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Modeling Candidate Selection with Multi-Quota Constraints (HMR-MMKP)

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Abstract— **Optimal selection under multiple constraints is a central problem in many applications of management science, such as planning, resource allocation, and system design. The classic variants of the Multiple-Choice Knapsack Problem (MCKP) and the Multiple-Choice Multidimensional Knapsack Problem (MM-MCKP) effectively address global and multiple capacities, while the Balanced Assignment Problem (BAP) ensures balanced assignments. However, these approaches do not handle nested hierarchical constraints or adaptive local quotas. We introduce the Hierarchical Multi-Quota and Adaptive Multiple-Choice Multidimensional Knapsack Problem (HMR-MMKP), a general model that simultaneously integrates hierarchical quotas, local lower bounds, and adaptive mechanisms. We prove its NP-completeness, analyze its bounds, and demonstrate its reduction to bounded flow problems, allowing the use of polynomial solvers to handle critical substructures. Theoretical comparisons with the state-of-the-art show that HMR-MMKP bridges the gap between classical multi-choice models, balanced assignments, and flow techniques, while remaining compatible with MILP formulations and guided heuristics. This contribution paves the way for hybrid solvers combining algorithmic robustness and structural flexibility.**

***Keywords*— HMR-MMKP, Multiple-choice Knapsack Problem, Multi-Quota Constraints, Candidate Selection, Expert System**

I. INTRODUCTION

Selection systems play a key role in the fair and optimal allocation of resources, while adhering to strict constraints and goals of diversity and transparency. Simultaneously considering multiple quotas, such as those based on categorization, grouping, or demographic distribution criteria, significantly increases the complexity of their modeling and algorithmic resolution. In this context, our study focuses on a case where resources must be allocated across multiple distinct groups, each of which is subdivided into subcategories. A key objective is to ensure a target distribution of resources, such as a specific proportion between different categories, while respecting distribution constraints. This scenario leads to a hierarchy of nested quotas, where the constraints related to the distribution between groups, subgroups, and subcategories interact in a complex way. Classical models, such as the *Multiple-Choice Multi-Dimensional Knapsack Problem (MM-MCKP),* well studied in the literature [6][8] are not capable of effectively handling: the nested hierarchy of quotas ($group \rightarrow subgroup \rightarrow subcategory$); and adaptive quotas, which allow for adjusting distribution values when the exact distribution becomes unfeasible. Indeed, although exact solutions, such as the algorithm proposed by Hifi et al. [7] efficiently address multiple capacity constraints, they do not account for the fine hierarchical interactions between quotas, nor adaptive mechanisms. Similarly, the balanced optimization approaches from the *Balanced Assignment Problem (BAP)* [4] focus on global balance but do not model nested hierarchical quotas with local bounds. These limitations raise a fundamental question: how can a combinatorial problem be formulated and solved, integrating both nested hierarchical constraints and adaptive constraints simultaneously? To address this issue, we introduce the *Hierarchical Multi-Ratio Multiple-Choice Multi-Dimensional Knapsack Problem (HMR-MMKP)*, a structured extension of the MM-MCKP that explicitly integrates hierarchical quotas and local adaptive mechanisms. We provide a complete formalization of the model and a theoretical analysis of its fundamental properties, including the proof of its NP-completeness. Beyond the introduction, the organization of this paper is as follows. Section 1 recalls the key definitions related to the *Knapsack Problem*. Section 2 presents the state of the art and identifies the limitations of existing approaches in dealing with nested and adaptive quotas. Section 3 introduces the formal modeling of the HMR-MMKP, detailing its hierarchical constraints and comparing it to existing models. Section 4 presents the theoretical properties, including

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the proof of NP-completeness, the analysis of bounds, and the combinatorial structure of the problem. Section 5 concludes and opens up research perspectives.

II. KEY DEFINITIONS

A. KP

The classic **Knapsack Problem (KP)** consists of selecting items with values and weights in order to maximize the total value under a single capacity constraint. Let n be the number of available items, with each item i having a value v_i and a weight w_i. The total capacity of the knapsack is W. The goal is to maximize the total value of the selected items while respecting the capacity constraint of the knapsack.
The mathematical formulation is as follows: $\max \sum_{i=1}^{n} v_i x_i$
subject to the constraint: $\sum_{i=1}^{n} w_i x_i \leq W$
with $x_i \in \{0,1\}$ representing the decision variable, where $x_i = 1$ if item i is selected, and $x_i = 0$ otherwise.

B. MCKP

In the **Multiple-Choice Knapsack Problem (MCKP),** items are grouped into classes, and exactly one item must be chosen from each class, subject to a capacity constraint. Let k be the number of classes, n_j the number of items in class j, and v_{ij} and w_{ij} respectively the value and weight of item i in class j. The total capacity of the knapsack is W. The goal is to maximize the total value of the selected items while respecting the capacity constraint.
The mathematical formulation is as follows: $\max \sum_{j=1}^{k} \sum_{i=1}^{n_j} v_{ij} x_{ij}$
subject to the constraint: $\sum_{j=1}^{k} \sum_{i=1}^{n_j} w_{ij} x_{ij} \leq W$
with the additional constraint: $\sum_{i=1}^{n_j} x_{ij} = 1 \quad \forall j \in \{1, 2, \dots, k\}$
where $x_{ij} \in \{0,1\}$ representing the selection of item i from class j (selecting exactly one item per class).

C. MM-MCKP

The **Multiple-Choice Multidimensional Knapsack Problem (MM-MCKP)** is a generalization of the MCKP, where multiple capacity constraints exist (rather than just one). Let m be the number of capacity constraints. Each item i in class j has a weight w_{ij} for each dimension (constraint) p, and a value v_{ij}. The goal is still to maximize the total value of the selected items while respecting multiple capacity constraints.
The mathematical formulation is as follows: $\max \sum_{j=1}^{k} \sum_{i=1}^{n_j} v_{ij} x_{ij}$
subject to the constraint: $\sum_{j=1}^{k} \sum_{i=1}^{n_j} w_{ij,p} x_{ij} \leq W_p \quad \forall p \in \{1, 2, \dots, m\}$
with the additional constraint: $\sum_{i=1}^{n_j} x_{ij} = 1 \quad \forall j \in \{1, 2, \dots, k\}$,
where $x_{ij} \in \{0,1\}$.
The decision variables x_{ij} select the items, while simultaneously respecting multiple capacity constraints across different dimensions (for example, multiple resources such as weight, value, volume, etc.).

III. STATE OF THE ART

The algorithmic literature on selection under constraints provides a structured overview, ranging from classical approaches to recent developments. For the *Multiple-Choice Knapsack Problem (MCKP),* [5] proposes a "minimal" algorithm based on an extended core and fast dynamic programming, marking a milestone for modern exact methods. The reference work [6] provides a comprehensive formalization of the MCKP and its variants, including multiple-choice and multidimensional extensions. For the *Multiple-Choice Multidimensional Knapsack Problem (MM-MCKP),* [8] analyzes the structure of the problem (linear relaxations, core) and compares ILP, metaheuristics, and hybrid approaches. The authors in [7] provide an exact algorithm for the MM-MCKP, while [9] introduce a Benders decomposition for the MCKP with setup constraints. More recently, [11] provides an in-depth survey of solution methods for the MCKP and their applications, highlighting the persistent challenges in contexts with complex constraints. On the side of the Balanced Assignment Problem (BAP) [3] sets the framework for balanced optimization and proposes an $O(n^4)$ algorithm for certain variants, while [4] systematize the LAP/BAP techniques (Hungarian, extensions, multi-index). The author of [10] introduces a physics-inspired approach for the *k*-cardinality assignment and its balanced versions, favoring global balance over nested local quotas. Finally, in the field of

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network flows, [1] proposes a strongly polynomial algorithm for minimum-cost flow, and the book by [2] exposes the standard reduction of flows with lower bounds to classical flows, paving the way for direct integration into solvers. In light of this state of the art, our contribution *HMR-MMKP* positions itself as a structured generalization: (i) to nest quotas at the cell level (family × category) with local lower bounds (by gender); (ii) to introduce an adaptation rule in case of insufficiency; (iii) to provide an algorithmic bridge to flows with bounds for polynomial feasibility, with certificates via maximum flow / minimum cut; (iv) to remain compatible with exact MILP formulations and guided heuristics. This approach seeks to bridge the gap between MCKP/MM-MCKP (multiple capacities without adaptive nested quotas), BAP (global balance without hierarchical local bounds), and flow methods (powerful but without integrated adaptive multi-choice models).

IV. Modeling the HMR-MMKP

The modeling of the *HMR-MMKP* is structured around three complementary stages. First, the case study presents the practical context of the problem and highlights the challenges related to the allocation of multiple resources. Then, a formal modeling translates this context into a mathematical program where each task has multiple choice alternatives subject to capacity constraints. Finally, a flow modeling provides a graphical representation of the problem, facilitating structural analysis and the exploration of suitable solution methods.

A. Case Study Presentation

In the context of a certification training delivery organization, a selection process is implemented to allocate a total of 7511 learners among four certification families: A, B, C, and D. The distribution of candidates must respect gender quotas and specific socio-academic categories in order to ensure an optimal allocation.

1) Candidate Categories

Candidates are divided into six distinct socio-academic categories, each corresponding to a level of education or a specific status:

- C_1: **CATEGORY 1** = Holders of DUT/BTS/License/Master/Doctorate,
- C_2: **CATEGORY 2** = Students in L3/M1/M2/PhD (not yet defended),
- C_3: **CATEGORY 3** = High School Diploma,
- C_4: **CATEGORY 4** = Others (self-employed, non-graduates, etc.),
- C_5: **CATEGORY 5** = Others (Students),
- C_6: **CATEGORY 6** = Others (Graduates without employment).

2) Candidate Distribution

The distribution of candidates among the different categories and certification families is given in the following table. A distinction is made between **female** and **man** for each category:

TABLE I
Distribution of candidates by certification family and socio-academic category, based on gender.

Category/ Family	C1	C2	C3	C4	C5	C6	Total
A	225	619	57	113	57	57	1128
female	158	434	40	80	40	40	792
male	67	185	17	33	17	17	336
B	1857	1238	413	413	0	207	4128
female	1300	867	290	290	0	145	2892
male	557	371	123	123	0	62	1236

Category/ Family	C1	C2	C3	C4	C5	C6	Total
C	563	507	0	57	0	0	1127
female	395	355	0	40	0	0	790

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male	168	152	0	17	0	0	337
D	225	563	57	169	57	57	1128
female	158	395	40	119	40	40	792
male	67	168	17	50	17	17	336

3) Basic Principle

The selection process is based on a quota system where, for each category C_i, candidates are sorted separately according to an evaluation function based on a score. The objective is to ensure a distribution that respects a target proportion of **70% female** and **30% male** for each certification family and category. Let N_i be the total number of candidates planned for category C_i. The planned number of women N_{if} in each category is defined as $N_{if} = 70\% \times N_i$. The planned number of men N_{im} is given by $N_{im} = N_i - N_{if}$. Then, the algorithm adjusts these quotas based on the actual selections and rebalances them if necessary to respect the gender proportions. For each category C_i, we initialize the variables N_{ifr}, representing the actual number of women selected, and N_{imr}, representing the actual number of men selected. We then calculate the selection remainder for each category, which corresponds to the difference between the planned and the actual number selected. If the gender quotas are not respected, we proceed to the complementary loop. In the complementary loop, the quotas are readjusted using the remainders to ensure that the target proportion of women and men is respected. The number of women to be reallocated is calculated as $R_{if} = 70\% \times R$, and the number of men to be reallocated is calculated as $R_{imr} = R - R_{ifr}$. The remainders are then adjusted, and the quotas for each category are completed until the sum of the remainders is zero. Finally, the final number of candidates selected for each category C_i is given by the sum of the selected female and male candidates, including those reallocated, i.e. $N_{ir} = N_{ifr} + N_{imr} + R_{ifr} + R_{imr}$. This final calculation ensures a distribution of candidates that complies with the initially defined gender quotas. This process will ensure a fair distribution of candidates while respecting the gender quotas and the capacity constraints imposed by each category and family. This model will efficiently manage nested quotas and adaptive readjustments, ensuring a fair and gender-proportional distribution. It can be extended to other sectors requiring allocation under complex constraints and adapted to diverse selection contexts.

4) Formal Modeling

Sets. Families $\mathcal{F} = \{1, \dots, 4\}$, categories $\mathcal{C} = \{1, \dots, 6\}$, genders $\mathcal{G} = \{f, m\}$, candidates $\mathcal{P}$.
For $p \in \mathcal{P}$, $i(p) \in \mathcal{C}$ (category), $g(p) \in \mathcal{G}$ (gender), $F_p \subseteq \mathcal{F}$ (eligible families).

Parameters. Quotas $N_{f,c} \in Z_{\geq 0}$, target ratios $(r_f, r_m) = (0.70\,; 0.30)$, scores $s_p \in R$ (or aggregated $v_{f,c,g}$), total capacity $Q_{\text{tot}}(e.g., 7511)$.

Variables. Assignments $x_{p,f} \in \{0,1\}$, counts $y_{f,c,g} \in Z_{\geq 0}$.

Objective. Maximize the total value $\mathbf{max} \sum_{p \in \mathcal{P}} \sum_{f \in F_p} s_p \, x_{p,f}$

Constraints.

$$\sum_{f \in F_p} x_{p,f} \leq 1 \qquad \forall p \in \mathcal{P} \text{ (uniqueness)} \qquad (1)$$

$$\sum_{p:\, i(p) = c,\, g(p) = g,\, f \in F_p} x_{p,f} = y_{f,c,g} \qquad \forall f, c, g \text{ (count link)} \qquad (2)$$

$$\sum_{g \in \mathcal{G}} y_{f,c,g} = N_{f,c} \qquad \forall f, c \text{ (cell quota)} \qquad (3)$$

$$y_{f,c,f} \geq \lfloor r_f N_{f,c} \rfloor, \; y_{f,c,m} \geq \lfloor r_m N_{f,c} \rfloor \qquad \forall f, c \text{ (target ratios)} \qquad (4)$$

$$\sum_{f,c,g} y_{f,c,g} \leq Q_{\text{tot}} \qquad \text{(total capacity)} \qquad (5)$$

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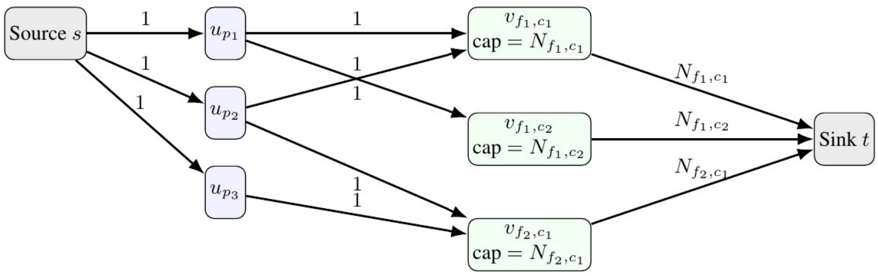
Adaptation. If the supply of one gender is insufficient locally, the missing places are shifted to the other gender while respecting (3).

5) *Flow Modeling*

Flow with Lower Bounds. For each arc $(a \to b)$ with lower bound L_{ab} and capacity U_{ab}, we set a new capacity $U'_{ab} = U_{ab} - L_{ab}$ and define node demands $d(\cdot)$:

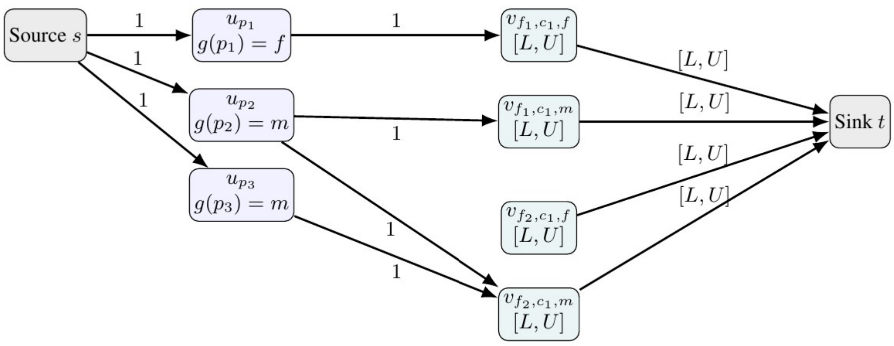
$$d(a) \mathrel{+}= L_{ab}\ ;\ d(b) \mathrel{-}= L_{ab}.$$

After transformation, we solve a standard flow by adding a super-source/super-sink to satisfy the demands $d(\cdot)$. A solution exists if and only if the flow covers all the demands; the final flow is reconstructed by adding L_{ab} to each original arc.



Arcs (s, u_p) with capacity 1; arcs $(u_p, v_{f,c})$ with capacity 1 if $i(p) = c$ and $f \in F_p$; arcs $(v_{f,c}, t)$ with capacity $N_{f,c}$.
A feasible assignment exists if and only if the maximum flow equals $T = \sum_{f,c} N_{f,c}$

Fig. 1 Aggregated maximum flow graph.



Each cell (f, c) is duplicated into (f, c, f) and (f, c, m).
The arcs to the sink have a *lower bound* $L = \lfloor r_g N_{f,c} \rfloor$ and an *upper bound* $U = N_{f,c}$

Fig. 2 Decomposition by genre.

V. Theoretical Properties

A. *Additional Results*

Theorem 1 (Strict Generalization). *The HMR-MMKP problem strictly generalizes the MM-MCKP problem.*

Proof. When all hierarchical and adaptive constraints are disabled, the HMR-MMKP reduces exactly to a classical MM-MCKP. However, there are instances of the HMR-MMKP (with nested or adaptive quotas) that cannot be formulated as a standard MM-MCKP without losing the optimal solution. A simple counterexample is given by two families and two categories, with minimum gender quotas: no MM-MCKP can impose these constraints by construction, whereas the HMR-MMKP can.

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Proposition 1 (Comparison with BAP). *The HMR-MMKP cannot be represented as a BAP problem without loss of generality when adaptive gender quotas are present.*

Proof. BAP formulations generally assume capacity constraints per node, without nested lower bounds dependent on an adaptive external parameter. Adaptive quotas introduce a global dependency that cannot be modelled by fixed capacities in BAP, as shown by an example where the optimal value changes based on the respect for gender quotas at the cell level.

Proposition 2 (Monotonicity). *If one of the quotas $N_{f,c}$ increases, the optimal value of the HMR-MMKP does not decrease.*

Proof. An increase in $N_{f,c}$ expands the set of feasible solutions, as it relaxes a constraint. By the definition of the optimum over a larger set, the optimal value cannot decrease.

Theorem 2 (Integer Relaxation Case). *If each family f is linked to a single category c and the eligibility graph is bipartite with no cross-coverings, the constraint matrix of the HMR-MMKP is totally unimodular, and the linear relaxation provides an integer solution.*

Proof. In this case, the problem reduces to a b-matching bipartite problem with quotas, whose incidence matrix is totally unimodular. Therefore, the extreme points of the LP polyhedron are integers, and the optimal solution of the relaxation is integer.

Proposition 3 (Flow Modeling with Lower Bounds). *Minimum gender quotas in the HMR-MMKP can be modelled as lower bounds on arcs in a flow network.*

Proof. For each cell (f,c) and gender g, we create an arc $(v_{f,c,g} \to t)$ with lower bound $L_{f,c,g} = \lfloor r_g N_{f,c} \rfloor$ and upper bound $U_{f,c,g} = N_{f,c}$. The classical transformation of flows with lower bounds reduces the problem to a standard flow, ensuring that the minimum quotas are respected in any feasible solution.

B. Complexity

Establish the intrinsic difficulty of the HMR-MMKP by reduction from the MCKP.

Theorem 3 (NP-hardness of HMR-MMKP*).* *The HMR-MMKP is NP-hard.*

Proof. We reduce an instance of the MCKP to the HMR-MMKP: create a single family f^*; associate each class C_j with a category c_j; for each item $a_{j,l}$, a candidate $p_{j,l}$ with $i(p_{j,1}) = c_j$, $F_{p_{j,1}} = \{f^*\}$, score $s_{p_{j,1}} = v_{j,1}$; set $N_{f^*,c_j} = 1$. Neutralize the ratios. Choosing a candidate per category is equivalent to selecting one item per class while maintaining optimality. Since the MCKP is NP-complete, the HMR-MMKP is NP-hard.

C. Global Feasibility via Maximum Flow

Provide a polynomial feasibility test (without scores)

Proposition 4 (Flow Criterion). *There exists a binary assignment x such that $\sum_{p:\, i(p)=c} x_{p,f} = N_{f,c}$ for all (f,c) if and only if the maximum flow in the bipartite network $(source\ \boldsymbol{s} \to candidates \to cells \to sink\ \boldsymbol{t})$ equals $T = \sum_{f,c} N_{f,c}$.*

Proof. Nodeu_p for each candidate, node $v_{f,c}$ for each cell, edges (s, u_p) with capacity 1, $(u_p, v_{f,c})$ with capacity 1 $if\ i(p) = c$ and $f \in F_p$, $(v_{f,c}, t)$ with capacity $N_{f,c}$.

Forward direction ($\to$) An integer assignment induces a flow of value T via the paths $s \to u_p \to v_{f,i(p)} \to t$

Reverse direction ($\leftarrow$) By the integrality of the flows, a total flow of value T defines an assignment saturating the quotas cell by cell. To integrate genders, we duplicate the $v_{f,c}$ to $v_{f,c,f}$ and $v_{f,c,m}$ with appropriate capacities.

D. Polynomial Case: Unique Membership

Identify a class of instances that can be solved efficiently.

Proposition 5. *If $|F_p| = 1$ for all $p \in \mathcal{P}$, the HMR-MMKP can be solved in $O(|\mathcal{P}|\, log|\mathcal{P}|)$ by local sorting within each cell (f,c).*

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Proof. The sets of eligible candidates by cell are disjoint; decisions are independent. Sorting candidates in each (f, c) by descending score and taking the $N_{f,c}$ best ones is optimal. The total cost is dominated by the sorting step.

E. Limit of a Local Procedure (Separate Gender + Reallocation)

Show that this procedure can be strictly suboptimal compared to the global optimum.

Proposition 6. *There exist instances where the procedure "local gender quotas then reallocation" results in an objective value strictly less than $Z^\star$.*

Proof. Consider a single category c, two families A, B with $N_{A,c} = N_{B,c} = 1$. Candidates: $(i) p_1$ female, $F_{p_1} = \{A\}$, score 51; $(ii) p_2$ male, $F_{p_2} = \{A, B\}$, score 100; (iii) p_3 male, $F_{p_3} = \{B\}$, score 99. Enforcing a local ratio $r_f = 0.5$, we select p_1 for A and p_3 for B (total 150). The global optimum selects p_2 for A and p_3 for B (total 199).

F. Bounds and Lagrangian Relaxation

Bound the optimal value and provide an exploitable upper bound.

Proposition 7 (Upper Bound). *An upper bound is obtained by $UB = \sum_{f,c} max_g\{v_{f,c,g}\}$ subject to the quota constraints.*

Proposition 8 (Lower Bound). *A lower bound is obtained by $LB = \sum_{f,c} \sum_g v_{f,c,g}^{(min)} \lfloor r_g N_{f,c} \rfloor$,*

where $v_{f,c,g}^{(min)}$ is the score of the worst candidate selected.

Proposition 9 (Dual Lagrangian Bound). *By dualizing the constraints $\sum_{p:\, i(p)=c} x_{p,f} = N_{f,c}$, the Lagrangian $L(\lambda) = max\{\sum_{p,f} s_p x_{p,f} - \sum_{f,c} \lambda_{f,c}(\sum_{p:i(p)=c} x_{p,f} - N_{f,c})\}$ bounds $Z^\star$ for all $\lambda \geq 0$, and $inf_{\lambda \geq 0} L(\lambda)$ is the best dual bound.*

Proof. For a feasible primal solution, the penalized term equals zero, so $\sum_{p,f} s_p x_{p,f} \leq L(\lambda)$. Taking the infimum over λ provides the best dual bound.

G. Local Feasibility under Adaptive Quotas

Characterize the feasibility of a cell (f, c) with the adaptation rule.

Proposition 10. *There exists a local assignment achieving dynamic adaptation if and only if*

$$cand_{f,c,f} + cand_{f,c,m} \geq N_{f,c}.$$

Proof. (Necessity) If the total number of candidates is $< N_{f,c}$, fulfilling $N_{f,c}$ is impossible.

(Sufficiency) Assign $\min\left(q_{f,c}^{g}, \mathrm{cand}_{f,c,g}\right)$ of the available gender g and then complete with the other gender until $N_{f,c}$. The condition guarantees total sufficiency.

VI. CONCLUSION

In conclusion, selection systems are crucial for ensuring a fair and optimal allocation of resources while respecting diversity and transparency objectives. However, the simultaneous management of multiple, nested, and adaptive quotas presents a major challenge for their modeling and algorithmic resolution. Classical models, such as the Multiple-Choice Multi-Dimensional Knapsack Problem (MM-MCKP), do not effectively handle the complexity of interactions between quotas. To address this need, we introduced the Hierarchical Multi-Ratio Multiple-Choice Multi-Dimensional Knapsack Problem (HMR-MMKP), an extension of the MM-MCKP that explicitly incorporates hierarchical quotas and adaptive mechanisms. This approach provides a complete formalization of the model and an in-depth theoretical analysis, including proof of its NP-completeness. It overcomes the limitations of existing approaches and constitutes a powerful tool for addressing complex allocation problems under multiple constraints. The next step in this research lies in the implementation of the HMR-MMKP model through a dedicated solver. This will involve developing a software solution using mathematical programming libraries (such as Gurobi, CPLEX, or linear programming tools in Python) to simulate realistic scenarios. Simulating large instances of the problem will allow testing the robustness and efficiency of the solver in terms of managing adaptive and nested quotas. Moreover,

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architectural development will be necessary to design a solver capable of handling both small and complex instances. This will include the implementation of a hybrid algorithm that could combine exact and heuristic techniques to solve the problem more efficiently. These efforts will pave the way for applying this method in various sectors requiring complex resource allocation, while ensuring an optimal and transparent distribution.

REFERENCES

[1] Orlin, J. B. (1993). *A Faster Strongly Polynomial Minimum Cost Flow Algorithm*. Operations Research, 41(2), 338–350. https://doi.org/10.1287/opre.41.2.338

[2] Ahuja, R. K., Magnanti, T. L., & Orlin, J. B. (1993). *Network Flows: Theory, Algorithms, and Applications*. Prentice Hall. ISBN: 978-0136175490

[3] Martello, S., & Toth, P. (1988). *A New Algorithm for the 0–1 Knapsack Problem*. Management Science, 30(2), 222–234. https://doi.org/10.1287/mnsc.34.5.633

[4] Burkard, R., Dell'Amico, M., & Martello, S. (2009). *Assignment Problems*. Society for Industrial and Applied Mathematics (SIAM). ISBN: 0898716632

[5] Pisinger, D. (1995). *A minimal algorithm for the 0–1 knapsack problem*. Operations Research, 45(5), 758–767. https://doi.org/10.1287/opre.45.5.758

[6] Kellerer, H., Pferschy, U., & Pisinger, D. (2004). *Knapsack Problems*. Springer. https://doi.org/10.1007/978-3-540-24777-7

[7] Hifi, M., Sadfi, S., & Sbihi, A. (2004). *An Exact Algorithm for the Multiple-choice Multidimensional Knapsack Problem*. [Online]. Available at: https://shs.hal.science/halshs-03322716/document

[8] Puchinger, J., Raidl, G. R., & Pferschy, U. (2010). *The Multidimensional Knapsack Problem: Structure and Algorithms*. INFORMS Journal on Computing, 22(2), 250–265. https://doi.org/10.1287/ijoc.1090.0344

[9] Boukhari, S., & Hifi, M. (2024). *An exact algorithm for the multiple-choice Knapsack problem with setups*. Computers & Industrial Engineering, 187, 109818. https://doi.org/10.1016/j.cie.2023.109818

[10] Koehl, P., & Orland, H. (2024). *Physicist's view on the unbalanced k-cardinality assignment problem*. Physical Review E, 110(1), 014108. https://doi.org/10.1103/PhysRevE.110.014108

[11] Szkaliczki, T. (2025). *Solution Methods for the Multiple-Choice Knapsack Problem and Their Applications*. Mathematics, 13(7), 1097. https://doi.org/10.3390/math13071097

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