters varying-iterative least squares (*fLPV-ILS*) algorithms.

The outline of the paper is as follows. The next Section details the differential equation representation of CT fractional LPV systems. In Section 3, the proposed methods for CT fractional LPV systems are detailed. Their performances are analyzed in Section 4 via a numerical example. Section 5 concludes the paper.

II. FRACTIONAL LPV SYSTEMS

A. Mathematical Background

A Single-Input-Single-Output (SISO) linear parameter varying fractional order system is governed by the following differential equation:

$$y_0(t)+\sum_{n=1}^{N} a_n(\rho_t)D^{\alpha_n}y_0(t)=\sum_{m=0}^{M} b_m(\rho_t)D^{\beta_m}u(t) \tag{1}$$

where differentiation orders are allowed to be non integer positive numbers and ordered for identifiability purposes:

$$\alpha_1<\cdots<\alpha_N,\ \beta_0<\cdots<\beta_M$$

$u(t)$ and $y_0(t)$ are respectively the input and free-noise output signals, D is the time-domain differential operator (also denoted p), $D=\dfrac{d}{dt}=p$.
$\rho_t:\mathbb{R}\rightarrow\mathbb{P}$ (the compact $\mathbb{P}\in\mathbb{R}^{n_{\mathbb{P}}}$ denotes the scheduling space) is the scheduling variable with $\rho_t=\rho(t)$.
The coefficients $a_n(\rho_t)$ and $b_m(\rho_t)$ are functions of ρ_t at time.
There exist two types of scheduling variable: scheduling variable with static dependence [17] and scheduling variable with dynamic dependence [10]. In this paper, the static dependence is considered.
The υ-order fractional derivative of a continuous-time function $f(t)$, relaxed at $t=0$, i.e. $f(t)=0$ for $t\leq 0$, is numerically evaluated using the Grünwald approximation [19]:

$$D^{\upsilon}f(t)\approx\frac{1}{h^{\upsilon}}\sum_{k=0}^{K}(-1)^k\begin{pmatrix}\upsilon\\ k\end{pmatrix}f(t-kh),\quad\forall t\in\mathbb{R}_+^* \tag{2}$$

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where $\upsilon \in \mathbb{R}_+^*$, $K = \lfloor \frac{t}{h} \rfloor$ with $\lfloor . \rfloor$ is the floor operator, h is the sampling period and $\left(\begin{array}{c} \upsilon \\ k \end{array} \right)$ is the Newton's binomial generalized to fractional orders:

$$\left(\begin{array}{c} \upsilon \\ k \end{array} \right) = \begin{cases} 1 & \text{, if } k = 0 \\ \frac{\upsilon(\upsilon-1)(\upsilon-2)\cdots(\upsilon-k+1)}{k!} & \text{, if } k > 0 \end{cases} \tag{3}$$

For time-domain simulation of fractional LPV systems, the Grünwald approximation defined by equation (2) is used.

B. Problem formulation

Let (S) the system modeled with a fractional continuous-time differential equation with a static scheduling dependence.

$$(S): \begin{cases} A(\rho_t, p) y_0(t) = B(\rho_t, p) u(t) \\ y(t) = y_0(t) + e_0(t) \end{cases} \tag{4}$$

where $y(t)$ is the measured output signal which is eventually corrupted by an additive white noise $e_0(t)$.

The ρ_t dependent polynomials A and B, witch contain the coefficients of fractional LPV differential equation, are defined by:

$$\begin{cases} A(\rho_t, p) = 1 + \sum\limits_{n=1}^{N} a_n(\rho_t) p^{\alpha_n} \\ B(\rho_t, p) = \sum\limits_{m=0}^{M} b_m(\rho_t) p^{\beta_m} \end{cases} \tag{5}$$

where

$$\begin{cases} a_n(\rho_t) = a_{n,0} + \sum\limits_{l=1}^{L} a_{n,l} f_l(\rho_t);\ n = 1, ..., N \\ b_m(\rho_t) = \sum\limits_{l=0}^{L} b_{m,l} f_l(\rho_t);\ m = 0, ..., M \end{cases} \tag{6}$$

$\{f_l\}_{l=0}^{L}$ is a memorphic [1] function of ρ_t with static dependence defined by:

$$f_l(\rho) = \rho_t^l, \quad l = 0, \cdots, L \tag{7}$$

Using equation (7), the coefficients given by (6) can be rewritten as:

$$\begin{cases} a_n(\rho_t) = a_{n,0} + a_{n,1}\rho_t + \cdots + a_{n,L}\rho_t^L \\ b_m(\rho_t) = b_{m,0} + b_{m,1}\rho_t + \cdots + b_{m,L}\rho_t^L \end{cases} \tag{8}$$

Thus, the fractional CT-IO-LPV system (S) is defined by:

$$(S): \begin{cases} y_0(t) + \sum\limits_{n=1}^{N} a_n(\rho_t) p^{\alpha_n} y_0(t) = \sum\limits_{m=0}^{M} b_m(\rho_t) p^{\beta_m} u(t) \\ y(t_k) = y_0(t_k) + e_0(t_k) \end{cases} \tag{9}$$

where $t_k = kh$ $(k \in \mathbb{Z})$.

The fractional differentiation orders are supposed *a priori* known. The objective is to estimate the coefficients of the fractional LPV differential equation (9), for each t_k $(k < N_t)$, using the set of available data $D_{N_t} = \{y(t_k), u(t_k), \rho(t_k)\}_{k=0}^{N_t}$ (N_t is the number of samples) sampled with a sampling period h.

[1] f is a memorphic function if it is in the form $f = \frac{g}{g_1}$ where g and g_1 are analytic functions with $g_1 \neq 0$.

The unknown parameters matrix is given by:

$$\theta = \begin{pmatrix} a_{1,0} & \cdots & a_{1,L} \\ \vdots & \ddots & \vdots \\ a_{N,0} & \cdots & a_{N,L} \\ b_{0,0} & \cdots & b_{0,L} \\ \vdots & \ddots & \vdots \\ b_{M,0} & \cdots & b_{M,L} \end{pmatrix} \tag{10}$$

Then, the problem is to consistently estimate the parameters matrix in the LPV framework.

III. Least squares based methods for fractional LPV system identification

The estimated output signal $\hat{y}(t_k)$ can be written as a linear regression form:

$$\hat{y}(t_k) = \Phi^T(t_k)\theta + e_\theta(t_k) \tag{11}$$

where $e_\theta(t_k)$ is the output error and $\Phi(t_k)$ the extended regression vector defined by:

$$\Phi(t_k) = \varphi^T \otimes F \tag{12}$$

where $\otimes$ design the kronecker product (also denoted a tensor product), F is given by the following equation:

$$F = [1\ f_1(\rho) \cdots f_L(\rho)] \tag{13}$$

and $\varphi(t_k)$ is defined by:

$$\begin{aligned} \varphi^T(t_k) = [& -p^{\alpha_1} y(t_k), -p^{\alpha_2} y(t_k), \cdots, -p^{\alpha_N} y(t_k), \\ & p^{\beta_0} u(t_k), \cdots, p^{\beta_M} u(t_k)] \end{aligned} \tag{14}$$

The regression vector defined by (14) contains fractional time-derivatives of the input and the noisy output signals. To built this regression vector, the compute of time-domain fractional derivatives is required.

The use of the Grünwald approximation (equation (2)) amplifies the additive noise effect. To solve this problem, the extension of the State Variable Filter (SVF) approach for fractional orders is proposed [1].

The extended regression vector, built from the filtered input and output signals, is given by:

$$\Phi_f(t_k) = F_\upsilon(s)\left[\varphi^T \otimes F\right] \tag{15}$$

with $F_\upsilon(s)$ is the transfer function of the fractional SVF defined by:

$$F_\upsilon(s) = s^\upsilon \left(\frac{\lambda}{\lambda + s} \right)^{\lfloor \alpha_N \rfloor + 1} \tag{16}$$

$\lfloor . \rfloor$ stands for the floor operator and λ is the cut-off frequency. λ should be chosen equal to, or larger than, the bandwidths of the system to be identified.

Equation (15) can be rewritten as:

$$\varphi_f = \left[-y_f^{\alpha_1}(t_k) - y_f^{\alpha_2}(t_k), \cdots, -y_f^{\alpha_N}(t_k), u_f^{\beta_0}(t_k), \cdots, u_f^{\beta_M}(t_k) \right]^T \tag{17}$$

where $y_f^{\alpha_n}$ and $u_f^{\beta_m}$ are the fractional filtered derivatives of the input and the measured output signals:

$$\begin{cases} y_f^{\alpha_n}(t_k) = F_{\alpha_n}(s)y(t_k);\ 1 \le n \le N \\ u_f^{\beta_m}(t_k) = F_{\beta_m}(s)u(t_k);\ 0 \le m \le M \end{cases} \tag{18}$$

A. Fractional-linear parameter varying-ordinary least squares algorithm (fLPV-OLS)

This method is inspired by the work developed in [12] for DT LPV system identification with rational models and extended in this section for fractional models.

The *fLPV-OLS* estimator is given by:

$$\hat{\theta}_{fLPV-OLS}(k) = \arg \min_{\theta \in \mathbb{R}^{n_\theta}} V(D_{N_t}, \theta) \tag{19}$$

where the cost function $V\{D_{N_t}, \theta\}$ is defined as:

$$V(D_{N_t}, \theta) = \frac{1}{N_t} \sum_{k=1}^{N_t} {\varepsilon_\theta}^2(t_k) \tag{20}$$

and based on the equation error:

$$\varepsilon_\theta(t_k) = y(t_k) - \hat{y}(t_k) \tag{21}$$

$\hat{y}(t_k)$ design the estimated output signal.

Then, the optimal estimator is given by:

$$\hat{\theta}_{fLPV-OLS} = \left[\frac{1}{N_t} \sum_{k=1}^{K} \hat{\Phi}_f(t_k) \hat{\Phi}_f^T(t_k) \right]^{-1} \left[\frac{1}{N_t} \sum_{k=1}^{K} \hat{\Phi}_f(t_k) y_f(t_k) \right] \tag{22}$$

This estimator is unbiased and consistent if:

$$\lim_{N_t \to \infty} \varepsilon_{\theta=0} \tag{23}$$

In the noisy framework, the OLS estimator gives a biased parameters. Particulary in the case of fractional LPV system identification, fractional derivatives take into account the hole past of the noisy output and the linear parameters variation. To obtain an unbiased estimation of the parameters matrix, an iterative technique is proposed and presented in the next section.

B. Fractional-linear parameter varying-iterative least squares algorithm (fLPV-ILS)

The *fLPV-ILS* algorithm is inspired by the work in [17] developed for CT-LPV system identification with rational models and the work developed in [7] for CT-LTI system identification with fractional models. It is extended in this section for CT-LPV system identification with fractional models.

Substituting the linear parameters by their expressions given by equation (6) in the fractional LPV system (S) equation (9) yields to:

$$(S): \begin{cases} y_0(t) + \sum_{n=1}^{N} a_{n,0} p^{\alpha_n} y_0(t) = - \sum_{n=1}^{N} \sum_{l=1}^{L} a_{n,l} f_l(\rho_t) p^{\alpha_n} y_0(t) + \\ \qquad \sum_{m=0}^{M} \sum_{l=0}^{L} b_{m,l} f_l(\rho_t) p^{\beta_m} u(t) \\ y(t_k) = y_0(t_k) + e_0(t_k) \end{cases} \tag{24}$$

Equation (24) can be rewritten as:

$$\begin{cases} F_0(p) y_0(t) = - \sum_{n=1}^{N} \sum_{l=1}^{L} a_{n,l} y_{\alpha_n,l}(t) + \sum_{m=0}^{M} \sum_{l=0}^{L} b_{m,l} u_{\beta_m,l}(t) \\ y(t_k) = y_0(t_k) + e_0(t_k) \end{cases} \tag{25}$$

where

$$F_0(p) = 1 + \sum_{n=1}^{N} a_{n,0} p^{\alpha_n} \tag{26}$$

and

$$\begin{cases} y_{\alpha_n,l}(t) = f_l(\rho_t) p^{\alpha_n} y_0(t);\ \{n,l\} \in \{1,...,N; 1,...,L\} \\ u_{\beta_m,l}(t) = f_l(\rho_t) p^{\beta_m} u(t);\ \{m,l\} \in \{0,...,M; 0,...,L\} \end{cases} \tag{27}$$

The system (S) can be rewritten as:

$$\begin{cases} y_0(t) = - \sum_{n=1}^{N} \sum_{l=1}^{L} \frac{a_{n,l}}{F_0(p)} y_{\alpha_n,l}(t) + \sum_{m=0}^{M} \sum_{l=0}^{L} \frac{b_{m,l}}{F_0(p)} u_{\beta_m,l}(t) \\ y(t_k) = y_0(t_k) + e_0(t_k) \end{cases} \tag{28}$$

The estimated output can be presented as a linear regression form:

$$\hat{y}(t_k) = \Phi^T(t_k)\theta + e_\theta(t_k) \tag{29}$$

where

$$\Phi(t_k) = \varphi^T(t_k) \otimes F \tag{30}$$

with φ as defined by (equation(eq17)) then, $\Phi_f(t_k)$ can be expressed as follow:

$$\begin{aligned} {\Phi_f}^T(t_k) = \Big[& -y_f^{\alpha_1,1}(t_k), \cdots, -y_f^{\alpha_N,1}(t_k), \cdots, \\ & -y_f^{\alpha_1,L}(t_k), \cdots, -y_f^{\alpha_N,L}(t_k), \\ & u_f^{\beta_0,1}(t_k), \cdots, u_f^{\beta_M,1}(t_k), \cdots, u_f^{\beta_0,L}(t_k), \cdots, u_f^{\beta_M,L}(t_k) \Big] \end{aligned} \tag{31}$$

$y_f^{\alpha_n,l}(t_k)$ and $u_f^{\beta_m,l}(t_k)$ are the fractional derivatives of the filtered input and the output signals are filtered using the following filter:

$$Q_0^i(s) = \frac{1}{F_0^i(s)} = \frac{1}{1 + \sum_{n=1}^{N} \hat{a}^i_{n,0} s^{\alpha_n}} \tag{32}$$

$Q_0^i(s)$ the filter transfer function which depend on the estimates at the iteration i.

The optimal parameter vector is given by:

$$\hat{\theta}^i_{fLPV-ILS} = \arg\min_\theta V^i(\theta) \tag{33}$$

At each iteration, the following cost function is minimized:

$$V^i_{fLPV-ILS}(\theta) = \frac{1}{N_t} \sum_{k=1}^{N_t} \frac{1}{2} \left(\varepsilon^i_\theta(t_k) \right)^2 \tag{34}$$

$\varepsilon^i_\theta(t_k)$ denotes the equation error at each iteration.

Minimising the cost function V^i at each iteration i leads to the *fLPV-ILS* estimator:

$$\hat{\theta}^i_{fLPV-ILS}(k) = \left[\frac{1}{N_t} \sum_{k=1}^{N_t} \Phi^i_f(t_k) {\Phi^i_f}^T(t_k) \right]^{-1} \left[\frac{1}{N_t} \sum_{k=1}^{N_t} \Phi^i_f(t_k) y_f(t_k) \right] \tag{35}$$

The proposed *fLPV-ILS* algorithm for fractional LPV model identification is summarized in 6 steps.

Step 1: $i=0$ Initialization

Compute the first estimate by applying the *fLPV-OLS* algorithm:

$$\hat{\theta}^{0}_{fLPV-ILS} = \hat{\theta}_{fLPV-OLS} \tag{36}$$

$i = i+1$

Step 2: Compute $\hat{y}(t_k)$ using the obtained auxiliary model $\hat{\theta}^{i-1}_{fLPV-ILS}$.

Step 3: Compute the continuous-time fractional estimated filter

$$F^{i}_{\upsilon}(s) = \frac{s^{\upsilon}}{1+\sum\limits_{n=1}^{N}\hat{a}^{i}_{n,0}s^{\alpha_n}} \tag{37}$$

to generate the estimates of the derivatives signals:

$$\begin{cases} \hat{y}^{\alpha_n,l}_{f}(t_k) = \dfrac{s^{\alpha_n}}{1+\sum\limits_{n=1}^{N}\hat{a}^{i}_{n,0}s^{\alpha_n}}\hat{y}(t_k) \\ u^{\beta_m,l}_{f}(t_k) = \dfrac{s^{\beta_m}}{1+\sum\limits_{i=1}^{N}\hat{a}^{i}_{n,0}s^{\alpha_n}}u(t_k) \end{cases} \tag{38}$$

Step 4: Built the filtered estimated regression vector $\Phi^{i}_{f}(t_k)$ (equation (31)).

Step 5: Compute the *fLPV-ILS* estimate (equation (35)).

Step 6: If $\hat{\theta}^{i+1}_{fLPV-ILS}$ has converged according to a specified convergence criterion

$$\frac{\left\|\hat{\theta}^{i+1}_{fLPV-ILS}-\hat{\theta}^{i}_{fLPV-ILS}\right\|}{\left\|\hat{\theta}^{i}_{fLPV-ILS}\right\|} < \xi \text{ where } \xi < 10^{-5}$$

or a maximum number of iterations is reached, then stop, else go to step 2.

IV. Simulation Results

The following fractional LPV system is considered to analyze the performances of the developed algorithms:

$$(S):\begin{cases} A(\rho_t,p) = 1+a_1(\rho_t)p^{\alpha} \\ B(\rho_t,p) = b_1(\rho_t) \end{cases} \tag{39}$$

where the coefficients, dependent on the scheduling parameter ρ_t, are chosen as :

$$\begin{cases} a_1(\rho_t) = 1-0.5\rho_t \\ b_1(\rho_t) = 2+\rho_t \end{cases} \tag{40}$$

where

$$\rho_t = \rho(t) = \sin(\frac{2\pi}{12}t) \tag{41}$$

The fractional order is *a priori* fixed to $\alpha = 0.5$ and only the linear coefficients are estimated using the proposed estimators. The true parameters matrix is defined by:

$$\theta_0 = [1,-0.5,2,1]^T \tag{42}$$

The input signal is chosen as a uniformly distributed sequence denoted $\mathscr{U}(-1,1)$. The output signal is contaminated with an additive white noise. The number of samples is $N_t = 500$, the sampling period is fixed to $h=0.1$ sec and the cut-off frequency in (16) is set to 100 rad/sec. Fig 1 shows the input, the scheduling variable and the noisy output signals.

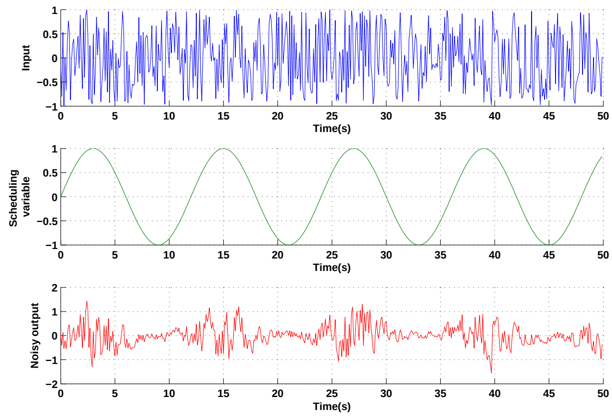


Fig. 1. Input/scheduling variable/free-noise output signals.

The performances of the proposed algorithms are assessed with $N_{mc} = 100$ runs of Monte Carlo simulation with different white noise realizations for $SNR = 30$ dB and $SNR = 15$ dB. For each realization, both the *fLPV-OLS* and the *fLPV-ILS* estimators are applied.

Simulation results are summarized respectively in Table I and II which contain the mean of estimates, the standard deviation of each parameter and the normalized relative quadratic error (NRQE) defined in (43) by:

$$NRQE = \sqrt{\frac{1}{N_{mc}}\sum_{i=1}^{N_{mc}}\frac{\left\|\hat{\theta}_i-\theta_0\right\|^2}{\left\|\theta_0\right\|^2}} \tag{43}$$

The distribution of the *fLPV-OLS* and *fLPV-ILS* estimates for $SNR = 30$ dB and $SNR = 15$ dB are plotted respectively

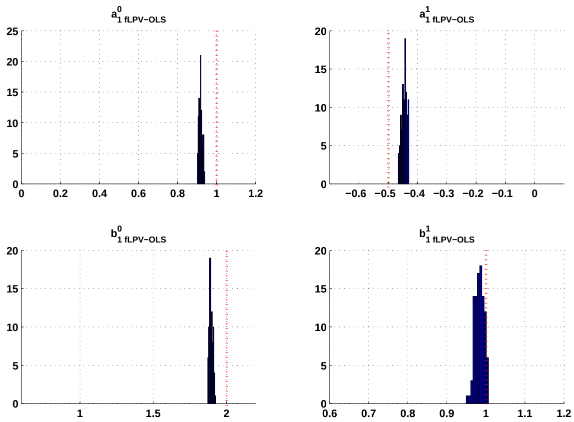
TABLE I
Monte Carlo simulation for $SNR = 30$ dB.

Method		*fLPV-OLS*		*fLPV-ILS*	
Parameter	True value	mean	std	mean	std
a^0_1	1	0,9178	0,0094	0,9965	0,00967
a^1_1	-0.5	-0,4448	0,0092	-0,4932	0,0092
b^0_1	2	1,8946	0,0118	1,9977	0,0112
a^1_1	1	0,9837	0,0113	0,9964	0,0099
NRQE		0,0588		0.0087	

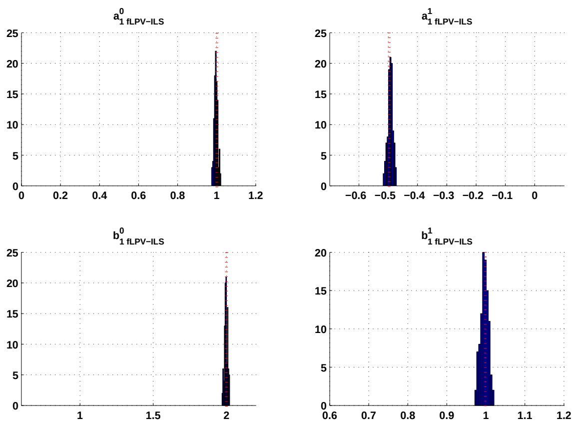
TABLE II
Monte Carlo simulation for $SNR = 15$ dB.

Method		*fLPV-OLS*		*fLPV-ILS*	
Parameter	True value	mean	std	mean	std
a^0_1	1	0,1608	0,0131	1,0037	0,0528
a^1_1	-0.5	-0,0102	0,0142	-0,4971	0,0462
b^0_1	2	0,8524	0,0257	2,0123	0,0638
a^1_1	1	0,6828	0,0260	1,0136	0,0605
NRQE		0.6150		0,0454	

in Fig.2 and Fig.3.



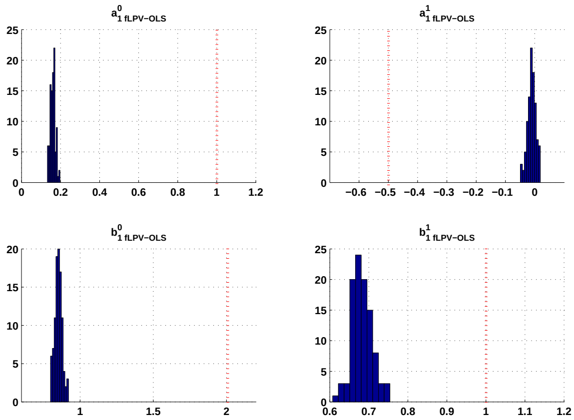
(a)



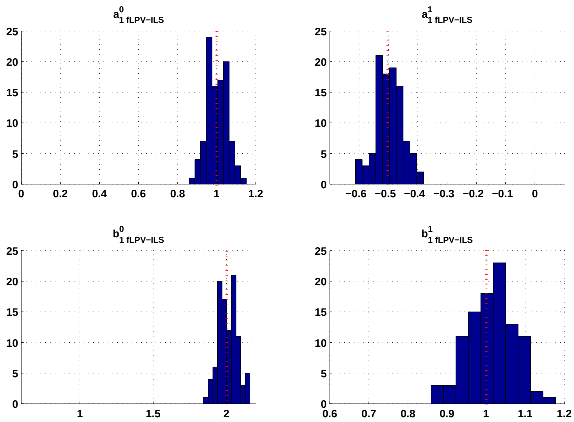
(b)

Fig. 2. Distribution of the estimates for SNR=30 dB ((a) *fLPV-OLS* and (b) *fLPV-ILS*).

The obtained results show that the *fLPV-ILS* estimator give an unbiased estimates, compared to the *fLPV-OLS* estimates, with small standard deviation for $SNR = 30$ dB and $SNR = 15$ dB. This result proves the efficiency and the consistency of the developed iterative algorithm.



(a)



(b)

Fig. 3. Distribution of the estimates for SNR=15 dB ((a) *fLPV-OLS* and (b) *fLPV-ILS*).

V. Conclusion

This paper proposed two new methods for the identification of a continuous-time linear parameter varying systems with fractional models in a noisy output context. The proposed methods are based on a linear regression form. For the the fractional-linear parameter varying-ordinary least squares (fLPV-OLS) estimator, time-domain derivatives are computed using a fractional State Variable Filter. A reformulation of the data generating system is made in order to applied the fractional-linear parameter varying-iterative least squares (fLPV-ILS) estimator. The performances of the proposed estimators have been assessed through a numerical example. Simulation results, show that the *fLPV-ILS* estimator gives good results and provide unbiased estimates compared to *fLPV-OLS* algorithm. Those results have been evaluated with the help of Monte Carlo simulation analysis.

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