10th International Conference on Control Engineering &Information Technology (CEIT-2025)
Proceedings Book Series –PBS- Vol 23, pp.44-52

MULTI- STAGE RECURSIVE LEAST SQUARES ALGORITHM for BOX–JENKINS SYSTEMS

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Abstract—in this paper, a new-type multi-stage recursive least-squares identification is proposed for identifying the system model parameters and the noise model parameters of Box–Jenkins Systems. The key is to decompose the original system into four smaller systems and then interactively estimating the parameters of each of those smaller systems. The dimensions of the relevant covariance matrices in each subsystem are reduced when compared to the Conventional recursive least-squares identification algorithm for Box–Jenkins systems, which results in a high computational efficiency for the suggested algorithm. Simulation results are presented to illustrate the effectiveness of the proposed algorithm.

Keywords — Multi-stage, Box–Jenkins, covariance matrices, computational efficiency.

I. INTRODUCTION

System identification is a methodology for building mathematical models of dynamic systems using measurements of the input and output signals of the system [1]. The process of system identification requires Measure the input and output signals from system in time or frequency domain. Such a methodology system identification was mainly developed for designing model-based control systems [2]. More generally, parameter estimation is at the heart of many signal processing applications aiming to extract information from signals, like radar, sonar, seismic, speech, communication, or biomedical (EEG, ECG, EMG) signals. Nowadays, dynamical models and identification methods play an important role in most of disciplines such as automatic control, signal processing, physics, economics, medicine, biology, ecology, seismology [3]. Recursive least squares (RLS) is an adaptive filter algorithm that recursively finds the coefficients that minimize a weighted linear least squares cost function relating to the input signals. This approach is in contrast to other algorithms such as the least mean squares (LMS) that aim to reduce the mean square error [4]. In the derivation of the RLS, the input signals are considered deterministic, while for the LMS and similar algorithms they are considered stochastic. Compared to most of its competitors, the RLS exhibits extremely fast convergence. However, this benefit comes at the cost of high computational complexity [5]. Recursive identification is a description of estimation algorithms where the parameter estimates are revised with each new measurement. Recursive identification is required when we want a new estimate of the parameter after each new measurement. Recursive identification relies on fast algorithms where the computational burden and required memory do not increase with time. It is well known that the least squares identification algorithm requires high computational burden because of the calculation of the large-dimensional covariance matrix and the identification algorithm has low estimation accuracy in parameters [6]. The Box-Jenkins Model is a mathematical model designed to forecast data ranges based on inputs from a specified time series. The Box-Jenkins Model can analyze several different types of time series data for forecasting purposes [7]. The decomposition approach is based on the reduction of the main identification problem to small sub problems, which are easy to solve. Filtered methods and multistage algorithms are some of the algorithms based on decomposition techniques [8]. The Box-Jenkins methodology comprises four steps Identification of process, Estimation of parameters, Verification, and, Forecasting. Model validation in the system specification process involves checking between measured data and required data. Model validation is usually defined to mean substantiation that a computerized model within its domain of applicability possesses a satisfactory range of accuracy consistent with the intended application of the model [9].

II. SYSTEM DESCIRPTION

In this study, a multi-stage recursive least squares algorithm for Box–Jenkins systems is presented.

$$(k) = \frac{B(z)}{A(z)}u(k) + \frac{C(z)}{D(z)}v(k) \tag{1}$$

ISSN: 2961-6611

$$(z) = 1 + a_1 z^{-1} + a_2 z^{-2} + \cdots + a_{na} z^{-na}$$

$$(z) = b + b_1 z^{-1} + b_2 z^{-2} + \cdots + b_{nb} z^{-nb}$$

$$(z) = 1 + c_1 z^{-1} + c_2 z^{-2} + \cdots + c_{nc} z^{-nc}$$

$$(z) = 1 + d_1 z^{-1} + d_2 z^{-2} + \cdots + d_{nd} z^{-nd}$$

In the system model illustrated in Fig.1, where $u(k)$ represents the system input sequence , $y(k)$ represents the system output sequence, $v(k)$ is a stochastic white noise with zero mean and variance σ², $A(z)$, $B(z)$, $C(z)$ and (z) are polynomials [10] in the unit backward shift operator $z^{-1}[i.e. z^{-1}\ (k) = y\ (k-1)]$ with known orders na, nb, nc and nd, the variables (k) and $w(k)$ denote the output of the system model and the noise model respectively.

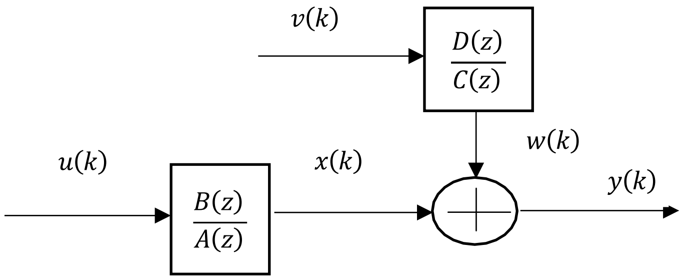

Fig. 1 The Box–Jenkins System

$$x(k) = \frac{B(z)}{A(z)} u(k)$$

$$w(k) = \frac{D(z)}{C(z)} v(k)$$

$$(k) = [1 - A(z)]x(k) + B(z)u(k)$$

$$x(k) = -\sum_{i=1}^{na} a_i x(k-i) + \sum_{i=1}^{nb} b_i u(k-i)$$

$$(k) = [1 - C(z)]w(k) + D(z)v(k)$$

$$w(k) = -\sum_{i=1}^{na} c_i w(k-i) + \sum_{i=1}^{nb} d_i v(k-i) + v(k)$$

The definitions of the parameter vectors Φ and θ are given as

$$\theta = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ 4 \end{bmatrix} \epsilon R^n = n_a + n_b + n_c + n_d$$

$$\theta_1 = [a_1, a_2, \ldots, a_{na}]^T$$

$$\theta_2 = [b_1, b_2, \ldots, b_{nb}]^T$$

$$\theta_3 = [c_1, c_2, \ldots, c_{nc}]^T$$

$$\theta_4 = [d_1, d_2, \ldots, d_{nd}]^T$$

ISSN: 2961-6611

$$\Phi = \begin{bmatrix} \varphi_1 \\ 2 \\ \varphi_3 \\ \varphi_4 \end{bmatrix}$$

Expressing the above equations in vector form, we get

$$(k) = \varphi_1^T \theta_1 + \varphi_2^T \theta_2 \tag{2}$$

$$\varphi_1 = [-(k-1), \ldots, -x(k-n_a)]$$

$$\varphi_2 = [(k-1), \ldots, u(k-n_b)]$$

$$(k) = \varphi_3^T \theta_3 + \varphi_4^T \theta_4 + v(k) \tag{3}$$

$$\varphi_3 = [-(k-1), \ldots, -w(k-n_c)]$$

$$\varphi_4 = [(k-1), \ldots, v(k-n_d)]$$

From (1), we have

$$y(k) = x(k) + w(k)$$

$$(k) = \varphi_1^T \theta_1 + \varphi_2^T \theta_2 + \varphi_3^T \theta_3 + \varphi_4^T \theta_4 + v(k) \tag{4}$$

This equation is the identification model of the BJ systems in (1).

$$(k) = [\varphi_1^T \ \varphi_2^T \ \varphi_3^T \ \varphi_4^T] \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ 4 \end{bmatrix} + v(k) \tag{5}$$

III. MULTI-STAGE RLS ALGORITHM

To formulate an MS-RLS algorithm for parameter estimation, the multi-stage identification method, which is based on the decomposition technique, necessitates defining an intermediate variable.

$$y_1(k) = y(k) - \varphi_2^T(k)\theta_2 - \varphi_3^T(k)\theta_3 - \varphi_4^T(k)\theta_4 \tag{6}$$

$$y_2(k) = y(k) - \varphi_1^T(k)\theta_1 - \varphi_3^T(k)\theta_3 - \varphi_4^T(k)\theta_4 \tag{7}$$

$$y_3(k) = y(k) - \varphi_1^T(k)\theta_1 - \varphi_2^T(k)\theta_2 - \varphi_4^T(k)\theta_4 \tag{8}$$

$$y_4(k) = y(k) - \varphi_1^T(k)\theta_1 - \varphi_2^T(k)\theta_2 - \varphi_3^T(k)\theta_3 \tag{9}$$

From (6) – (9), equation (4) can be decomposed into four sub-identification models,

$$y(k) = \varphi_i^T(k)\theta_i + v(k), \qquad i = 1,2,3,4. \tag{10}$$

These includes the parameters vectors $\theta_1, \theta_2, \theta_3$ and θ_4 .Then, four criterion functions are defined as,

$$J(\theta_i) := \sum_{j=1}^{k} [y_i(j) - \varphi_i^T(j)\theta_i]^2, \qquad i = 1,2,3,4.$$

Let the partial derivatives of $J(\theta_i), i = 1,2,3,4$ with respect to θ_i be zero

$$\left.\frac{\partial J_i(\theta_i)}{\partial \theta_i}\right|_{\theta_i=\theta_i(k)} = -2\varphi_i(j) \sum_{j=1}^{k} [y_i(j) - \varphi_i^T(j)\theta_i(k)]$$

$$= 0, \qquad i = 1,2,3,4. \tag{11}$$

ISSN: 2961-6611

$$Let\ \theta(k) := \begin{bmatrix} \theta_1(k) \\ \theta_2(k) \\ \theta_3(k) \\ \theta_4(k) \end{bmatrix} \in \mathbb{R}^n\ be\ the\ estimate\ of\ \theta = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \end{bmatrix} \in \mathbb{R}^n$$

The RLS algorithm for computing $\theta_1(k), \theta_2(k), \theta_3(k)$ and $\theta_4(k)$ can be obtained as follows:

$$\theta_1(k) = \theta_1(k-1) + L_1(k)\,[\,y_1(k) - \varphi_1^T(k)\,\theta_1(k-1)] \tag{12}$$

$$L_1(k) = P_1(k-1)\,\varphi_1(k)[\,1 + \varphi_1^T(k)P_1(k\ \ 1)\varphi_1(k)]^{-1} \tag{13}$$

$$P_1(k) = [\,I - L_1(k)\varphi_1^T(k)]P_1(k-1) \tag{14}$$

$$\theta_2(k) = \theta_2(k-1) + L_2(k)\,[\,y_2(k) - \varphi_2^T(k)\ \theta_2(k-1)] \tag{15}$$

$$L_2(k) = P_2(k-1)\,\varphi_2(k)[\,1 + \varphi_2^T(k)P_2(k\ \ 1)\varphi_2(k)]^{-1} \tag{16}$$

$$P_2(k) = [\,I - L_2(k)\varphi_2^T(k)]P_2(k-1) \tag{17}$$

$$\theta_3(k) = \theta_3(k-1) + L_3(k)[\,y_3(k) - \varphi_3^T(k)\ \theta_3(k-1)] \tag{18}$$

$$L_3(k) = P_3(k-1)\,\varphi_3(k)[\,1 + \varphi_3^T(k)P_3(k-1)\varphi_3(k)]^{-1} \tag{19}$$

$$P_3(k) = [\,I - L_3(k)\varphi_3^T(k)]P_3(k-1) \tag{20}$$

$$\theta_4(k) = \theta_4(k-1) + L_4(k)\,[\,y_4(k) - \varphi_4^T(k)\ \theta_4 \tag{21}$$

$$L_4(k) = P_4(k-1)\,\varphi_4(k)\,[\,1 + \varphi_4^T(k)P_4(k-1)\varphi_4(k)]^{-1} \tag{22}$$

$$P_4(k) = [\,I - L_4(k)\varphi_4^T(k)]P_4(k-1) \tag{23}$$

The following RLS algorithm is derived by substituting equations (6), (9) into (12), (15), (18), and (21)

$$\theta_1(k) = \theta_1(k-1) + L_1(k)[\,y(k) - \varphi_2^T(k)\theta_2 - \varphi_3^T(k)\theta_3 - \varphi_4^T(k)\theta_4 - \varphi_1^T(k)\ \theta_1(k-1)] \tag{24}$$

$$\theta_2(k) = \theta_2(k-1) + L_2(k)[\,y(k) - \varphi_1^T(k)\theta_1 - \varphi_3^T(k)\theta_3 - \varphi_4^T(k)\theta_4 - \varphi_2^T(k)\ \theta_2(k-1)] \tag{25}$$

$$\theta_3(k) = \theta_3(k-1) + L_3(k)[y(k) - \varphi_1^T(k)\theta_1 - \varphi_2^T(k)\theta_2 - \varphi_4^T(k)\theta_4 - \varphi_3^T(k)\ \theta_3(k-1)] \tag{26}$$

$$\theta_4(k) = \theta_4(k-1) + L_4(k)[\,y(k) - \varphi_1^T(k)\theta_1 - \varphi_2^T(k)\theta_2 - \varphi_3^T(k)\theta_3 - \varphi_4^T(k)\ \theta_4(k-1)] \tag{27}$$

Equations (24)-(27) contain the vectors of unknown parameters $\theta_1, \theta_2, \theta_3, \theta_4$ and hence the vectors of unknown parameters $\theta_1, \theta_2, \theta_3, \theta_4$ must be replaced by their corresponding estimates $\theta(k-1)$. We have

$$\theta_1(k) = \theta_1(k-1) + L_1(k)[\,y(k) - \varphi_2^T(k)\theta_2(k-1) - \varphi_3^T(k)\theta_3(k-1) - \varphi_4^T(k)\theta_4(k-1)\varphi_1^T(k)\ \theta_1(k-1)] \tag{28}$$

$$\theta_2(k) = \theta_2(k-1) + L_2(k)[\,y(k) - \varphi_1^T(k)\theta_1(k-1) - \varphi_3^T(k)\theta_3(k-1) - \varphi_4^T(k)\theta_4(k-1)\varphi_2^T(k)\ \theta_2(k-1)] \tag{29}$$

$$\theta_3(k) = \theta_3(k-1) + L_3(k)[\,y(k) - \varphi_1^T(k)\theta_1(k-1) - \varphi_2^T(k)\theta_2(k-1) - \varphi_4^T(k)\theta_4(k-1)\varphi_3^T(k)\ \theta_3(k-1)] \tag{30}$$

$$\theta_4(k) = \theta_4(k-1) + L_4(k)[\,y(k) - \varphi_1^T(k)\theta_1(k-1) - \varphi_2^T(k)\theta_2(k-1) - \varphi_3^T(k)\theta_3(k-1)\varphi_4^T(k)\ \theta_4(k-1)] \tag{31}$$

A challenge arises because the information vectors $\varphi_1(k), \varphi_3(k) and\ \varphi_4(k)$ include the unknown inner variables $(k-i), w(k-i)$, and the noise terms $v(k-i)$, making it impossible to estimate $\theta_1(k), \theta_2(k), \theta_3(k)$, and$\theta_4(k)$.The proposed method is based on the auxiliary model identification approach. Let $\hat{x}(k)$, $\hat{w}(k)$ and $\hat{v}(k)$represent the estimates of $x(k), w(k), and\ v(k)$, respectively, and define:

$$\hat{\Phi} = [\hat{\varphi}_1, \varphi_2, \hat{\varphi}_3, \hat{\varphi}_4]^T$$

ISSN: 2961-6611

$$\hat{\varphi}_1\,(\,) = \;[-\hat{x}(k-1), \ldots - \hat{x}(k-n_a)]^T \;\epsilon\; R^{na} \tag{32}$$

$$\varphi_2\,(\,) = [u(k-1), \ldots, u(k-n_b)]^T \;\epsilon\; R^{nb} \tag{33}$$

$$\hat{\varphi}_3\,(\,) = [-\hat{w}(k-1), \ldots, -\hat{w}(k-n_c)]^T R^{nc} \tag{34}$$

$$\hat{\varphi}_4\,(k) = \;[\hat{v}(k-1), \ldots, \hat{v}(k-n_d)]^T \epsilon\; R^{nd} \tag{35}$$

Based on equations (2), (4), the estimates $x\hat{}(k), w\hat{}(k), and\ v\hat{}(k)$ can be determined using the following equations:

$$\hat{x}(k) = \hat{\varphi}_1^{\,T}\,\theta_1 + \;\hat{\varphi}_1^{\,T}\,\theta_2 \tag{36}$$

$$\hat{w}(k) = y(k) - \hat{x}(k) \tag{39}$$

$$\hat{v}(k) = \hat{w}(k) - \hat{\varphi}_3^{\,T}\,\theta_3 - \;\hat{\varphi}_4^{\,T}\,\theta_4 \tag{40}$$

By substituting $\varphi_1(k), \varphi_3(k)$ and $\varphi_4(k)$ in (28)–(31) with their corresponding estimates $\hat{\varphi}_1$, $\hat{\varphi}_3$, and $\hat{\varphi}_4$ we can outline the multi-stage RLS identification algorithm for estimating the parameter vectors θ_1, θ_2 , θ_3, and θ_4 of the BJ models as follows [11]:

$$\theta_1\,(k) = \theta_1\,(k-1) + L_1(k)[\,y(k) - \varphi_2^{\,T}(k)\theta_2\,(k-1) - \hat{\varphi}_3^{\,T}(k)\theta_3\,(k-1) - \hat{\varphi}_4^{\,T}(k)\theta_4\,(k-1) - \hat{\varphi}_1^{\,T}(k)\,\theta_1\,(k-1)] \tag{41}$$

$$L_1(k) = P_1(k\,-\,1)\,\hat{\varphi}_1\,(k)[\,1 + \;\hat{\varphi}_1^T(k)P_1(k\,-\,1)\hat{\varphi}_1\,(k)]^{-1} \tag{42}$$

$$P_1(k) = [\,I - \;L_1(k)\hat{\varphi}_1^T(k)]P_1(k\,-\,1), P\,(0) = \alpha\, I \tag{43}$$

$$\theta_2\,(k) = \theta_2(k-1) + L_2(k)[\,y(k) - \hat{\varphi}_1^{\,T}(k)\theta_1\,(k-1) - \hat{\varphi}_3^{\,T}(k)\,\theta_3\,(k-1) - \hat{\varphi}_4^{\,T}(k)\theta_4\,(k-1) - \varphi_2^{\,T}\,(k)\,\theta_2\,(k-1)] \tag{44}$$

$$L_2(k) = P_2(k\,-\,1)\,\hat{\varphi}_2\,(k)[\,1 + \;\varphi_2^T(k)P_2(k\,-\,1)\varphi_2\,(k)]^{-1} \tag{45}$$

$$P_2(k) \;= [\,I - \;L_2(k)\varphi_2^T(k)]P_2(k\,-\,1)\;P\,(0) = \alpha\, I \tag{46}$$

$$\hat{\varphi}_1\,(\,) = \;[-\hat{x}(k-1), \ldots - \hat{x}(k-n_a)]^T \;\epsilon\; R^{na} \tag{48}$$

$$\varphi_2\,(\,) = [-u(k-1), \ldots - u(k-n_b)]^T \;\epsilon\; R^{nb} \tag{49}$$

$$\theta_3\,(k) = \theta_3(k-1) + L_3(k)\,[y(k) - \hat{\varphi}_1^{\,T}(k)\theta_1\,(k-1) - \varphi_2^{\,T}(k)\theta_2\,(k-1) - \hat{\varphi}_4^{\,T}\,(k)\theta_4\,(k-1) - \hat{\varphi}_3^{\,T}\,(k)\,\theta_3(k-1)] \tag{50}$$

$$L_3(k) = P_3(k\,-\,1)\;\;\hat{\varphi}_3\,(k)[\,1 + \;\hat{\varphi}_3^T(k)P_3(k\,-\,1)\;\hat{\varphi}_3\,(k)]^{-1} \tag{51}$$

$$P_3(k) \;= [\,I - \;L_3(k)\hat{\varphi}_3^T(k)]P_3(k\,-\,1)\;, \qquad (0) = \alpha\, I \tag{52}$$

$$\theta_4\,(k) = \theta_4(k-1) + L_4(k)\,[\,y(k) - \hat{\varphi}_1^{\,T}(k)\theta_1\,(k-1) - \varphi_2^{\,T}(k)\theta_2\,(k-1) - \hat{\varphi}_3^{\,T}(k)\theta_3\,(k-1) - \hat{\varphi}_4^{\,T}(k)\;\;\theta_4\,(k-1)] \tag{53}$$

$$L_4(k) = P_4(k\,-\,1)\,\hat{\varphi}_4\,(k)[\,1 + \;\hat{\varphi}_4^T(k)P_4(k\,-\,1)\hat{\varphi}_4\,(k)]^{-1} \tag{54}$$

$$P_4(k) \;= [\,I - \;L_4(k)\hat{\varphi}_4^T(k)]P_4(k\,-\,1)\quad (0) = \alpha\, I \tag{55}$$

IV. MODELVALIDATION

Ensuring the accuracy and reliability of a model's predictions, model validation is the last stage of the system identification process [12]. In this study, the statistical approach of prediction error mean (PEM) is utilized to validate the mathematical model [13].

$$Prediction\ Error\ Mean\ (PEM) = \frac{1}{N}\sum_{=1}^{N}\quad e\,(k) \tag{56}$$

Where:
PE : Prediction Error Mean
N: Total number of data points

ISSN: 2961-6611

(k): Prediction error for the k-the data point

V. SIMULATION RESULTS

The suggested algorithm's effectiveness is demonstrated by considering the following Box-Jenkins system, second-order system with:

$$(k) = \frac{0.215z^{-1} + 0.635z^{-2}}{1 + 0.141z^{-1} + 0.18z^{-2}}u(k) + \frac{1 - 0.19z^{-1}}{1 + 0.63z^{-1}}n(k)$$

The input is produced as a white sequence with $m = 0$ and$\sigma^2 = 1$, while (k) is produced as a Gaussian white noise with $m = 0$ and variance $\sigma^2 = 0.20$. PEM has been used to assess the model's validation. PEM with time sequences is displayed in Fig. 2. As can be seen, PEM decrease with increasing sequence length, indicating that this approach is highly effective.

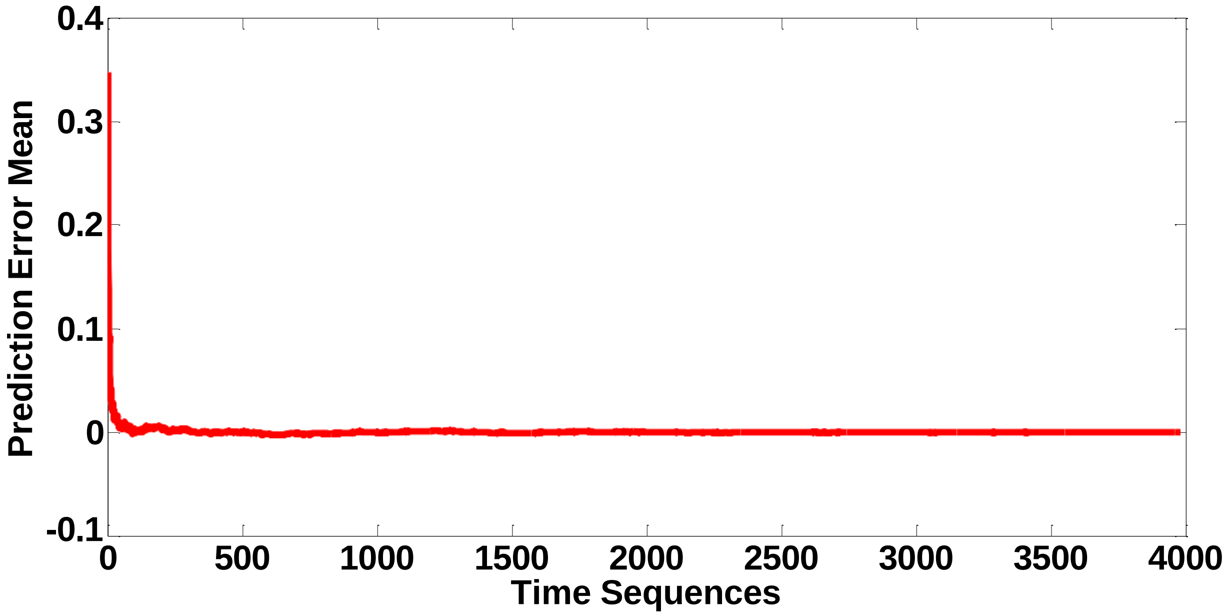

Fig. 2 Prediction Error Mean

The estimated output of this algorithm and the actual output have been plotted together with their error (Residual) as displays in Fig. 3.

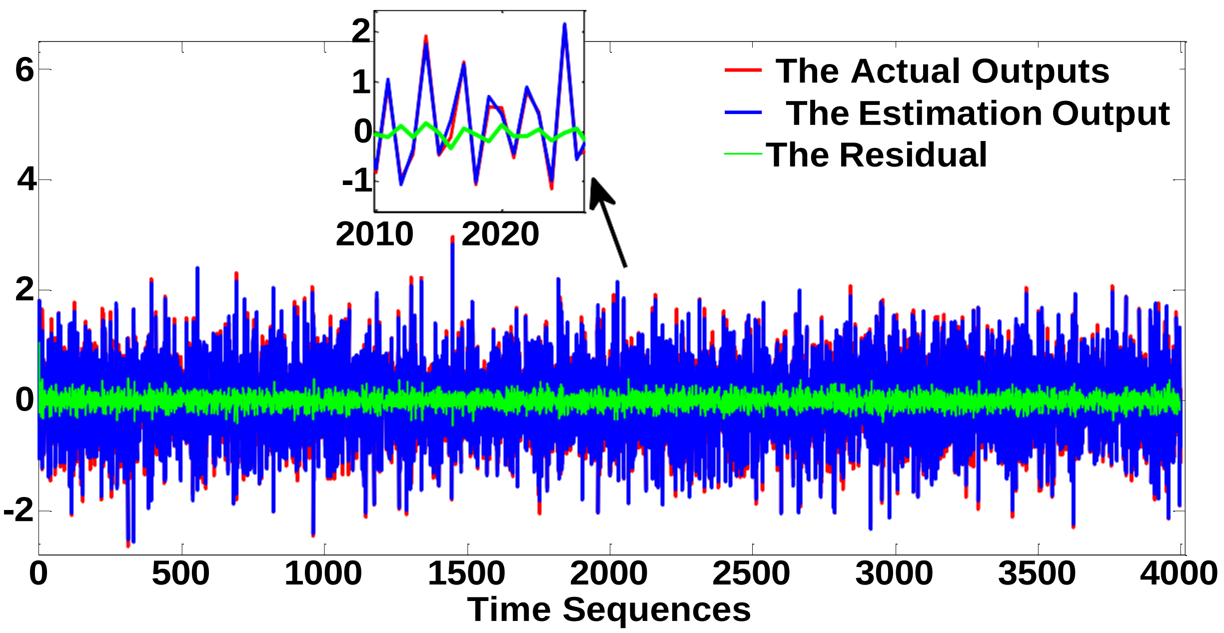

Fig. 3 The estimated output and the actual output with their error (Residual)

ISSN: 2961-6611

The figure shows that the model's efficiency is high because the estimated output closely follow the actual output and is quite near to zero. Appling the root mean square errors (RMSE) method with various noise variances, when ($\sigma^2 = 0.1^2$) and ($\sigma^2 = 0.8^2$) in order to demonstrate the performance of the algorithm. The following formula is used to calculate RMSE.

$$RMS - Error = \sqrt{\frac{\sum_{i=1}^{N}\left(\hat{}_i - y_i\right)^2}{N}} \qquad (57)$$

Fig. 4 illustrates the plotted RMSE.

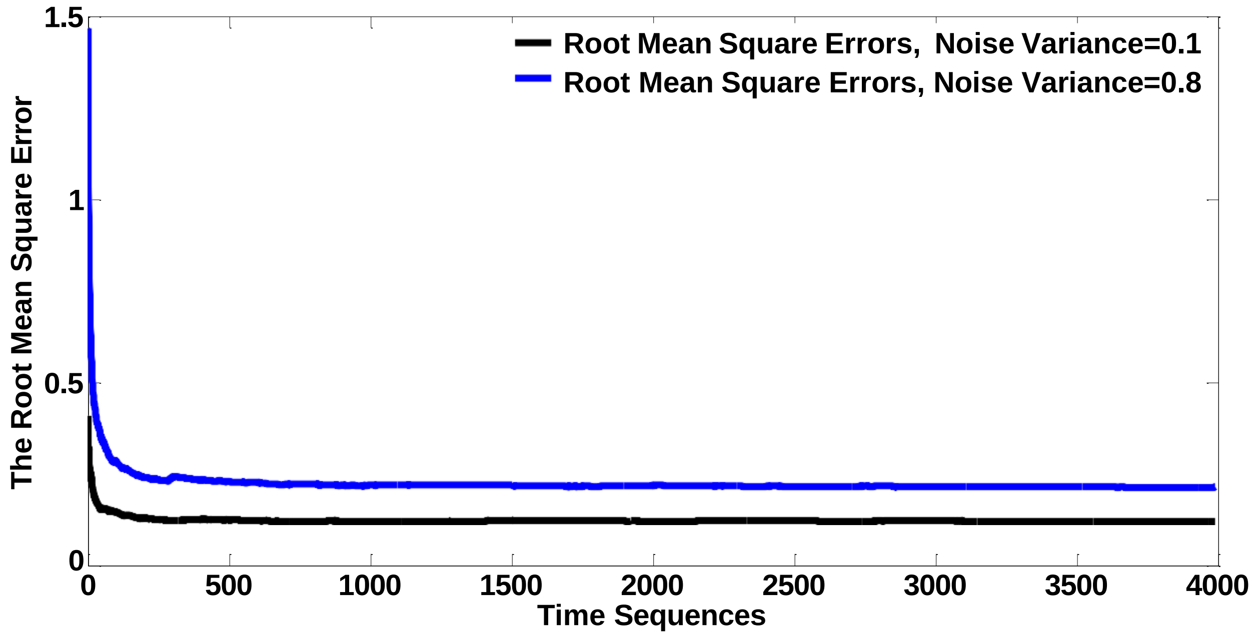

Fig. 4 The root mean square errors with different noise variance (0.1) and (0.8)

The following result has been reached: generally speaking, system performance improves when noise variance decreases.

VI. CONCLUSIONS

The main contribution of this work to multi -stage recursive least squares algorithm for Box– Jenkins Systems has been derived. Decomposing the original system into four subsystems and then interactively estimating each subsystem's parameters is the goal. The analysis indicates that the proposed MS-RLS algorithm has high performance and requires less computational load. Prediction error mean has been applied for validation of this approach and the result indicates that the proposed algorithm is effective. The proposed algorithm can be extended to other linear or nonlinear systems.

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ISSN: 2961-6611