

Robust tracking control of wheeled mobile robot subject to uncertainties

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Abstract— This article introduces a robust tracking control method specifically designed for wheeled mobile robot (WMR), which tackles various sources of uncertainty such as wind disturbances and slipping. By applying the principles of the differential flatness methodology, the inherently under-actuated dynamics of WMR are transformed into a more manageable linear canonical form, facilitating the development of a stabilizing feedback controller. To effectively handle uncertainties arising from wheel slip and wind disturbances, the proposed feedback controller incorporates sliding mode control (SMC). However, the escalation of uncertainties may intensify chattering phenomena within the SMC framework, attributable to increased control inputs. To address this issue, a boundary layer surrounding the switching surface is introduced, implementing a continuous control law aimed at mitigating chattering effects. The stability properties of the closed-loop system are established using Lyapunov theory. Comprehensive numerical simulations are performed on a WMR system to evaluate the effectiveness and performance of the proposed control strategy.

Index Terms—Differential flatness, Sliding mode control, Wheeled mobile robot.

I. INTRODUCTION

Robotic systems are complex machines designed to interact with their environment autonomously or under human control. They consist of mechanical components, sensors, actuators, and computational algorithms. These systems use artificial intelligence and advanced algorithms to perceive their surroundings, make decisions, and execute tasks efficiently.

Recently, wheeled mobile robots have significantly expanded their applications, streamlining tasks and enhancing efficiency in industrial logistics [1], healthcare [2], agriculture [3], and surveillance [4]. Their adaptability and versatility make them vital across various domains, fueling ongoing advancements in automation and productivity. However, due to the diverse applications of wheeled mobile robots, precise control algorithm design is crucial, considering the non-linearity of their kinematic models. This complex challenge piques the interest of numerous researchers, underscoring the importance and relevance of this field.

In the past few decades, significant progress has been achieved in the field of tracking control for WMR through the application of nonlinear control theory. Among these methodologies, linearization controllers, such as the flatness controller [5], have emerged as a popular approach, greatly simplifying the controller design process. The flatness property is a technique used to characterize the dynamic behavior of nonlinear underactuated models by identifying a set of fundamental system variables known as flat outputs. Achieving flatness enables the expression of both the system's state and input variables as functions of the flat outputs and their derivatives, eliminating the necessity for an integration process. Moreover, flatness control converts the nonlinear model into the canonical Brunovsky form [6], simplifying the concept of a feedback controller capable of achieving precise trajectory tracking. The ease of controlling a linear system in contrast to an underactuated nonlinear system has motivated researchers to employ the principles of flatness in the planning and tracking of trajectories for robotic systems. In [7], a technique for generating optimal trajectories is presented, employing the transcription method, flatness, and b-spline curves. As a result, the

utilization of flatness enables a decrease in the number of variables in trajectory optimization, leading to improved computational efficiency. Moreover, in [8], Helling combined flatness with a predictive control strategy to achieve real-time trajectory tracking for underactuated marine surface vehicles. Additionally, Salah [9] developed a method for generating upper coverage trajectories for mobile robots, taking advantage of the benefits offered by flatness.

There is always a difference between the mathematical model describing the movement of WMR and reality. This difference is due to environmental phenomena neglected during modeling, such as wind and slipping. The question that arises is how flatness control applied to WMR can ensure the accurate tracking of a desired trajectory despite the presence of uncertainties.

To resolve this problem, a robust feedback controller must be combined with flatness, taking into account the impact of uncertainties on the model. Up to the present, there have been limited methods in the literature concerning the robustness issues of flatness systems. Among these approaches, sliding mode control (SMC) has been successfully utilized in a variety of systems [10], [11].

SMC is a robust control technique employed to handle dynamic systems amidst uncertainties and disturbances. Fundamentally, SMC aims to steer the system state onto a predetermined sliding surface within the state space. Once the system resides on this surface, its behavior becomes constrained, facilitating effective regulation. SMC achieves this by employing discontinuous control actions, referred to as switching control, which dynamically alternate between different control laws. This switching mechanism ensures that the system remains on the sliding surface, enhancing robustness against external influences. However, despite its effectiveness, SMC is associated with a phenomenon called chattering, characterized by rapid switching between control actions near the sliding surface. While chattering can theoretically improve tracking accuracy, it can lead to practical issues such as mechanical wear and high-frequency oscillations. To address this problem, numerous approaches have been suggested in the existing literature, such as high-order SMC [12], boundary layer techniques [13], and the active adaptive continuous nonsingular terminal sliding mode algorithm [14]. A frequently utilized approach for mitigating the chattering phenomenon involves incorporating the boundary layer technique within SMC. The primary contribution of this article lies in leveraging the simplicity provided by the flatness control concept and the robustness achieved through the integration of boundary layer SMC to establish a high-performance and resilient tracking controller for WMR. This design aims to address the challenges posed by wheel slip and wind disturbances.

Initially, the paper emphasizes the role of flatness control in converting a WMR into a linear canonical and decoupled form, thus laying the groundwork for the development of a stabilizing feedback controller. Subsequently, it introduces a robust feedback control approach based on boundary layer SMC to ensure precise trajectory tracking despite the presence of uncertainties.

The rest of this article follows this structure: Section 2 offers a thorough examination of the flatness control technique for WMR. The detailed description of the proposed robust tracking controller can be found in Section 3. Section 4 introduces and analyzes the simulation results. Lastly, Section 5 concludes the paper by summarizing the main findings and suggesting potential future avenues of research.

II. FLATNESS-BASED TRACKING CONTROL

In our study, we analyzed a differential two-wheeled mobile robot (Fig. 1) that consists of two independent active wheels and a third passive wheel. This robotic system is widely regarded as an effective trade-off between control ease and the degrees of freedom that enable the robot to meet mobility requirements.

The configuration of the mobile robot with wheels can be described by the vector $g_r = [x, y, \theta]$. In this notation, x and y represent the coordinates of the robot's center position in the stationary frame (O, X, Y), while θ represents the orientation angle of the robot. The state equation of the WMR kinematic model, neglecting uncertainties, is represented as follows:

$$\begin{aligned}\dot{x} &= \cos(\theta)v \\ \dot{y} &= \sin(\theta)v \\ \dot{\theta} &= \omega\end{aligned}\tag{1}$$

The robot's translational and rotational velocities are denoted by v and ω respectively. The angular velocities of the right and left wheels (ω_r and ω_l) can be defined as functions of the robot's translational and rotational velocities as follows:

$$v = \left(\frac{\omega_r + \omega_l}{2} \right) r \quad (2)$$

$$\omega = \left(\frac{\omega_r - \omega_l}{2b} \right) r \quad (3)$$

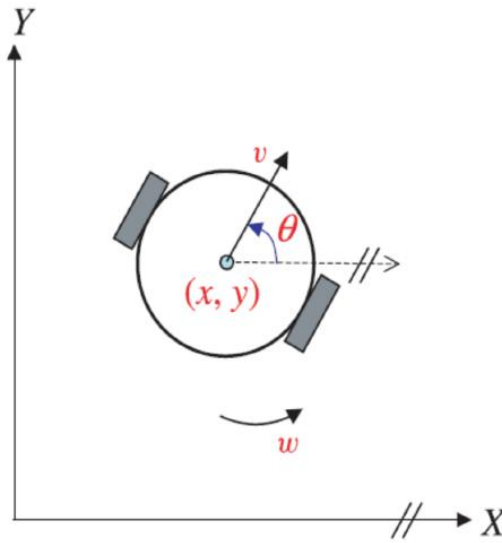


Fig 1. Two-wheeled mobile robot

The variables r and $2b$ represent the radius and distance between the wheels, respectively. The non-holonomic limitation is defined as follows, based on the non-slip requirement:

$$\dot{x} \sin \theta - \dot{y} \cos \theta = 0 \quad (4)$$

The accuracy of the tracking will be guaranteed through the flatness property, which involves describing all system states and inputs, as well as their finite time derivatives, within the framework of a flat output. Considering the following nonlinear system:

$$\dot{x} = f(x, u) \quad (5)$$

where $x \in \mathbb{R}^n$ and $u \in \mathbb{R}^m$ represent the state and the input vector.

The nonlinear system (5) is differentially flat if there exists an output λ in the following form:

$$\lambda = \xi(x, u, \dot{u}, \dots, u^{(c)}) \in \mathbb{R}^m \quad (6)$$

such that the state and the input can be expressed as follows:

$$x = \kappa_1(\lambda, \dot{\lambda}, \ddot{\lambda}, \dots, \lambda^{(a)}) \quad (7)$$

$$u = \kappa_2(\lambda, \dot{\lambda}, \ddot{\lambda}, \dots, \lambda^{(a+1)}) \quad (8)$$

where a and c are finite multi-indices, and ξ , κ_1 and κ_2 are smooth vector functions of the output vector λ and its derivatives. The flatness property permits us to calculate diffeomorphism and feedback linearization, which transforms the nonlinear system into a controllable linear system where the flat outputs depict the state vector.

Several studies in the literature, including ryu [15], have shown that the WMR kinematic modeling can be defined as a differentially flat model, where the positional coordinates denoted as $\lambda = [\lambda_{11}, \lambda_{21}]^T = [x, y]^T$ serve as the flat outputs. Therefore, the entire set of state and control components pertaining to the WMR system are expressed using the flat variable λ and its derivatives, as demonstrated below:

$$\theta = \arctan \frac{\dot{\lambda}_{21}}{\dot{\lambda}_{11}} \quad (9)$$

$$v = \sqrt{\dot{\lambda}_{11}^2 + \dot{\lambda}_{21}^2} \quad (10)$$

$$\omega = \frac{\dot{\lambda}_{11} \ddot{\lambda}_{21} - \ddot{\lambda}_{11} \dot{\lambda}_{21}}{\dot{\lambda}_{11}^2 + \dot{\lambda}_{21}^2} \quad (11)$$

The differentially flat nature of the WMR's kinematic model has been demonstrated in the literature by various researchers [15]. This implies that all the states and controls of the kinematic WMR model can be expressed as functions of λ and its derivatives. However, the non-invertible relationship between the control input vectors w and v and the highest derivatives of the flat output limits the development of static feedback linearization for the nonlinear WMR. To address this constraint, we incorporate the control input v into the kinematic model Equation (1) by treating it as an additional state. As a result, we obtain a revised system that can be defined as follows:

$$\begin{aligned} \dot{x} &= \cos(\theta) v \\ \dot{y} &= \sin(\theta) v \\ \dot{v} &= u_{r1} \\ \dot{\theta} &= u_{r2} \end{aligned} \quad (12)$$

The state and control inputs of the modified system defined by Equation (12) are represented by $X_r = [x, y, v, \theta]^T$ and $u_{r1} = \dot{v}$ and $u_{r2} = \dot{\omega}$. In order to establish a bijective relationship between the inputs u_{r1} , u_{r2} , and higher-order derivatives of $\alpha_{11} = x$, $\alpha_{11} = y$, we apply successive differentiations to the flat outputs until at least one of the input variables appears in the resulting expressions, as illustrated below:

$$\begin{bmatrix} \ddot{\lambda}_{11} \\ \ddot{\lambda}_{21} \end{bmatrix} = B_{rob} \begin{bmatrix} u_{r1} \\ u_{r2} \end{bmatrix} \quad (13)$$

Where B_{rob} is described as follows:

$$B_{rob} = \begin{bmatrix} \cos(\theta) & -v \sin(\theta) \\ \sin(\theta) & v \cos(\theta) \end{bmatrix} \quad (14)$$

The matrix B_{rob} is not singular if $v \neq 0$. In this case, we can define the control as follows:

$$\begin{bmatrix} u_{r1} \\ u_{r2} \end{bmatrix} = B_{rob}^{-1} \begin{bmatrix} \ddot{\lambda}_{11} \\ \ddot{\lambda}_{21} \end{bmatrix} \quad (15)$$

To arrive at the linearized system, referred to as the Burnovsky Form (BF), we can substitute the control input (15) into equation (13). This substitution yields the following modified expression:

$$(BF_1) \begin{cases} \dot{\lambda}_{11} = \lambda_{12} \\ \dot{\lambda}_{12} = v_1 \\ Y_1 = \lambda_{11} = x \end{cases} \quad (BF_2) \begin{cases} \dot{\lambda}_{21} = \lambda_{22} \\ \dot{\lambda}_{22} = v_2 \\ Y_2 = \lambda_{21} = y \end{cases} \quad (16)$$

Where v_1, v_2 represent a suitable feedback controller defined as follows:

$$v_1 = \ddot{\lambda}_{xd} - \sigma_{x2}(\lambda_{12} - \dot{\lambda}_{xd}) - \sigma_{x1}(\lambda_{11} - \lambda_{xd}) \quad (17)$$

$$v_2 = \ddot{\lambda}_{yd} - \sigma_{y2}(\lambda_{22} - \dot{\lambda}_{yd}) - \sigma_{y1}(\lambda_{21} - \lambda_{yd}) \quad (18)$$

Where λ_{xd} and λ_{yd} denote the desired trajectories for the flat output λ_{11} and λ_{21} , respectively. Meanwhile, the controller gains are represented by σ_{x1} , σ_{x2} , σ_{y1} , and σ_{y2} . The polynomial of the Burnovsky system (16) can be defined as follows:

$$s^2 + \sigma_{x2}s + \sigma_{x1} = s^2 + 2m_x\epsilon_{xc} + \epsilon_{xc}^2 \quad (19)$$

$$s^2 + \sigma_{y2}s + \sigma_{y1} = s^2 + 2m_y\epsilon_{yc} + \epsilon_{yc}^2 \quad (20)$$

Where the parameters m_x and m_y are the damping coefficients, and ϵ_{xc} and ϵ_{yc} are the frequencies in equations (19) and (20). We can calculate the controller gain as follows:

$$\sigma_{x1} = \epsilon_{xc}^2, \sigma_{x2} = 2m_x\epsilon_{xc}, \sigma_{y1} = \epsilon_{yc}^2, \sigma_{y2} = 2m_y\epsilon_{yc} \quad (21)$$

By integrating the feedback law, as described in equations (17- 18), into the system (15), we can express the Flatness-Based Tracking Control (FBTC) utilized for the mobile robot in the following manner:

$$\begin{bmatrix} u_{FBTCx} \\ u_{FBTCy} \end{bmatrix} = B_{rob}^{-1} \begin{bmatrix} \ddot{\lambda}_{xd} - \sigma_{x2}\dot{e}_1 - \sigma_{x1}e_1 \\ \ddot{\lambda}_{yd} - \sigma_{y2}\dot{e}_2 - \sigma_{y1}e_2 \end{bmatrix} \quad (22)$$

where $e_1 = \lambda_{11} - \lambda_{xd}$ and $e_2 = \lambda_{21} - \lambda_{yd}$.

In ideal conditions where uncertainties such as wind and wheel slip are negligible in the kinematic model of the WMR, the control input defined by equation (22) can achieve satisfactory tracking performance for the desired trajectory.

However, it is practically impossible to have a model that accurately represents the real-world movement of the robot in all environmental conditions. As a result, the following section will focus on developing a robust tracking control for a WMR kinematic model that is subject to uncertainties.

III. FLATNESS BASED SLIDING TRACKING CONTROL

In order to account for real-world conditions, we consider uncertainties such as slippage and external environmental disturbances when describing the kinematic model of WMR. As a result, the model is defined differently, as shown below:

$$\begin{cases} \dot{x} = \cos(\theta)v + v_t \cos(\theta) + v_s \sin(\theta) + p_x \\ \dot{y} = \sin(\theta)v + v_t \sin(\theta) - v_s \cos(\theta) + p_y \\ \dot{\theta} = \omega + \omega_s \end{cases} \quad (23)$$

The variables p_x and p_y represent the external environmental disturbances, indicating the potential influences from the surrounding conditions. On the other hand, v_t and v_s represent the slip velocities, where v_t denotes the slip velocity along the forward direction, and v_t represents the slip velocity normal to it. Additionally, ω_s denotes the angular slip velocity.

According to [15], it is assumed that the slippage phenomenon can be defined and bounded as follows:

$$v_t(t) = v_s(t) = \omega_s(t) = \kappa_1 v(t) \quad (24)$$

$$\|v_t\| \leq \varepsilon_1 \|v\|, \|v_s\| \leq \varepsilon_2 \|v\|, \omega_s \leq \varepsilon_3 \quad (25)$$

where κ_1 , ε_1 , ε_2 and ε_3 are a positive constants.

Assuming that λ_{xd} and λ_{yd} are the reference trajectories for λ_{11} and λ_{21} , respectively, we can define the error dynamics as $e_i = \lambda_{i1} - \lambda_{id}$ for $i = 1, 2$. To achieve convergence of the tracking error e_i to zero in the presence of uncertainties, we employ a sliding mode control approach that relies on the principles of the flatness law. By incorporating this control strategy, we aim to ensure robust and accurate tracking performance even in the face of system uncertainties. The design of the SMC involves two essential stages: the choice of the sliding surface and the development of the control law.

These steps play a crucial role in establishing an effective and stable sliding mode control strategy. The selection of the sliding surface determines the desired system behavior and convergence properties, while the design of the control law focuses on generating control signals that guide the system towards the desired sliding surface and ensure its maintenance on that surface. In the context of the tracking example for the Wheeled Mobile Robot (WMR), we make use of the sliding variable $\sigma_r = [s_x, s_y]^T$ to represent the tracking error. To define the sliding surface, we consider the desired tracking behavior and express it as follows, taking into account the specific requirements of the system:

$$s_x = \dot{e}_1 + \beta_1 e_1 \quad (26)$$

$$s_y = \dot{e}_2 + \beta_2 e_2 \quad (27)$$

Where the gains β_1 and β_2 can be selected using poleplacement techniques to ensure the asymptotic convergence of the tracking errors $e_1 = \lambda_{11} - \lambda_{xd}$ and $e_2 = \lambda_{21} - \lambda_{yd}$ to zero. In this tracking example, the sliding variable $\sigma_r = [s_x, s_y]^T$ is chosen as the tracking error. Therefore, the sliding surface for the WMR can be defined as follows:

$$\dot{e}_1 + \beta_1 e_1 = 0 \quad (28)$$

$$\dot{e}_2 + \beta_2 e_2 = 0 \quad (29)$$

As suggested by mauledux [16], to guarantee that the sliding surface $\sigma_r = 0$ is attractive, we can enforce the dynamics of σ_r as follows:

$$\dot{\sigma}_r = \kappa_i \operatorname{sgn}(\sigma_r) \quad (30)$$

Where the standard signum function is denoted by sgn , and κ_i ($i = 1, 2$) is a constant. One approach to proving the error dynamics stability is to analyze the following Lyapunov function:

$$V_s = \frac{1}{2} \dot{\sigma}_r \sigma_r \quad (31)$$

The derivative of V_s is defined as follows:

$$V_s = \sigma_r \dot{\sigma}_r \quad (32)$$

We can conclude that V_s is a positive function and its derivative \dot{V}_s is negative or zero. Hence, the system exhibits asymptotic Lyapunov stability. Using equations (26), (27) and (30) we obtain:

$$-\kappa_1 \operatorname{sgn}(s_x) = \ddot{e}_1 + \beta_1 \dot{e}_1 \quad (33)$$

$$-\kappa_2 \operatorname{sgn}(s_y) = \ddot{e}_2 + \beta_2 \dot{e}_2 \quad (34)$$

As a result, by using equations (33) and (34), we can obtain:

$$\ddot{\lambda}_{11} = \ddot{\lambda}_{xd} - \beta_1 \dot{e}_1 - \kappa_1 \operatorname{sgn}(s_x) \quad (35)$$

$$\ddot{\lambda}_{21} = \ddot{\lambda}_{yd} - \beta_2 \dot{e}_2 - \kappa_2 \operatorname{sgn}(s_y) \quad (36)$$

Substituting ($\ddot{\lambda}_{11}$) and ($\ddot{\lambda}_{21}$) with their new expressions defined by equations (33) and (34) in the control defined by 15, the flatness sliding tracking controller (FSTC) applied to WMR is defined as follows:

$$\begin{bmatrix} u_{FSMCx} \\ u_{FSMCy} \end{bmatrix} = B_{rob}^{-1} \begin{bmatrix} \ddot{\lambda}_{xd} - \beta_1 \dot{e}_1 - \kappa_1 \operatorname{sgn}(s_x) \\ \ddot{\lambda}_{yd} - \beta_2 \dot{e}_2 - \kappa_2 \operatorname{sgn}(s_y) \end{bmatrix} \quad (37)$$

The FSTC defined by equations (37) contains a discontinuous control term due to the function $\operatorname{sgn}(\sigma)$. Although selecting sufficiently large values for κ_1 and κ_2 can achieve convergence to sliding variable in limited time and provide robustness against perturbations, it also causes the phenomenon of chattering. Thus, to avoid this problem, the function $\operatorname{sgn}(\sigma_r)$ can be replaced by the function Sat defined as follows:

$$Sat(\sigma_r) \begin{cases} \frac{\sigma_r}{a_s} & \text{if } |\sigma_r| \leq a_s \\ \operatorname{sgn}(\sigma_r) & \text{if } |\sigma_r| > a_s \end{cases} \quad (38)$$

With a_s being the width of the threshold of the saturation function.

IV. SIMULATION RESULTS

This section conducts simulation tests to verify the effectiveness and superiority of the proposed controller, Flatness Sliding Mode Control (FSMC), denoted by Equation (37-38), over Flatness-Based Tracking Control (FBTC) outlined in Equation (22). The parameters of the WMR are $r = 0.1\text{m}$, $b = 0.15\text{m}$. The controller design parameters of FBTC and FSMC are chosen as $m_x = m_y = 1$, $\epsilon_{xc} = \epsilon_{yc} = 2$, $\beta_1 = \beta_2 = 5$ and $\kappa_1 = \kappa_2 = 10$, $a_s = 0.2$. In this

simulation, we consider that slip velocities v_t and v_s can be up to 30% of the forward speed. So $\kappa_1 = 0.3$. Moreover, the WMR is subjected to sinusoidal wind perturbation, defined as follows:

$$p_x = p_y = 3 \cos(2t) \text{ m/sec}, \omega_s = 2 \text{ rad/sec} \quad (39)$$

The reference trajectory considered is a circle, which is defined by the following equation:

$$x_r = \cos(t), y_r = \sin(t) \quad (40)$$

Fig 2 illustrates the comparative performance of uncertain WMR systems employing different control strategies, namely FBTC and FSMC. Upon analyzing the simulation data presented in this figure, it is evident that when affected by slippage and external disturbances, the WMR system deviates significantly from its intended trajectory, making FBTC ineffective as a controller. In contrast, the incorporation of the FSMC controller, utilizing the discontinuous term of the sliding mode, effectively mitigates the effects of uncertainty, ensuring stability in the closed-loop control system. As depicted in Fig 3, the occurrence of chattering in the FSMC control signals notably diminishes, suggesting that the proposed control method achieves superior trajectory tracking while avoiding chattering. This enhancement in tracking performance is especially remarkable when confronted with aggressive disturbances.

For a quantitative assessment of the tracking capabilities of the Wheeled Mobile Robot (WMR), we have utilized two metrics: the Integral Absolute Error (IAE) and a control effort performance index, as suggested in existing literature, to facilitate comparison. Additionally, we introduce another nonlinear control method, namely Backstepping Sliding Mode Control (BSMC), alongside FBTC, to provide a comprehensive comparison with the proposed control strategy. The Integral Absolute Error (IAE) is computed for each control strategy according to the following procedure:

$$IAE_i = \int_0^{t_f} |e_i(t)| dt. \quad e_i(t) = \lambda_i(t) - \lambda_{id}(t) \quad (42)$$

where t_f is the total simulation duration and $i = 1, 2$ represents the position in the x and y direction, respectively. The control effort is given as follows:

$$P_{avg} = \frac{1}{N} \sum_{k=1}^N u^2(k) \quad (42)$$

where N indicates the total count of samples. The associated key performance indicators IAE and P_{avg} for both strategies are provided in Table I. After examining the data depicted in the table, it becomes evident that the FSMC controller exhibits superior tracking performance when contrasted with the FBTC and BSMC approaches. Although its tracking capability is nearly equivalent to that of the BSMC, the FSMC demands minimal effort to attain its objectives in comparison. The enhanced efficiency of the FSMC over the BSMC can be attributed to the advantages of flatness control, which streamline controller design by transforming the nonlinear system into a linear one.

TABLE I: IAE and P_{avg} performance indexes

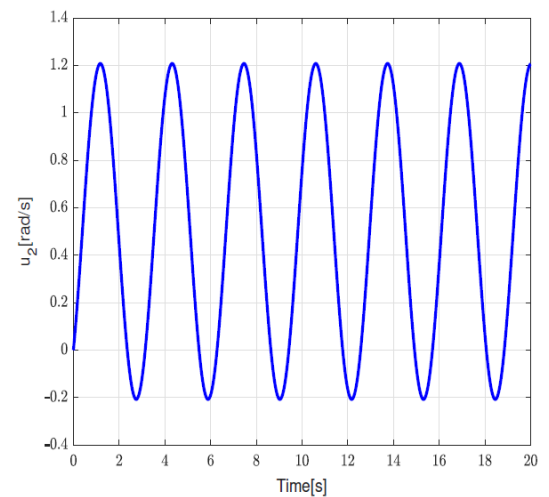
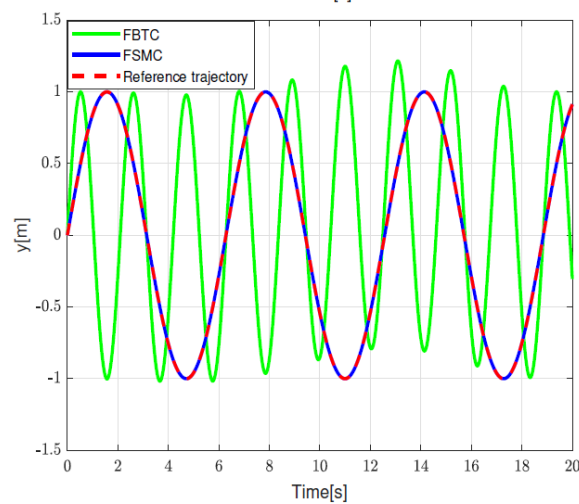
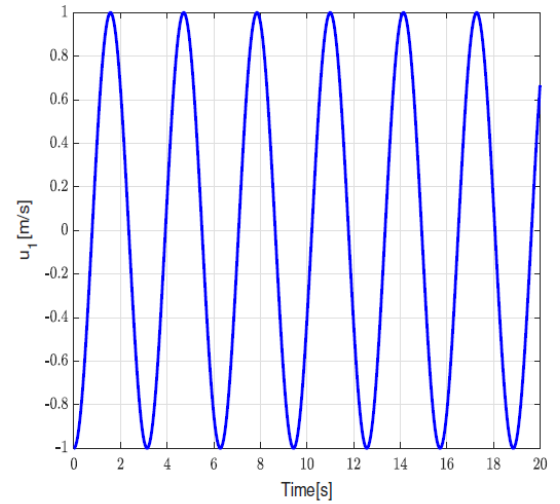
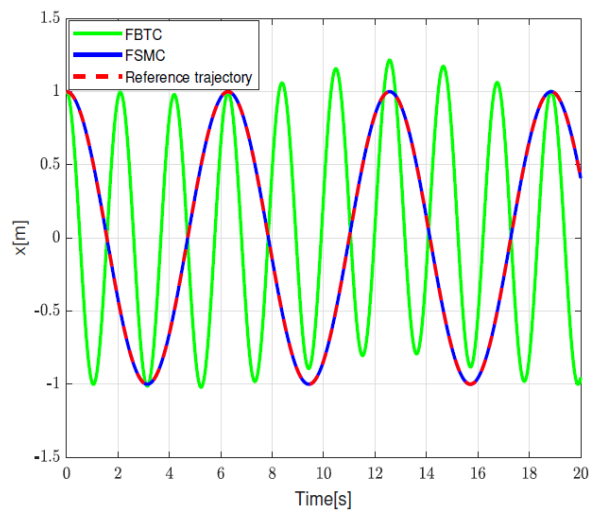
Index	FBTC	BSMC	FSMC
IAE	3.2127	0.08	0.0427
P_{avg}	0.75	1.1266	0.035

V. CONCLUSION

This paper tackles the challenge of robust trajectory tracking for the Wheeled Mobile Robot (WMR). Utilizing the flatness-based control method, the nonlinear kinematic model of the WMR is transformed into a canonical form, simplifying the implementation of a feedback controller. This controller leverages sliding mode techniques to enhance system performance and robustness. Simulation results demonstrate the efficacy of the FSMC control approach in improving trajectory tracking performance, even in scenarios with variations in wheel slip and external wind disturbances. Future research will explore disturbances with unknown bounds impacting the WMR.

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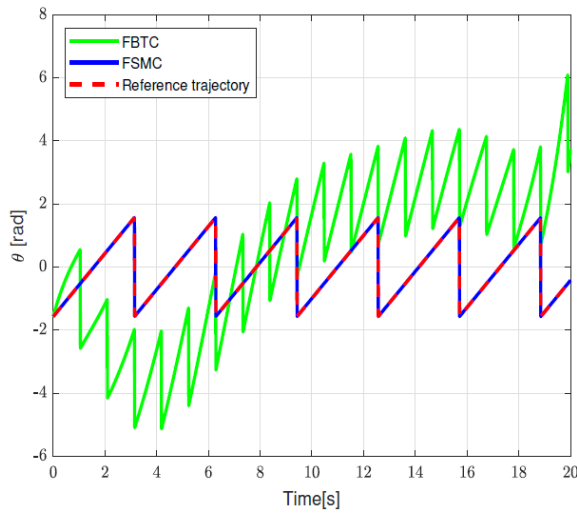


Fig 2. Results of tracking simulation for wheeled mobile robot

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