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Stabilization and Robust Tracking of a Nonlinear Mechatronic System via Passivity-Based Control

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Abstract— In this paper, we examine the application of interconnection and damping assignment (IDA), through a novel formulation of passivity-based control (PBC) to the stabilization problem of mechatronic System, which necessitates kinetic and potential energy modification. Characterizing a class of systems for which IDA-PBC produces a smooth asymptotically stabilizing controller with a guaranteed domain of attraction is the primary goal of this work. We provide a stabilizing control rule for a class of systems inside the scope of IDA-PBC based on port-Hamiltonian system modeling for networks of mechanical systems. Simulation results demonstrate the effectiveness of the proposed strategy in controlling a permanent magnet DC motor (PMDC).

Keywords— **Passivity-based control (PBC), IDA-PBC, Mechatronic systems, Port-Hamiltonian system.**

I. Introduction

Mechatronic systems, such as precision stages, robotic arms and electromechanical actuators, are difficult to control because of their inherent complexity, which includes elasticity, coupled dynamics, nonlinear friction and persistent model uncertainty [8], [10]. High-performance stabilization and stable trajectory tracking in the presence of these disturbances is a basic goal [3], [4], but it is frequently beyond the reach of traditional linear control techniques, which may not be as robust as they need to be and may necessitate time-consuming, expensive gain scheduling [11]. Nonetheless, Passivity-Based Control (PBC), one of the more recent nonlinear control theories [8], is notable for its physically intuitive basis in energy principles. PBC attempts to alter the system's inherent energy landscape to get the intended stability features, as opposed to only suppressing faults [2], [12]. Nevertheless, for several practical applications, the PBC technique alone may not always be adequate to prescribe a performance in terms of stability [1], [7]. To accomplish a problem from applications involving mechanical systems that demand high precision, for example, it is necessary to guarantee a required performance in terms of other indices (such as oscillations and rate of convergence) [13]. Interconnection and Damping Assignment (IDA-PBC) is a very sophisticated and methodical approach inside the PBC framework [5], [6], [14]. To achieve asymptotic stabilization, the control engineer can independently assign interconnection (energy flow) and damping (energy dissipation) attributes using this technique, which offers an organized design process to enforce a desired port-Hamiltonian structure on the closed-loop system [9], [15]. The resultant controllers frequently have outstanding performance and are naturally robust. In this study, an IDA-PBC technique for robust trajectory tracking and global stabilization of a nonlinear mechatronic system is designed, analyzed, and validated. We begin by obtaining the system's port-Hamiltonian model and clearly describing its nonlinearities. The main contribution is the realization of a nonlinear control rule that assigns a desired closed-loop energy function by solving the partial differential equations (PDEs) resulting from the energy-shaping and damping injection processes using the IDA-PBC methodology. In addition to guarantee the global asymptotic stability of an equilibrium point, we show that

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the suggested controller may be expanded to solve the trajectory tracking problem via a canonical transformation. The IDA-PBC controller's performance is assessed by using high-fidelity numerical simulations. Significantly lower sensitivity of changes in system parameters, outstanding disturbance rejection and enhanced transient performance are to really confirm by the results. The reminder of this paper is organized as follows: Section II provides essential background on the port-Hamiltonian representation and the IDA-PBC methodology. Section III details the IDA-PBC controller design and stability analysis. Section IV presents and discusses the simulation results. Conclusions and future research directions are outlined in Section V.

II. PRELIMINARIES

This section provides the essential background on the port-Hamiltonian representation that underpins the subsequent developments. The port-Hamiltonian framework offers a systematic and physically grounded approach for modeling complex dynamic systems. Consider a general nonlinear dynamical system, whose dynamics can be described by the following state-space representation: [1]:

$$\begin{cases} \dot{x}(t) = f(x(t), u(t)) \\ y(t) = h(x(t), u(t)) \end{cases} \qquad (1)$$

Where $x(t) \in R^n$ is the state vector, $u(t) \in R^m$ is the control input and $y(t) \in \mathrm{R}^m$ is the output.

A. *Definition* [2]:

System (1) is said to be passive if there exists a storage function $H(x) \ge 0$ (interpreted as stored energy) such that:

$$\dot{H}(x(t)) \le u^T(t) y(t) \qquad \forall t \ge 0 \qquad (2)$$

This relation means that the system's internal energy cannot increase faster than the energy supplied by the external input u.

B. *Port-Hamiltonian systems* [2]

A port-Hamiltonian system extends Hamiltonian systems by including energy exchanges with the environment through input and output ports, while preserving the system's energy structure.
An important class of passive systems is given by the port-Hamiltonian (PCH) framework, defined as:

$$\begin{cases} \dot{x} = [J(x) - R_d(x)]\nabla_x H(x) + g(x)u \\ y = g^T(x)\nabla_x H(x) \end{cases} \qquad (3)$$

Where $H(x)$ is the total stored energy the stored energy (Hamiltonian), $J(x) = -J^T(x)$ is the skew-symmetric interconnexion matrix, $R_d(x) = R_d^T(x) \ge 0$ is the dissipation matrix, $g(x)$ is the input distribution matrix and ∇_x Gradient of the Hamiltonian with respect to x.

C. *Dirac structure and general formulation*

The formulation (3) can be derived from the Dirac structure, which describes the power-conserving interconnections of dynamical systems. The general relation is:

$$\begin{bmatrix} \dot{x} \\ y \end{bmatrix} = \begin{bmatrix} J(x) - R_d(x) & g(x) \\ -g^T(x) & 0 \end{bmatrix} \begin{bmatrix} \nabla_x H(x) \\ u \end{bmatrix} \qquad (4)$$

This representation ensures that the energy balance is automatically preserved, with dissipation governed by $R_d(x)$. 8

D. *Port-Hamiltonian formulation of the DC motor*

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To illustrate the theoretical framework, we apply it to a permanent-magnet direct current motor as follows:

$$\begin{cases} \dot{x} = [J(x) - R(x)]\nabla_x + d + gu, \\ y = g^T(x)\nabla_x \end{cases} \quad (5)$$

With: $J(x) = \begin{bmatrix} 0 & -K \\ K & 0 \end{bmatrix} = -J^T$, $R(x) = \begin{bmatrix} R_a & 0 \\ 0 & b \end{bmatrix} = R^T$, $d = \begin{bmatrix} 0 \\ -C_r \end{bmatrix}$, $g = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

Where R_a and b the armature resistance and the friction coefficient represent respectively. The state variables of the system are $x = [x_1 \, x_2]^T$ with:

$$x_1 = p_m = J\omega; x_2 = \phi_m = L_a i \quad (6)$$

u is the input vector and y is the output vector. L_a and J are, respectively, the motor inductance and the inertia referred to the motor shaft. ω is the angular speed of the motor (rad/s), i is the armature current (A), p_m is the mechanical angular momentum (or angular impulse) and ϕ_m is the armature magnetic flux.

The total energy of the system is the sum of the mechanical energy and the electrical energy. It can be represented as a Hamiltonian function: $H(x) = \frac{1}{2} x^T D^{-1} x$ (7)

The development of this function gives: $H(x) = \frac{1}{2}[J\omega \quad L_a i]\begin{bmatrix} \frac{1}{J} & 0 \\ 0 & \frac{1}{L_a} \end{bmatrix}\begin{bmatrix} J\omega \\ L_a i \end{bmatrix}$ (8)

$$H(x) = \frac{1}{2} J\omega^2 + \frac{1}{2} L_a i^2 \text{ ; Where: } \frac{\partial H}{\partial \phi_m} = \frac{\phi_m}{L_a} = i, \qquad \frac{\partial H}{\partial p_m} = \frac{p_m}{J} = \omega \quad (9)$$

III. STABILISATION PROBLEM

The main idea is to design a control law input $u(t)$ in such a manner that the desired equilibrium $x_r = (i_r, \omega_r)$, associated with a reference speed ω_r is asymptotically stable. Moreover, the control law preserves the system's energetic structure (PBC principle) and the stability is robust against load torque disturbances $T_L(t)$ .The IDA-PBC control relies on the passivity framework and has been successfully applied to various fields. The key idea is to solve the energy-balance PDE by selecting appropriate matrices of interconnection $J_d(x)$ damping $R_d(x)$ and $H_d(x)$ the desired energy function.

A. *energy-balance Partial Differential Equation*

A key step in the proposed methodology is obtained by the port-Hamiltonian model:

$$\dot{x}(t) = [J_d(x) - R_d(x)]\nabla_x H_d(x) \quad (10)$$

Where $H_d(x) = \frac{1}{2} L(i - i_r)^2 + \frac{1}{2} J(\omega - \omega_r)^2$ is the desired Hamiltonian, $J_d(x) = -J_d^T(x) = \begin{bmatrix} 0 & -K_e \\ K_t & 0 \end{bmatrix}$

is the desired interconnection matrix. $R_d(x) = R_d^T(x) = \begin{bmatrix} R + k_i & 0 \\ 0 & f + k_\omega \end{bmatrix} \geq 0$ is a desired damping matrix.

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B. Cooperative IDA-PBC

Consider a general nonlinear plant: $\dot{x}(t) = f(x) + g(x)u(x)$ (11)

We assume that there exist matrices $J = J_d^T, R = R_d^T \geq 0$ and the desired Hamiltonian equation H_d.

The closed-loop system must satisfy: $f(x) + g(x)u(x) = \left[J_d(x) - R_d(x)\right]\nabla_x H_d(x)$ (12)

The feedback control law is obtained through equation (12):

$$u(x) = \left(g^T(x)g(x)\right)^{-1} g^T(x)\left(\left[J_d(x) - R_d(x)\right]\nabla_x H_d(x) - f(x)\right) \quad (13)$$

Whose equilibrium x_r is asymptotically stable, provided $k_i, k_\omega > 0$.

The controller is designed through Interconnection and Damping Assignment (IDA-PBC). The desired Hamiltonian function is defined by: $H_d(x) = \frac{1}{2L}(\phi_m - \phi_m^*)^2 + \frac{1}{2j}(p_m - p_m^*)^2$ (14)

$$\frac{\partial H}{\partial \phi_m} = \frac{\phi_m}{L_a} = i, \qquad \frac{\partial H}{\partial p_m} = \frac{p_m}{J} = \omega$$

Where $\phi_m{}^*$ and $p_m{}^*$ are, respectively, the reference flux and the reference generalized momentum. We define the variable $x_d = x - x^*$. To ensure asymptotic stability and the desired control, one must solve the following equation to find the control signal u : $(J_d - R_d)\frac{\partial H_d(x)}{\partial x_d} = (J - R)\frac{\partial H(x)}{\partial x} + d + gu$ (15)

With J_d and R_d being the matrices fixed by the designer to ensure closed-loop control via IDA-PBC. Thus, we have:

$$\begin{bmatrix} -r_{d1} & -J_d \\ J_d & -r_{d2} \end{bmatrix}\begin{bmatrix} i - i^* \\ \omega - \omega^* \end{bmatrix} = \begin{bmatrix} -R_a & -K \\ K & -b \end{bmatrix}\begin{bmatrix} i \\ \omega \end{bmatrix} + d + gu \quad (16)$$

For the steady-state regime, we have: $L\frac{di}{dt} = -R_a i - K\omega + u = 0 \Rightarrow \quad u^* = R_a i^* + K\omega^*$ (17)

$$J\frac{d\omega}{dt} = Ki - b\omega^* - C_r = 0 \quad (18)$$

It follows that: $i^* = \frac{1}{K}(b\omega^* + C_r)$ (19)

Based on Equation (16), one obtains: $J_d(i - i^*) - r_{d2}(\omega - \omega^*) = Ki - b\omega - C_r$ (20)

If we take $J_d = K$ and $r_{d2} = b$, we get: $K(i - i^*) - b(\omega - \omega^*) = Ki - b\omega - C_r$ (21)

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Hence: $$-Ki^* + b\omega^* = -C_r \quad (22)$$

Which gives the value of the reference current i^* by: $i^* = \frac{1}{K}(b\omega^* + C_r)$ (23)

$$-r_{d1}(i-i^*) - K(\omega-\omega^*) = R_a i - K\omega + u \Rightarrow \quad u = -r_{d1}(i-i^*) - K(\omega-\omega^*) + R_a i$$
$$\Rightarrow \quad u = -r_{d1}(i-i^*) - K\omega^* + R_a i \quad (24)$$

By replacing i^* from equation (23) into the above formula (24), we obtain the expression of the control signal:

$$u = (R_a - r_{d1})i + \left(\frac{r_{d1}}{K}b + K\right)\omega^* + \frac{r_{d1}}{K}C_r \quad (25)$$

IV. NUMERICAL SIMULATIONS AND RESULTS

The desired speed is set to then, in the interval $0s < t \le 4s$, it is $120 rad/s$ then, in the interval $4 \le t \le 10$ it is $150 rad/s$. In figure.1 (Fig1) and Figure.2 (Fig2), simulations are presented for three values of $r_{d1} = 0.5, 3, 5$. We can observe that the value $r_{d1} = 3$ provides the best performance (overshoot less than 2% and a response time of $0.01s$). The resisting torque is $C_r = 0.5 N.m$

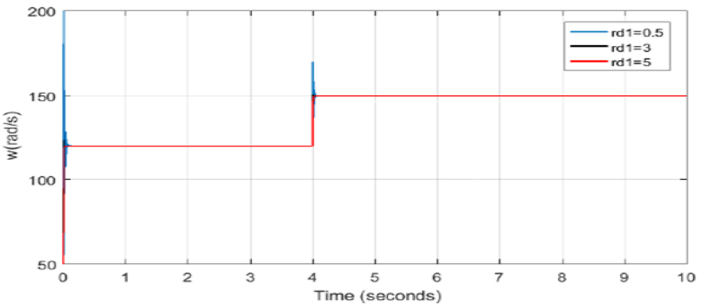

Fig.1: Mechanical speed with the IDA-PBC control

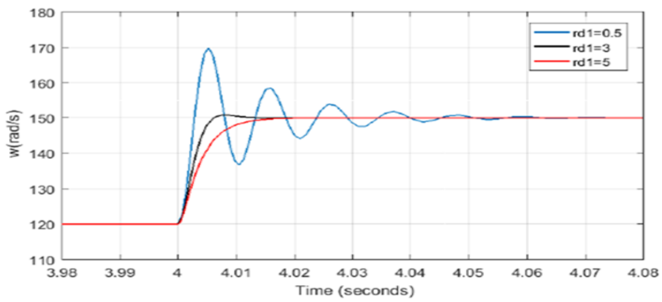

Fig.2: Zoom at mechanical speed

The figure (1) shows the motor speed response for different damping parameters $r_{d1} = 0.5, 3, 5$. The reference speed changes from 120 rad/s to 150 rad/s under a load torque of 0.5 N·m. It illustrates how the parameter r_{d1} affects the response: $r_{d1} = 3$ gives the best compromise — fast rise time, small overshoot, and good tracking.

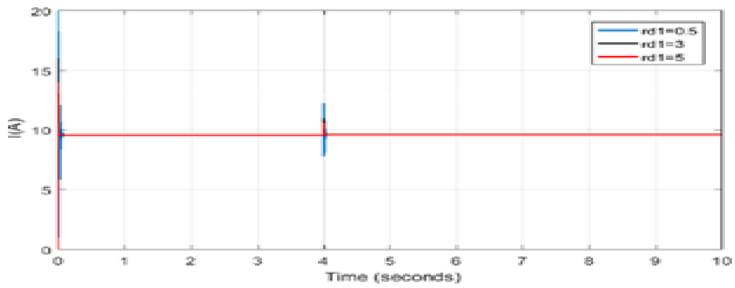

Fig.3: Armature current with IDA-PBC control.

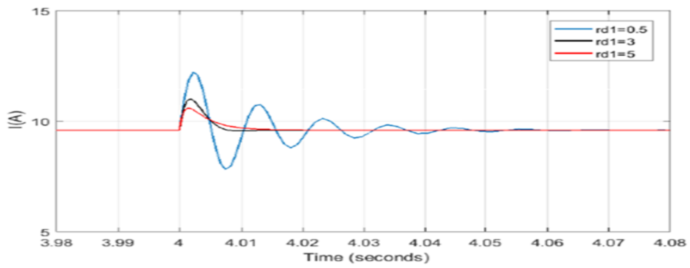

Fig. 4: Zoom on the motor current

A close-up of the transient part of figure (3), showing small oscillations and overshoot. It confirms that $r_{d1} = 3$ ensures smooth and well-damped convergence to the reference speed.

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The figure (3) shows the current $i(t)$ response for the same cases. The current increases during acceleration and decreases once the desired speed is reached. The value of r_{d1} influences the amplitude and duration of current peaks a moderate value limits both overshoot and current stress.

A detailed view of the current transient in figure (4), used to evaluate the smoothness of the control action. It shows that with $r_{d1} = 3$, the current variation remains within acceptable limits, ensuring safe operation of the motor and power converter. The simulations demonstrate that the **IDA-PBC controller** provides excellent tracking performance. By properly tuning the damping parameter r_{d1}, the system achieves **fast, stable, and energy-efficient behavior**, with reduced overshoot and moderate current transients.

V. CONCLUSIONS

In this paper, passivity-based control strategies for a Permanent Magnet DC Motor (PMDCM) were presented and implemented. The study demonstrated the effectiveness of the Interconnection and Damping Assignment Passivity- Based Control (IDA-PBC) approach for stabilizing and controlling nonlinear mechatronic systems. Based on the port- Hamiltonian framework, the proposed method guarantees asymptotic stability and robustness while preserving the system's intrinsic energy structure. Simulation results for the PMDC motor confirm a fast dynamic response, minimal overshoot, and strong disturbance rejection capability.

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