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Enhancing Target Tracking Accuracy in Multistatic Radar Systems Using an Adaptive Algorithm for Waveform and Cartesian Estimate Selection

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Abstract— **Target tracking in multistatic radar systems is challenging due to the impact of waveform selection and the transformation of elliptic measurements into Cartesian estimates. This paper proposes a novel algorithm that jointly optimizes the transmitted waveform and Cartesian estimates to minimize tracking error. The algorithm utilizes the Cramer-Rao Lower Bound (CRLB) for Cartesian estimates derived from elliptic measurements (time delay, Doppler shift, and angle of arrival). A single transmitter and multiple receivers dynamically select the optimal waveform and estimate based on the target's predicted state. Simulations demonstrate that the proposed algorithm significantly outperforms fixed configurations, improving tracking accuracy while reducing power and bandwidth consumption.**

Keywords: Multistatic radar, Adaptive waveform selection, Target tracking, CRLB.

I. Introduction

Target tracking in multistatic radar systems has garnered significant attention in recent years due to its advantages over monostatic radar, such as increased coverage, immunity to jamming, and spatial diversity [1, 2]. In multistatic radar systems, transmitters and receivers are spatially separated, leading to improved target detection and tracking performance. However, the performance of these systems heavily depends on the transmitted waveform, radar geometry, and the accuracy of target state estimates [3, 4].

One of the main challenges in multistatic radar systems is the transformation of elliptic measurements—such as time delay, Doppler shift, and angle of arrival—into Cartesian estimates of the target's position and velocity. These Cartesian estimates are essential for accurate target tracking using Linear Kalman Filters (LKF) [5]. However, their accuracy is significantly affected by both the transmitted waveform and the transformation method used. Traditional approaches often rely on fixed waveforms and fixed Cartesian estimates, which may not adapt well to dynamic tracking scenarios where the target's position and velocity evolve over time [6].

To address this challenge, this paper proposes a novel algorithm for the **joint adaptive selection of the transmitted waveform and Cartesian estimate**. The algorithm is designed to minimize the Mean Square Error (MSE) of the tracking process by leveraging the Cramer-Rao Lower Bound (CRLB) of Cartesian estimates. The CRLB provides a theoretical lower bound on the variance of any unbiased estimator and is used here to evaluate the performance of various waveform and estimate configurations [7]. By dynamically selecting the optimal waveform and Cartesian estimate based on the target's predicted state, the algorithm improves tracking accuracy while reducing the communication bandwidth and power consumption between the transmitter and receivers.

The main contributions of this paper are as follows:

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- **Derivation of the CRLB for Cartesian Estimates** using a transformation method that incorporates waveform parameters, target motion, and radar geometry.
- **Proposal of a joint adaptive selection scheme** that dynamically chooses both the waveform and Cartesian estimate to minimize the tracking MSE.
- **Performance evaluation through simulations**, showing that the proposed algorithm outperforms traditional fixed-configuration methods in terms of accuracy, efficiency, and system stability.

This study contributes to the ongoing development of intelligent, resource-efficient multistatic radar systems capable of adapting to dynamic environments.

II. SYSTEM MODEL AND PROPOSED ALGORITHM

1. Multistatic Radar System:

We consider a multistatic radar system consisting of a single transmitter (Tx) and N receivers (R_{X1} ,R_{X2} ..., R_{XN}) distributed at different locations. The system is designed to track a moving target in a two-dimensional space. Each receiver is capable of measuring:
Time delay (τ_i).
Doppler shift (v_i).
Angle of arrival (θ_i).
These measurements are referred to as elliptic measurements and are transformed into Cartesian estimates of the target's position (x, y) and velocity ($\dot{x}$, $\dot{y}$) using specific mathematical transformations.

2. Elliptic Measurements:

The elliptic measurements for each receiver i are defined as follows:

$$\tau_i = \frac{R_i - L_i}{c}, v_i = \frac{f_c}{c} R_i, \theta_i = atan2(y - yR_i, x - xR_i) \quad (1)$$

Where:

1. $R_i = R_T + R_{Ri}$: The total distance from the transmitter to the target and then to receiver i,
2. $R_T = \sqrt{x^2 + y_2}$: The distance from the transmitter to the target.
3. $R_{Ri} = \sqrt{(x - x_{Ri})^2 + (y - y_{Ri})^2}$: The distance from the target to receiver i,
4. $L_i = \sqrt{(x_{Ri}^2 + y_{Ri}^2}$: The distance between the transmitter and receiver i ,
5. c : The speed of light.
6. f_c: The carrier frequency of the transmitted signal.

3. Cartesian Estimates: -

To extract position and velocity from the elliptic measurements, we define three distinct transformation methods:

- Transformation 1 (y_1):Uses $\tau_1, \theta_1, v_1, v_2$.
- Transformation 2 (y_2):Uses $\tau_2, \theta_2, v_1, v_2$.
- Transformation 3 (y_3):Uses $\theta_1\ \theta_2, v_1, v_2$.

That target position (x, y) is computed using the intersection of elliptic measurement, while the target velocity ($\dot{x}$, $\dot{y}$) is computed using Doppler shift measurement.

4. Cramer-Rao Lower Bound (CRLB):

These measurements are referred to as elliptic measurements and are transformed into Cartesian estimates of the target's position (x, y) and velocity ($\dot{x}$, $\dot{y}$) using specific mathematical transformations.

$$\boldsymbol{C}_{\mathbf{y1}} = \frac{\partial \mathbf{h}l(\mathbf{z}l)}{\partial \mathbf{z}l} \mathbf{Cz}l \frac{\partial \mathbf{h}l(\mathbf{z}l)T}{\partial \mathbf{z}l} \quad (2)$$

where:

- $\mathbf{C}\mathbf{y}l$: The CRLB for the Cartesian estimates.

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- $\mathbf{C}\mathbf{z}l$: The CRLB for the elliptic measurements.
- $\frac{\partial \mathbf{h}l(\mathbf{z}l)T}{\partial \mathbf{z}l}$: The Jacobian matrix of the transformation $\mathrm{h}l$.

This relationship quantifies how transformation quality depends on measurement sensitivity and radar geometry.

5. *Proposed Algorithm:* The proposed algorithm consists of two main phases:

- Target Tracking: A linear Kalman filter (LKF) is used to track the target based on the Cartesian estimates.
- Adaptive Selection: The optimal transmitted waveform and Cartesian estimate are selected to minimize the tracking mean-square error (MSE). This is achieved by minimizing the trace of the error covariance matrix of the target state estimate:

$$\{m_{k+1}^{*}, \boldsymbol{\psi}_{k+1}^{*} = \ argmin_{m,\psi}\, Tr(P_{K+1|k+1}\ (m,\psi)) \qquad (3)$$

where:

- m_{k+1}^{*}:The index of the optimal Cartesian estimate.
- ψ_{k+1}^{*}: The parameters of the optimal waveform.
- $P_{K+1|k+1}\ (m,\psi)$: The error covariance matrix of the target state estimate.

6. Advantages of the Proposed Algorithm:

- Reduced Power and Bandwidth Consumption: Only one Cartesian estimate is used at each time step.
- Improved Tracking Performance: The tracking performance is dynamically optimized based on the target's state.
- Stability: The use of a linear Kalman filter (LKF) avoids the divergence issues associated with nonlinear filters.

III. SIMULATION SETUP

To evaluate the performance of the proposed algorithm, a simulation was conducted in a multistatic radar environment. The configuration includes specific transmitter/receiver locations, target trajectory, waveform parameters, and noise models.

- Transmitter and Receiver Locations:
  - Transmitter (Tx): Located at the origin [0,0] meters.
  - Receivers (Rx): Three receivers located at the following positions:
    - Rx1: [20000,0] meters.
    - Rx2: [10000,20000] meters.
    - Rx3: [0,15000] meters.

This setup provides spatial diversity and ensures sufficient geometric variation for evaluating the effectiveness of different Cartesian estimates.

- Target Trajectory:
  - Initial Position: [15000,10000] meters.
  - Initial Velocity: [−150,100] meters/second.
  - Target Path: The target is tracked along a predefined trajectory, where the true target positions are almost identical to the estimates generated by the algorithm.

The trajectory was designed to evaluate tracking performance under dynamic motion conditions.

- Transmitted Waveform Parameters:
  - Waveform Type: Gaussian Linear Frequency Modulated (Gaussian-LFM).
  - Carrier Frequency (fc): 12.5 GHz.

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- Pulse Length (λ): Ranges between 40 and 80 microseconds.
- Frequency Sweep (**Δ_F**): Ranges between 0.1 and 0.5 MHz.

These waveform settings affect the resolution of time delay and Doppler shift, thereby impacting position and velocity estimates.

- Signal-to-Noise Ratio (SNR):
  - The SNR at each receiver is modeled using the following relationship:

$$SNR_{i,k} = \frac{R_0^4}{(R_{Tk}R_{Rik})^2} \tag{4}$$

where R_0=70000 meters.

This SNR model reflects realistic signal degradation over distance and is crucial for simulating measurement noise.

- Angle Measurement Error**:**
  - The standard deviation of the angle of arrival measurement (σ_θ): 0.2 radians.

This value accounts for angular uncertainty, which influences the accuracy of the angle-based Cartesian estimates.

IV. SIMULATION RESULTS AND DISCUSSION

This section presents the performance analysis of the proposed algorithm based on simulated scenarios. The results focus on how waveform characteristics—particularly pulse length—affect the accuracy of position and velocity estimates. The Cramer-Rao Lower Bound (CRLB) is used as a performance metric for evaluating each Cartesian estimate.. The figure 1 illustrates the relationship between Pulse Length (λ) measured in microseconds (μs) and the Cramer-Rao Lower Bound (CRLB) for position and velocity estimates in a multistatic radar system. The figure is divided into two main sections:

1**.** *CRLB for Position (RCRLB Position):*

- Y-axis: Represents the CRLB for position, which indicates the minimum achievable error in estimating the target's position. Lower values correspond to higher accuracy.
- X-axis**:** Represents the pulse length (λ) in microseconds (μs).
- Curves**:** The different curves (*V*1, *V*2, *V*3) represent various position estimates based on time delay and angle-of-arrival measurements from different receivers.

Observations:

- Effect of Pulse Length on Position CRLB**:**
  1. For position estimates (*V*1, *V*2, *V*3), the CRLB decreases with shorter pulse lengths (e.g., 10 μs). This is because shorter pulses provide higher resolution in time delay measurements, improving position estimation accuracy.
  2. Conversely, longer pulse lengths (e.g., 40 μs) increase the CRLB for position, indicating reduced accuracy.
- Differences Between Estimates (*V1, V2, V3*)**:**
  1. *V*3 shows lower CRLB values compared to *V*1 and *V*2, especially with shorter pulses. This is because *V*3 relies on angle-of-arrival measurements from two receivers, providing higher accuracy.

2. *CRLB for Velocity (RCRLB Velocity):*

- Y-axis: Represents the CRLB for velocity, indicating the minimum achievable error in estimating the target's velocity.
- X-axis: Represents the pulse length (λ) in microseconds (μs).

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- Curves**:** The different curves ($y1$, $y2$, $y3$) represent various velocity estimates based on Doppler shift measurements from different receivers.

Observations:

- Effect of Pulse Length on Velocity CRLB:
  - For velocity estimates ($y1$, $y2$, $y3$), the CRLB decreases with longer pulse lengths (e.g., 40 µs). Longer pulses provide better Doppler resolution, improving velocity estimation accuracy.
  - Shorter pulse lengths (e.g., 10 µs) increase the CRLB for velocity, indicating reduced accuracy.
- Differences Between Estimates **(**$y1$, $y2$, $y3$**)**:
  - $y3$ shows lower CRLB values compared to y1$y1$ and $y2$, especially with longer pulses. This is because $y3$ relies on Doppler shift measurements from two receivers, providing higher accuracy.

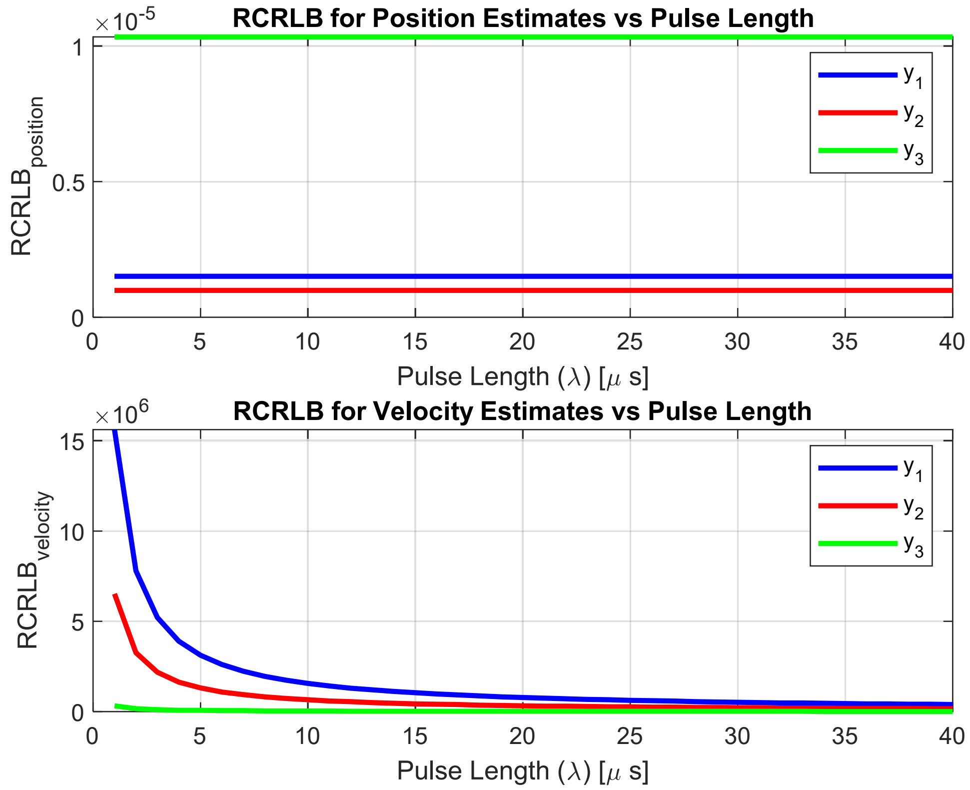

Fig.1 RCRLB vs pulse Length

The Figure2 presents the Root Cramer-Rao Lower Bound (RCRLB) for position estimates in a multistatic radar system, focusing on the relationship between the target's position and the accuracy of the position estimates. The figure is divided into two main sections:

1. *RCRLB for Position:*

- Y-axis: Represents the RCRLB for position, which indicates the minimum achievable error in estimating the target's position. Lower values correspond to higher accuracy.
- X-axis: Represents the target's x-position in meters (m).

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- Curves**:** The different curves (*V*1, *V*2, *V*3) represent various position estimates based on time delay and angle-of-arrival measurements from different receivers.

Observations:

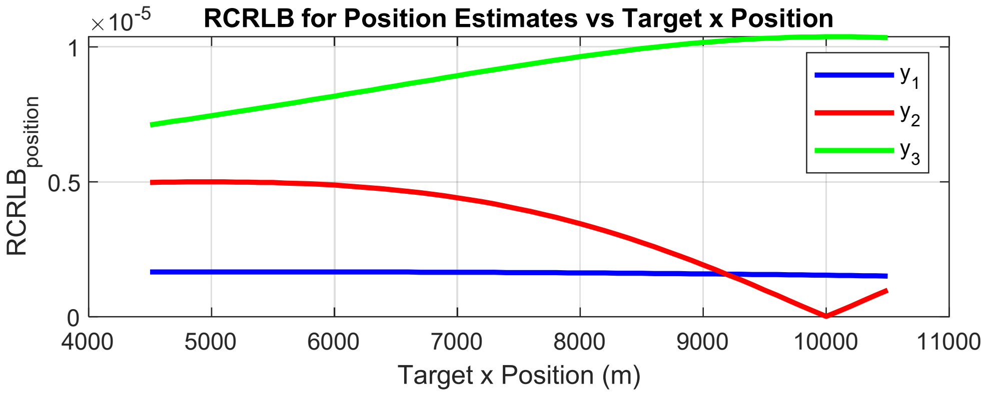
- Effect of Target Position on RCRLB**:**
   i. The RCRLB for position varies with the target's x-position. As the target moves further from the origin (e.g., from 4000 m to 11000 m), the RCRLB generally increases, indicating reduced accuracy in position estimation.
   ii. This increase in RCRLB is due to the geometric dilution of precision (GDOP), where the accuracy of position estimates degrades as the target moves further from the radar system.
- Differences Between Estimates (V1, V2, V3):
   i. *V*3 shows lower RCRLB values compared to *V*1 and *V*2 across all target positions. This is because *V*3 relies on angle-of-arrival measurements from two receivers, providing higher accuracy.
   ii. *V*1 and *V*2, which rely on measurements from a single receiver, show higher RCRLB values, especially at greater distances.

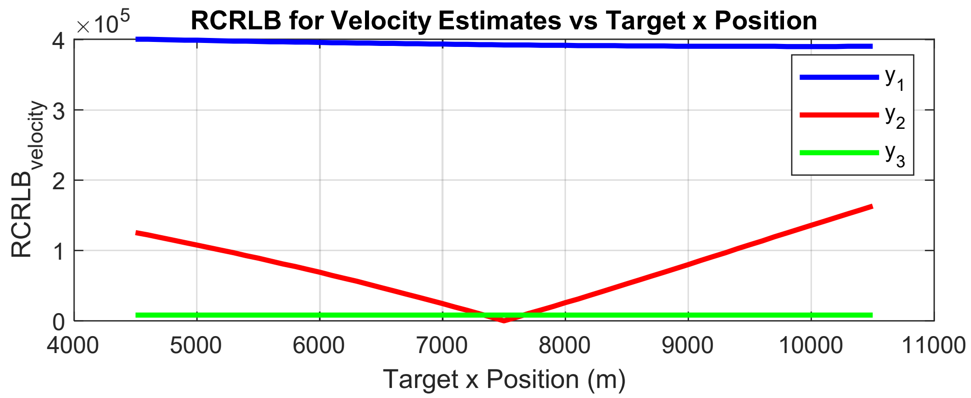
2. *Target x-Position (m):*

- Y-axis: Represents the target's x-position in meters (m).
- X-axis: Represents the target's x-position in meters (m), repeated for clarity.
- Curves**:** The curves represent the same position estimates (*V*1, *V*2, *V*3) as in the first section, but with a focus on the target's x-position.

Observations:

- Consistency in RCRLB Trends**:**
   - The trends observed in the first section are consistent here. The RCRLB increases as the target moves further from the origin, and *V*3 consistently outperforms *V*1 and *V*2 in terms of accuracy.
   - The repeated x-axis emphasizes the importance of the target's position relative to the radar





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system in determining the accuracy of position estimates.

Fig.2 RCRLB vs Target x position

V. CONCLUSION:

This paper presents a novel algorithm designed to enhance target tracking in multistatic radar systems by jointly adapting the transmitted waveform and Cartesian estimate based on the target’s predicted state. The core of the approach lies in leveraging the Cramer-Rao Lower Bound (CRLB) as a performance metric to guide the dynamic selection process. The key results are:

1. Improved Tracking Accuracy: The algorithm dynamically adapts to changes in the target's state and radar geometry, enhancing tracking accuracy.
2. Reduced Power and Bandwidth Consumption: Only one Cartesian estimate is selected at each step, reducing power and bandwidth usage.
3. System Stability: The use of a linear Kalman filter (LKF) ensures stable performance without the divergence issues associated with nonlinear filters.

Compared to conventional methods that rely on fixed configurations, the proposed approach delivers significant improvements in both estimation precision and resource efficiency, particularly in dynamic tracking scenarios where the target's behavior and geometry change over time. Future research could further enhance the algorithm using advanced signal processing techniques to increase accuracy and reliability.

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