10th International Conference on Control Engineering &Information Technology (CEIT-2025)
Proceedings Book Series –PBS- Vol 23, pp.31-43

M-DECOMPOSED RECURSIVE GENERALIZED EXTENDED LEAST SQUARES ALGORITHM For CARARMA SYSTEMS

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***Abstract*—This paper concerns the identification problem for large-scale controlled autoregressive autoregressive moving average (CARARMA) systems. Based on the decomposition technique, the M-decomposed recursive generalized extended least squares algorithm is developed to identify the parameters of large-scale (CARARMA) systems. The complication of the identification is that the noise information vector contains unknown variables. The method used to address this challenge is substituting these variables with their respective estimates, and the M-Decomposed RGELS algorithm is derived. The proposed algorithm has high computational efficiency compared to the traditional RGELS Algorithm because the covariance matrix of the main system in the traditional RGELS Algorithm is divided into M sub-covariance matrices. The proposed algorithm enhances the accuracy of parameter estimation while reducing the computational load, Finally, The simulation results demonstrate that the proposed algorithm has higher estimation accuracy under low noise variance, and model validation methods illustrates that the (SMAPE) progressively decreases, when the time sequence increases, which indicating a high level of algorithm performance.**

***Keywords*— parameter estimation, RGELS method, model Validation, large-scale system, M-decomposed.**

I. INTRODUCTION

System Identification is a process in control engineering and signal processing where mathematical models are developed to represent the behavior of a dynamic system. The dynamic system could be a physical system, a biological system, an economic process, or any system that changes over time. It relies on collecting data from the dynamic system. This data includes input signals (stimuli or control inputs) and output signals (responses or measurements). Mathematical models are developed to describe the relationship between the input and output of the dynamic system. These models could be in the form of differential equations, transfer functions, state-space representations, or other mathematical formulations [1]. The procedure entails the estimation of the parameters of a mathematical model based on the gathered input-output data. This can be accomplished through a range of techniques, including least squares estimation, recursive estimation, or optimization algorithms. [2]. System identification is an important branch in the field of modern control and is an important method to establish systematic mathematical models from the combination of observation data and prior knowledge, This methodology has found extensive application across various fields for several decades, particularly in areas such as controller design and system analysis. A key component of system identification is parameter identification, which involves estimating parameters based on measurable data. The techniques for parameter estimation are versatile and can be utilized in numerous applications [3]. The main idea of the coupling identification concept is to decompose the original system into several subsystems and to estimate the parameters based on the coupled parameter relationships between these subsystems. In the field of system modeling and control. For instance, Yao and Ding derived a Two-stage least squares-based iterative identification algorithm for controlled autoregressive autoregressive moving average (CARARMA) systems [4]. Munya and Nasar Aldian derive an extended three-stage recursive identification algorithm of MISO for (CARARMA) systems [5]. The recursive identification and iterative identification methods are basic. The recursive least squares methods are the

ISSN: 2961-6611

10th International Conference on Control Engineering &Information Technology (CEIT-2025)
Proceedings Book Series –PBS- Vol 23, pp.31-43

commonly used parameter estimation approaches among many different parameter estimation techniques Recently, the recursive least squares (RLS) algorithm is always combined with other methods to identify complex systems. Parameter estimation plays an important role in control theory and application, because a robust controller usually has the assumption that the parameters of the systems should be known prior There exist lots of identification algorithms, e.g., the least squares (LS) algorithm [6]. The least square method is widely used for parameter estimation of time domain data. The least squares (LS) approach has widespread applications in many fields, such as statistics, numerical analysis, and engineering. In this method, the least squares estimate of the coefficients is obtained by way of minimizing the sum of the squares of the error terms where this sum is called the cost function, and error or residual itself is defined as the difference between the desired output and the estimated output [7]. One of the disadvantages of offline techniques is that they cannot take into account the feedback of the changes that the parameterization produces onto the parameterization itself and cannot be used when the array does not have an inverse [8]. Recursive methods have widely been used in signal filtering, modeling dynamic systems, and solving matrix equations. Unlike the Recursive estimation is a method for estimating the parameters of a system model in real-time, where new data is constantly arriving. This is in contrast to batch estimation methods, which estimate the parameters based on a fixed set of data. In recursive estimation, the parameter estimates are updated as each new data point arrives. This allows for real-time adaptive control, where the system parameters can be estimated and updated online, and without the need to stop the system and collect a new batch of data. In adaptive controllers, the observations are obtained sequentially in real-time. It is then desirable to make the computation of the least squares estimate recursively to save the computation time. In recursive (also called on-line) identification methods, the parameter estimates are computed recursively in time. Although the LS and RLS algorithms are capable of estimating system parameters, they are constrained by their inability to estimate the parameter of the noise model and their poor computing efficiency [9]. The Recursive Generalized Extended Least Squares (RGELS) algorithm is a powerful tool for real-time, dynamic regression modeling in environments where the relationships between variables and their associated errors change over time. Incorporating recursion and extended least squares, allows for efficient and accurate updates to model parameters as new data arrives. This paper is structured as follows. In Section 2 introduces the system description and its identification model is given. In Section 3 M- decomposed RGELS for 8th order system. In Section 4 presents model validation. In Section 5 simulation results. Finally, the conclusions are given in section 6.

II. SYSTEM DESCRIPTION

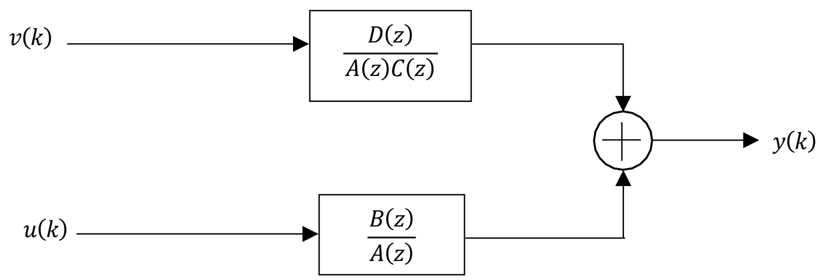
The discrete-time, linear, time-invariant system is considered as depicted in Fig. 1, explained by (CARARMA), which is represented as [10]:

$$A(z)y(k) = \sum_{i=1}^{8} B(z)u(k) + \frac{D(z)}{C(z)}v(k), \qquad (1)$$

y(k) output sequences, u(k) input sequences, and v(n) is a stochastic white noise. $A(z)$, $B(z)$, $C(z)$ and $D(z)$ are polynomials and represented as:

$$A(z) = 1 + a_1z^{-1} + a_2z^{-2} + \cdots + a_{n_a}z^{-na}, \qquad B(z) = b_1z^{-1} + b_2z^{-2} + \cdots + b_{nb}z^{-nb},$$

$$C(z) = 1 + c_1z^{-1} + c_2z^{-2} + \cdots + c_{n_c}z^{-nc}, \qquad D(z) = 1 + d_1z^{-1} + d_2z^{-2} + \cdots + d_{n_d}z^{-nd},$$



ISSN: 2961-6611

Fig. 1. The system described SISO models

The parameter vectors θ are defined as:

$$\theta = [\theta_s^T \quad \theta_n^T], \qquad \varphi_{(n)} = [\varphi_s^T(n) \quad \varphi_n^T(n)], \qquad \theta_s = [a_1\ a_2 \dots a_{n_a}, b_1\ b_2 \dots b_{nb}]^T,$$

$$\varphi_s^T(k) = [-y(k-1), -y(k-2), \dots - y(k-n_a), u(k-1), u(k-2), \dots u(k-n_b)],$$

This equation can be written as:

$$y_s(k) = \varphi_s^T(k)\theta_s,$$

$$y_s(k) = -(a_1 y(k-1) + a_2 y(k-2) + \dots + a_{n_a} y(k-n_a)) + b_1 u(k-1) + b_2 u(k-2) + \dots + b_{nb} u(k-n_b), \tag{2}$$

The noise identification model is:

$$w(k) = \frac{D(z)}{C(z)} v(k), \tag{3}$$

And therefore

$$w(k) = (1 - C(z))w(k) + D(z)v(k), \tag{4}$$

$$\theta_n = [c_1\ c_2 \dots c_{n_w}, d_1\ d_2 \dots d_{n_d}]^T,$$

$$\varphi_n^T(k) = [-w(k-1), -w(k-2), \dots - w(k-n_c), v(k-1), v(k-2), \dots v(k-n_d)]^T$$

And it can be written in a linear regression form as [11]:

$$w(k) = \varphi_n^T(k)\theta_n + v(k), \tag{5}$$

$$w(k) = -c_1 w(k-1) - c_2 w(k-2) \dots - c_{n_w} w(k-n_c) + d_1 v(k-1) + d_2 v(k-2) + \dots + d_{n_d} v(k-n_d) + v(k), \tag{6}$$

Finally, y(k) Totlal can be written as

$$y(k) = \varphi_s^T(k)\theta_s + \varphi_n^T(k)\theta_n + v(k), \tag{7}$$

$$y(k) = \varphi^T(k)\theta + v(k), \tag{8}$$

III. M- Decomposed RGELS For 8th Order System

In this section, M-decomposed has been develop based Recursive Generalized Extended Least Squares (M-RGELS) algorithm which can fully use the new arrived data.

$$m = \frac{\text{number of total parameters}}{M}, \tag{9}$$

number of total parameters = 16
M=4 number of Matrix
m=4 number of parameters

From Fig. 1. It can be described in a linear regression form as

$$y(k) = \varphi_1^T(k)\theta_1 + \varphi_2^T(k)\theta_2 + \varphi_3^T(k)\theta_3 + \varphi_4^T(k)\theta_4 + w(k), \tag{10}$$

$$\theta_1 = [a_1, \dots, a_m]^T, \qquad \theta_2 = [a_{(n_a-m+1)}, \dots, a_{na}]^T,$$

ISSN: 2961-6611

10th International Conference on Control Engineering &Information Technology (CEIT-2025)
Proceedings Book Series –PBS- Vol 23, pp.31-43

$$\theta_3 = [b_1, \dots, b_m]^T, \qquad \theta_4 = [b_{(n_b-m+1)}, \dots, b_{n_b}]^T,$$

$$\varphi_1(k) = [-y(k-1), -y(k-2), \dots, -y(k-m)]^T, \qquad \varphi_2(k) = [-y(k-(n_a-m+1)), \dots, -y(k-n_a)]^T,$$

$$\varphi_3(k) = [u(k-1), u(k-2), \dots, u(k-m)]^T, \qquad \varphi_4(k) = [u(k-(n_b-m+1)), \dots, u(k-n_b)]^T,$$

Suppose n_c=2 and $n_d = 2$

$$w(k) = -c_1 w(k-1) - c_2 w(k-2) + d_1 v(k-1) + d_2 v(k-2) + v(k), \tag{11}$$

And it can be written in a linear regression form as

$$w(k) = \varphi_5^T(k)\theta_5 + v(k), \tag{12}$$

Where

$$\varphi_5^T(k) = [-w(k-1), -w(k-2), v(k-1), v(k-2)]^T, \qquad \theta_5 = [c_1\ c_2, d_1\ d_2]^T,$$

Finally, y(k) can be written as [8] :

$$y(k) = \varphi_1^T(k)\theta_1 + \varphi_2^T(k)\theta_2 + \cdots + \varphi_M^T(k)\theta_M + \varphi_5^T(k)\theta_5 + v(k), \tag{13}$$

$$y_1 = -y(k-1), -y(k-2), \dots, -y(k-m).\begin{bmatrix} a_1 \\ \vdots \\ a_m \end{bmatrix},$$

$$y_2 = -y(k-(n_a-m+1)), \dots, -y(k-n_a).\begin{bmatrix} a_{(n_a-m+1)} \\ \vdots \\ a_{n_a} \end{bmatrix},$$

$$y_3 = u_1(k-1), u_1(k-2), \dots, u_1(k-m).\begin{bmatrix} b_1 \\ \vdots \\ b_m \end{bmatrix},$$

$$y_4 = u_1(k-(n_b-m+1)), \dots, u_1(k-n_b).\begin{bmatrix} b_{(n_b-m+1)} \\ \vdots \\ b_{nb} \end{bmatrix},$$

$$y_1(k) = y(k) - \varphi_2^T(k)\theta_2 - \cdots - \varphi_{M+1}^T(k)\theta_{M+1}, \tag{14}$$

$$y_2(k) = y(k) - \varphi_1^T(k)\theta_1 - \varphi_3^T(k)\theta_3 - \cdots - \varphi_{M+1}^T(k)\theta_{M+1}, \tag{15}$$

$$y_3(k) = y(k) - \varphi_1^T(k)\theta_1 - \varphi_2^T(k)\theta_2 - \varphi_4^T(k)\theta_4 - \cdots - \varphi_{M+1}^T(k)\theta_{M+1}, \tag{16}$$

$$y_4(k) = y(k) - \varphi_1^T(k)\theta_1 - \varphi_2^T(k)\theta_2 - \varphi_3^T(k)\theta_3 - \varphi_{M+1}^T(k)\theta_{M+1}, \tag{17}$$

$$y_5(k) = y(k) - \varphi_1^T(k)\theta_1 - \varphi_2^T(k)\theta_2 - \varphi_3^T(k)\theta_3 - \varphi_4^T(k)\theta_4, \tag{18}$$

From (14) – (18), equation (13) can be decomposed into five M- Decomposed identification models,

$$y_i(k) = \varphi_i^T(k)\theta_i + v(k), \qquad i = 1,2,3,4,5 \tag{19}$$

These includes the parameters vectors $\theta_1, \theta_2, \theta_3, \theta_4$ and θ_5 Then, five criterion functions are defined as,

$$J_i(\theta_i) := \sum_{j=1}^{k} [y_i(j) - \varphi_i^T(\theta_i)]^2, \qquad i = 1,2,3,4,5,$$

ISSN: 2961-6611

Let the partial derivation of $J_i(\theta_i), i = 1,2,3,4,5$ with respect to θ_i be zero

$$\left.\frac{\partial J_i(\theta_i)}{\partial \theta_i}\right|_{\theta_i=\hat{\theta}_i(k)} = -2\varphi_i(j)\sum_{j=1}^{k}\left[y_i(j) - \varphi_i^T(j)\hat{\theta}_i(k)\right] = 0, \; i = 1,2,3,4,5\,, \tag{20}$$

Let $\hat{\theta}(k) = [\hat{\theta}_1(k)\ \hat{\theta}_2(k)\ \hat{\theta}_3(k)\ \hat{\theta}_4(k)\ \hat{\theta}_5(k)]^T \in \mathbb{R}^n$ be the estimate of $\theta = [\theta_1\ \theta_2\ \theta_3\ \theta_4\ \theta_5\ ]^T \in \mathbb{R}^n$

Minimizing the criterion functions and as a result, RGELS algorithm can be obtained for computing $\hat{\theta}_i(k)$:

$$\hat{\theta}_i = \hat{\theta}_i(k-1) + L_i(k)[y_i(k) - \varphi_i^T(k)\hat{\theta}_i(k-1)], \tag{21}$$

$$L_i(k) = p_i(k-1)\varphi_i(k)[1 + \varphi_i^T(k)P_i(k-1)\varphi_i(k)]^{-1},$$

$$p_i(k) = [I - L_i(k)\varphi_i^T(k)]p_i(k-1), \qquad P_i(0) = p_0 I,\ i = 1,2,3,4,5,$$

The equations from (14)-(18) are substituted into equation (21) with $i = 1,2,3,4,5$, obtains

$$\hat{\theta}_1(k) = \hat{\theta}_1(k-1) + L_1(k)[y(k) - \varphi_2^T(k)\theta_2 - \varphi_3^T(k)\theta_3 - \varphi_4^T(k)\theta_4 - \varphi_5^T(k)\theta_5 - \varphi_1^T(k)\hat{\theta}_1(k-1)], \tag{22}$$

$$\hat{\theta}_2(k) = \hat{\theta}_2(k-1) + L_2(k)\ [y(k) - \varphi_1^T(k)\theta_1 - \varphi_3^T(k)\theta_3 - \varphi_4^T(k)\theta_4 - \varphi_5^T(k)\theta_5 - \varphi_2^T(k)\hat{\theta}_2(k-1)], \tag{23}$$

$$\hat{\theta}_3(k) = \hat{\theta}_3(k-1) + L_3(k)\ [y(k) - \varphi_1^T(k)\theta_1 - \varphi_2^T(k)\theta_2 - \varphi_4^T(k)\theta_4 - \varphi_5^T(k)\theta_5 - \varphi_3^T(k)\hat{\theta}_3(k-1)], \tag{24}$$

$$\hat{\theta}_4(k) = \hat{\theta}_4(k-1) + L_4(k)\ [y(k) - \varphi_1^T(k)\theta_1 - \varphi_2^T(k)\theta_2 - \varphi_3^T(k)\theta_3 - \varphi_5^T(k)\theta_5 - \varphi_4^T(k)\hat{\theta}_4(k-1)], \tag{25}$$

$$\hat{\theta}_5(k) = \hat{\theta}_5(k-1) + L_5(k)\ [y(k) - \varphi_1^T(k)\theta_1 - \varphi_2^T(k)\theta_2 - \varphi_3^T(k)\theta_3 - \varphi_4^T(k)\theta_4(k) - \varphi_5^T(k)\hat{\theta}_5(k-1)] \tag{26}$$

Equations (22) – (26) include the unknown parameter $\theta_i\ i = 1,2,3,4,5$. The Replacement of the unknown θ_i in (22)–(26) with their estimates $\hat{\theta}_i(k-1)$ is the solution:

$$\begin{aligned}\hat{\theta}_1(k) &= \hat{\theta}_1(k-1) + L_1(k)\ [y(k) - \varphi_2^T(k)\hat{\theta}_2(k-1) - \varphi_3^T(k)\hat{\theta}_3(k-1) - \varphi_4^T(k)\hat{\theta}_4(k-1) - \\ &\quad \varphi_5^T(k)\hat{\theta}_5(k-1) - \varphi_1^T(k)\hat{\theta}_1(k-1)], \\ &= \hat{\theta}_1(k-1) + L_1(k)\ [y(k) - \varphi^T(k)\hat{\theta}(k-1)],\end{aligned}$$

$$\begin{aligned}\hat{\theta}_2(k) &= \hat{\theta}_2(k-1) + L_2(k)\ [y(k) - \varphi_1^T(k)\hat{\theta}_1(k-1) - \varphi_3^T(k)\hat{\theta}_3(k-1) - \varphi_4^T(k)\hat{\theta}_4(k-1) - \\ &\quad \varphi_5^T(k)\hat{\theta}_5(k-1) - \varphi_2^T(k)\hat{\theta}_2(k-1)] \\ &= \hat{\theta}_2(k-1) + L_2(k)\ [y(k) - \varphi^T(k)\hat{\theta}(k-1)],\end{aligned}$$

$$\begin{aligned}\hat{\theta}_3(k) &= \hat{\theta}_3(k-1) + L_3(k)\ [y(k) - \varphi_1^T(k)\hat{\theta}_1(k-1) - \varphi_2^T(k)\hat{\theta}_2(k-1) - \varphi_4^T(k)\hat{\theta}_4(k-1) - \\ &\quad \varphi_5^T(k)\hat{\theta}_5(k-1) - \varphi_3^T(k)\hat{\theta}_3(k-1)] \\ &= \hat{\theta}_3(k-1) + L_3(k)\ [y(k) - \varphi^T(k)\hat{\theta}(k-1)],\end{aligned}$$

$$\begin{aligned}\hat{\theta}_4(k) &= \hat{\theta}_4(k-1) + L_4(k)\ [y(k) - \varphi_1^T(k)\hat{\theta}_1(k-1) - \varphi_2^T(k)\hat{\theta}_2(k-1) - \varphi_3^T(k)\hat{\theta}_3(k-1) - \\ &\quad \varphi_5^T(k)\hat{\theta}_5(k-1) - \varphi_4^T(k)\hat{\theta}_4(k-1)] \\ &= \hat{\theta}_4(k-1) + L_4(k)\ [y(k) - \varphi^T(k)\hat{\theta}(k-1)],\end{aligned}$$

$$\begin{aligned}\hat{\theta}_5(k) &= \hat{\theta}_5(k-1) + L_5(k)\ [y(k) - \varphi_1^T(k)\hat{\theta}_1(k-1) - \varphi_2^T(k)\hat{\theta}_2(k-1) - \varphi_3^T(k)\hat{\theta}_3(k-1) - \\ &\quad \varphi_4^T(k)\hat{\theta}_4(k-1) - \varphi_5^T(k)\hat{\theta}_5(k-1)] \\ &= \hat{\theta}_5(k-1) + L_5(k)\ [y(k) - \varphi^T(k)\hat{\theta}(k-1)],\end{aligned}$$

ISSN: 2961-6611

10th International Conference on Control Engineering &Information Technology (CEIT-2025)
Proceedings Book Series –PBS- Vol 23, pp.31-43

$\varphi_5(k)$ includes the unknown noise terms $w(k-i)$ and $v(k-i)$, and thus these estimated parameters cannot be generated based on the above algorithms in (22)–(26). The solution is to Replacing $w(k-i)$ and $v(k-i)$ with their estimates $\hat{w}(k-i)$ and $\hat{v}(k-i)$, and obtain

$$\hat{\varphi}_5(k) = [-\hat{w}(k-1), -\hat{w}(k-2), \hat{v}(k-1), \hat{v}(k-2)]^T, \qquad \hat{\varphi}(k) = \begin{bmatrix} \varphi_1 \\ \varphi_2 \\ \varphi_3 \\ \varphi_4 \\ \hat{\varphi}_5 \end{bmatrix} \in \mathbb{R}^n,$$

Now, equation (10) can be written as,

$$w(k) = y(k) - \varphi_1^T(k)\theta_1 - \varphi_2^T(k)\theta_2 - \varphi_3^T(k)\theta_3 - \varphi_4^T(k)\theta_4, \qquad v(k) = w(k) - \varphi_5^T(k)\theta_5,$$

The estimated $w(k)$ and $v(k)$ can be computed by

$$\hat{w}(k) = y(k) - \varphi_1^T(k)\hat{\theta}_1 - \varphi_2^T(k)\hat{\theta}_2 - \varphi_3^T(k)\hat{\theta}_3 - \varphi_4^T(k)\hat{\theta}_4, \qquad \hat{v}(k) = \hat{w}(k) - \hat{\varphi}_5(k)\hat{\theta}_5,$$

Thus, M-Decomposed Recursive Generalized Extended parameter estimation algorithm for computing the estimated parameters and of CARARMA model is obtained as

$$\hat{\theta}_1(k) = \hat{\theta}_1(k-1) + L_1(k)[y(k) - \hat{\varphi}^T(k)\hat{\theta}(k-1)], \tag{27}$$

$$L_1(k) = p_1(k-1)\varphi_1(k)[1 + \varphi_1^T(k)P_1(k-1)\varphi_1(k)]^{-1}, \tag{28}$$

$$P_1(k) = [I - L_1(k)\varphi_1^T(k)]P_1(k-1), p_1(0) = P_0 I_{n_a}, \tag{29}$$

$$\varphi_1(k) = [-y(k-1), -y(k-2), \dots, -y(k-m)]^T \in \mathbb{R}^{na}, \tag{30}$$

$$\theta_2(k) = \theta_2(k-1) + L_2(k)[y(k) - \hat{\varphi}^T(k)\theta(k-1)], \tag{31}$$

$$L_2(k) = p_2(k-1)\varphi_2(k)[1 + \varphi_2^T(k)P_2(k-1)\varphi_2(k)]^{-1}, \tag{32}$$

$$P_2(k) = [I - L_2(k)\varphi_2^T(k)]P_2(k-1), p_2(0) = P_0 I_{n_a}, \tag{33}$$

$$\varphi_2(k) = -y(n - (n_a - m + 1)), \dots, -y(n - n_a)]^T \in \mathbb{R}^{na}, \tag{34}$$

$$\theta_3(k) = \theta_3(k-1) + L_3(k)[y(k) - \hat{\varphi}^T(k)\theta(k-1)], \tag{35}$$

$$L_3(k) = p_3(k-1)\varphi_3(k)[1 + \varphi_3^T(k)P_3(k-1)\varphi_3(k)]^{-1}, \tag{36}$$

$$P_3(k) = [I - L_3(k)\varphi_3^T(k)]P_3(k-1), p_3(0) = P_0 I_{n_b}, \tag{37}$$

$$\varphi_3(n) = [u_1(n-1), u_1(n-2), \dots, u_1(n-m)]^T \in \mathbb{R}^{nb}, \tag{38}$$

$$\theta_4(k) = \theta_4(k-1) + L_4(k)[y(k) - \hat{\varphi}^T(k)\theta(k-1)], \tag{39}$$

$$L_4(k) = p_4(k-1)\varphi_4(k)[1 + \varphi_4^T(k)P_4(k-1)\varphi_4(k)]^{-1}, \tag{40}$$

$$P_4(k) = [I - L_4(k)\varphi_4^T(k)]P_4(k-1), p_4(0) = P_0 I_{n_b}, \tag{41}$$

$$\varphi_4(n) = [u_1(n - (n_b - m + 1)), \dots, u_1(n - n_b)]^T \in \mathbb{R}^{nb}, \tag{42}$$

$$\theta_5(k) = \theta_5(k-1) + L_5(k)[y(k) - \hat{\varphi}^T(k)\theta(k-1)], \tag{43}$$

$$L_5(k) = p_5(k-1)\hat{\varphi}_5(k)[1 + \hat{\varphi}_5^T(k)P_5(k-1)\hat{\varphi}_5(k)]^{-1}, \tag{44}$$

ISSN: 2961-6611

$$P_5(k) = [I - L_5(k)\hat{\varphi}_5(k)]P_5(k-1), p_5(0) = P_0 I_{n_{c+n}\ d} \quad (45)$$

$$\hat{w}(k) = y(k) - \varphi_1^T(k)\theta_1 - \varphi_2^T(k)\theta_2 - \varphi_3^T(k)\theta_3 - \varphi_4^T(k)\hat{\theta}_4, \quad (46)$$

$$\hat{v}(k) = \hat{w}(k) - \hat{\varphi}_5^T(k)\hat{\theta}_5, \quad (47)$$

$$\hat{\varphi}_5(k) = \begin{bmatrix} -\hat{w}(k-1), -\hat{w}(k-2) \\ \hat{v}(k-1), \hat{v}(k-2) \end{bmatrix}^T, \quad (48)$$

$$\hat{\varphi}(k) = [\hat{\varphi}_1^T(k), \hat{\varphi}_2^T(k), \hat{\varphi}_3^T(k), \hat{\varphi}_4^T(k), \hat{\varphi}_5^T(k)]^T \in \mathbb{R}^n, \quad (49)$$

$$\hat{\theta}(k) = [\hat{\theta}_1^T(k), \hat{\theta}_2^T(k), \hat{\theta}_3^T(k), \hat{\theta}_4^T(k), \hat{\theta}_5^T(k)]^T \in \mathbb{R}^n, \quad (50)$$

IV. MODEL VALIDATION

Model validation is a crucial aspect of simulation model development. In this section, the effectiveness of the proposed algorithm will be proved using Symmetric Mean Absolute Percentage Error (SMAPE) is selected. Generally, SMAPE is commonly used to validate the model by comparing the predicted values to the actual values [12].

It is defined as

$$\text{SMAPE} = \frac{1}{S}\sum_{n=1}^{S} \frac{|y(n) - \hat{y}(n)|}{(|y(n)| + |\hat{y}(n)|)/2}, \quad (51)$$

where n represents the time step, y(n) represents the target output, $\hat{y}(n)$ represents the predicted output, and S represents the number of samples. SMAPE can minimize the impact of data size and units, allowing it to reflect the relative magnitude of errors effectively. A higher SMAPE value indicates a greater discrepancy between target and predicted values, signifying poorer predictive performance.

V. SIMULATION RESULTS

The effectiveness of the suggested algorithm is demonstrated by considering the following eight-order system:

$$(z) = 1 + 0.192z^{-1} + 0.15z^{-2} + 0.032z^{-3} + 0.20\,z^{-4} + 0.12z^{-5} + 0.021z^{-6} + 0.10z^{-7} + 0.14z^{-8}$$

$$(z) = 0.008z^{-1} + 0.13z^{-2} + 0.14z^{-3} + 0.03\,z^{-4} + 0.16z^{-5} + 0.92z^{-6} + 0.4z^{-7} + 0.43z^{-8}$$

$$(z) = 1 + 0.09z^{-1} + 0.57z^{-2} \qquad D(z) = 1 + 0.16z^{-1} + 0.27z^{-2}$$

$$\theta_1 = [0.192, 0.15, 0.032, 0.20]^T, \quad \theta_2 = [0.12, 0.021, 0.10, 0.14]^T, \quad \theta_3 = [0.008, 0.13, 0.14, 0.03]^T,$$

$$\theta_4 = [0.16, 0.92, 0.4, 0.43]^T, \qquad \theta_5 = [0.09, 0.57, 0.16, 0.27]^T,$$

u(k) is taken as a white sequence with a Gaussian distribution of $m = 0$ and $\sigma^2 = 1^2$ v(k) is produced a Gaussian white noise with $m = 0$ and $\sigma^2 = 0.1^2$. In order to assess this algorithm's effectiveness, the algorithm's predicted output is plotted together with its actual output and the residual as demonstrated in Fig. 2

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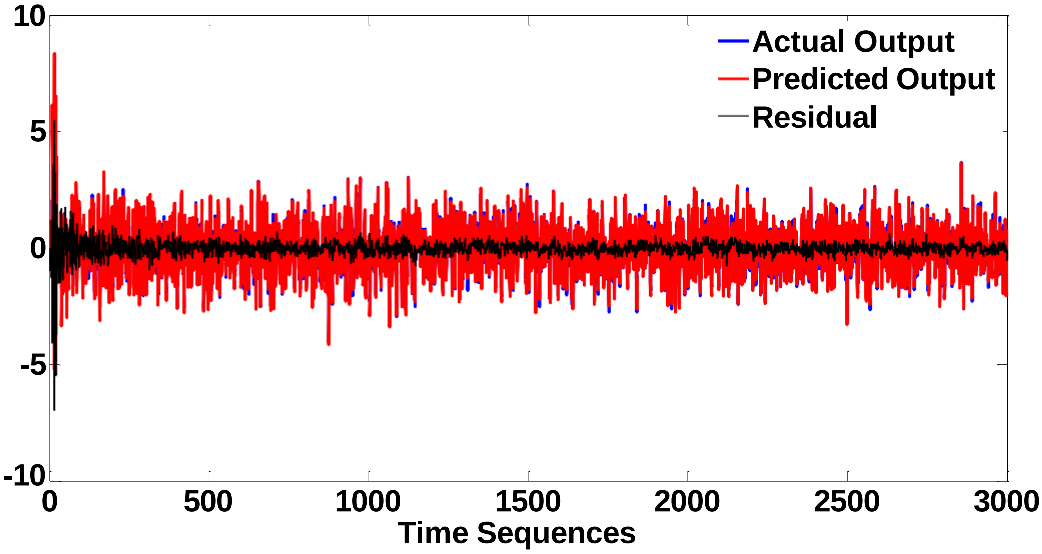

Fig. 2. Residual of SISO system using RGELS algorithm.

This figure shows that the predicted output of this suggested algorithm is very close to the actual output and the residual is very close to zero which means, the effectiveness of this algorithm is high.

For further explanation, the window between n = 850 and n = 900 has been used as provided in Fig. 3

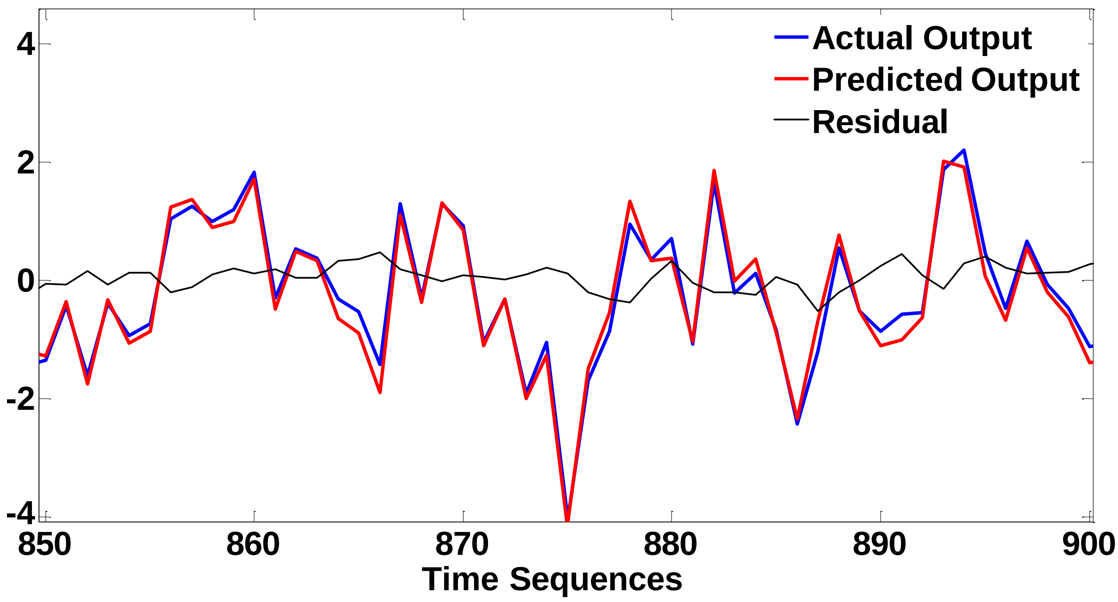

Fig. 3. Residual of SISO system using RGELS algorithm between n = 850 and n = 900.

This figure indicates Data from n =850 to n =900 has been taken to show the accuracy of the identified algorithm. The figure shows the predicted outputs follow the actual outputs. which concludes that the estimated model fits the actual data.

The estimation error is calculated using the formula below, and is demonstrated in Fig. 4 [10]:

$$\delta = \sqrt{\frac{\|\hat{\theta}_1-\theta_1\|^2+\|\hat{\theta}_2-\theta_2\|^2+\|\hat{\theta}_3-\theta_3\|^2+\|\hat{\theta}_4-\theta_4\|^2}{\|\theta_1\|^2+\|\theta_2\|^2+\|\theta_3\|^2+\|\theta_4\|^2}}, \quad (52)$$

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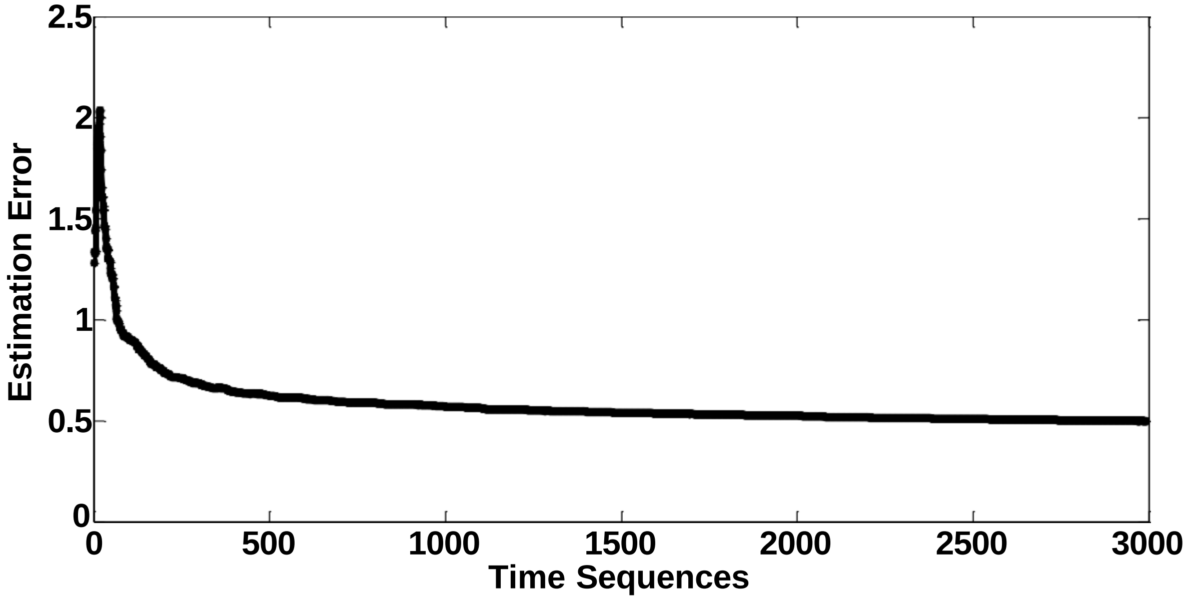

Fig. 4. The parameter estimation error of the RGELS algorithm.

The figure illustrates that the estimation errors decrease with the number of sequences increases. This demonstrates the efficacy of the suggested algorithm. Fig. 5 shows the validation of this technique utilized (SMAPE):

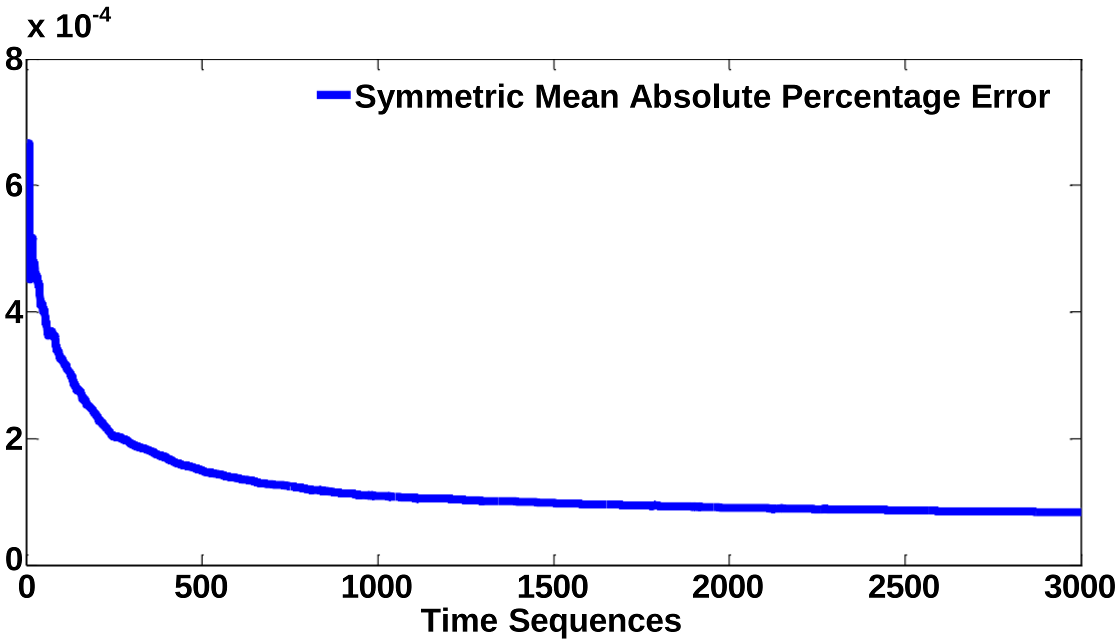

Fig. 5. Symmetric Mean Absolute Percentage Error (SMAPE) of RGELS algorithm versus the sequences.

From the figure, (SMAPE) gets smaller and smaller as the duration of the sequence increases. This suggests that the algorithm's performance is high.

Applying this algorithm with the different noise variance, when ($\sigma^2 = 0.1^2$) and ($\sigma^2 = 0.6^2$) to estimate the parameters of this system and based on this result, the parameter estimation errors have been computed and plotted as demonstrated in fig. 6. In addition to, the residuals have been plotted too as shown in Fig. 7.

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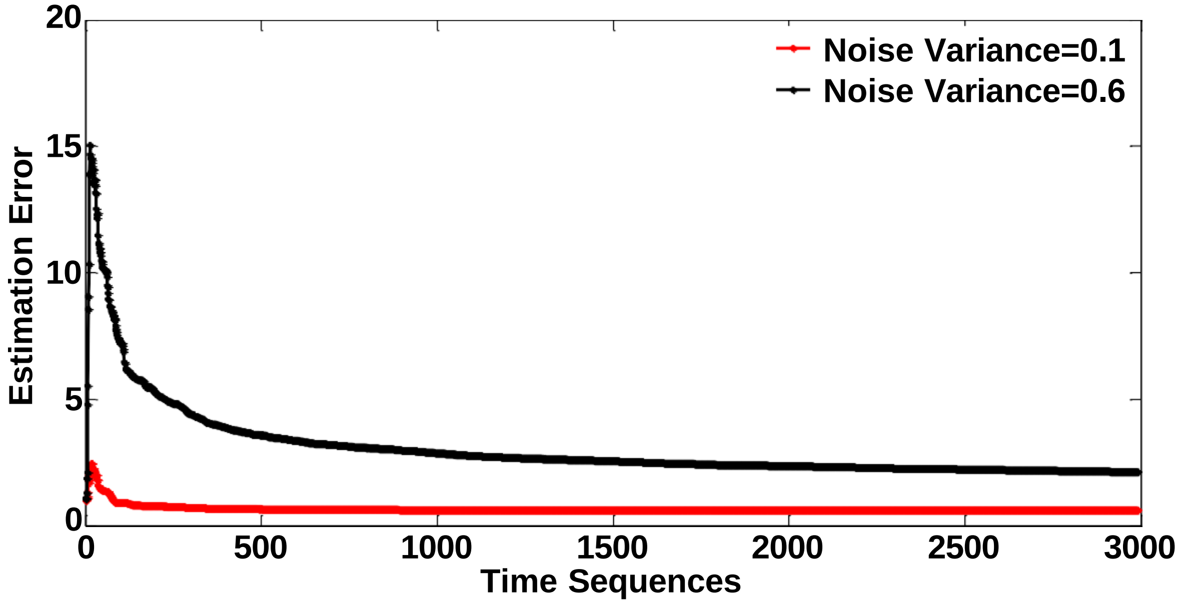

Fig. 6. RGELS algorithm with different noise (0.1) and (0.6) versus the sequences.

From this figure, the parameter estimation errors with noise variance (($\sigma^2 = 0.1^2$) is lower than the parameter estimation errors with noise variance (($\sigma^2 = 0.6^2$). Thus, the following conclusion has been drawn, in general, the parameter accuracies increase under decreasing noise variance.

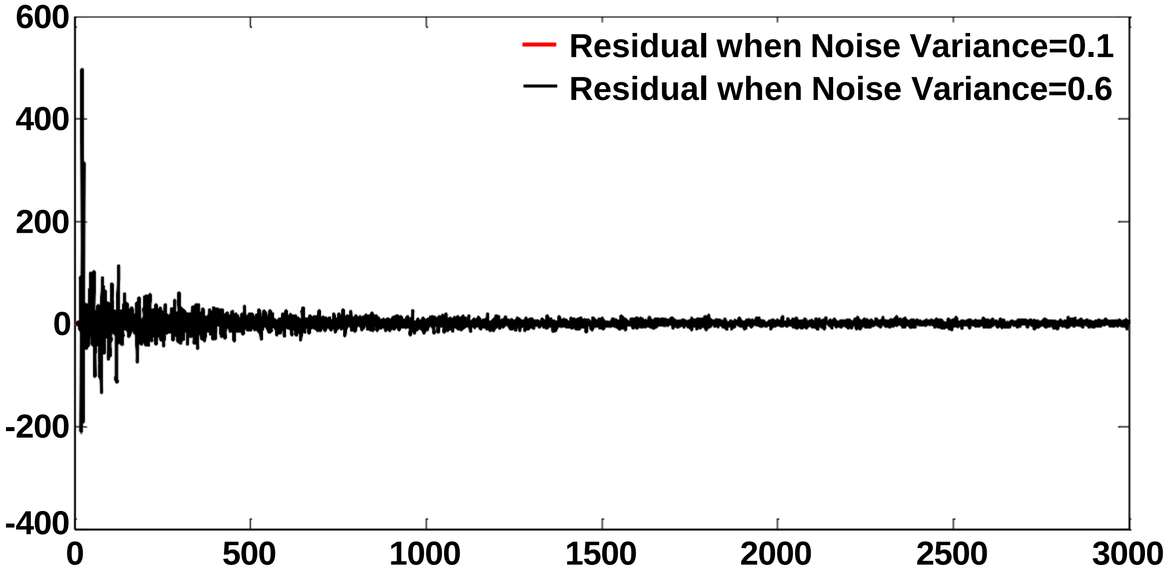

Fig. 7. Show that the noise variance (($\sigma^2 = 0.1^2$) is lower than the parameter estimation errors with noise variance (($\sigma^2 = 0.6^2$).

The window between n = 1000 and n = 1100 has been utilized for additional explanation, as seen in Fig. 8.

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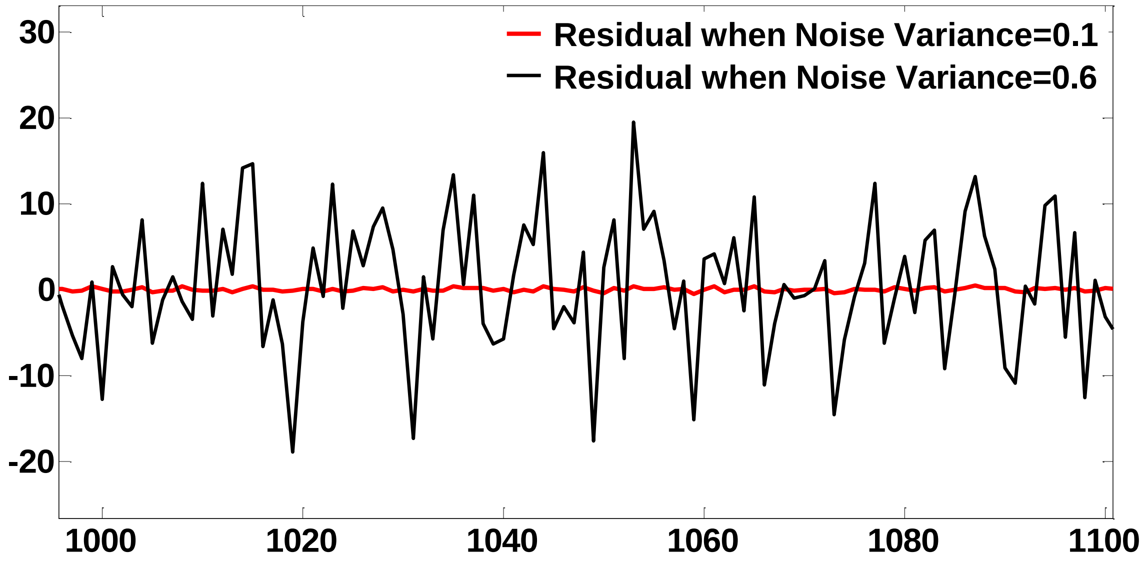

Fig. 8. Show the Residual when the noise variance (0.1^2) and (0.6^2) between n = 1000 and n = 1100.

The figure illustrates that the residual signal intensity increases with the noise variance increasing. Thus, it becomes more challenging for the proposed algorithm to accurately estimate the actual output.

VI. CONCLUSIONS

In this paper, the M-decomposed recursive generalized extended least squares algorithm has been derived for large-scale controlled autoregressive autoregressive moving average (CARARMA) systems. By using the M-Decomposed technique, the main system has been divided into M-subsystems and the dimension of the information vector considerably decreases. The main contribution of the proposed identification method has a high computation efficiency for large-scale systems compared with conventional recursive generalized extended least squares algorithm. The simulation results indicate that the proposed algorithm achieves higher estimation accuracies under lower noise variance, and the effectiveness of the proposed method can be improved by increasing the time sequences. The methods used in this paper can be extended to study the identification of other linear systems, nonlinear systems, multiple-input single-output systems, or multiple-input multiple-output systems.

REFERENCES

[1] F. Stratulat and F. Ionescu, Linear Control Systems: Analysis and Synthesis-Theory and Applications, Steinbeis-Edition, Berlin, Germany, 2009.
[2] Lennart Ljung, System Identification Theory For The User, Second Edition, prentice hall ptr Upper Saddle River, NJ 07458. 1999.
[3] H. Ma, J. Pan, L. Lv, G. Xu, F. Ding, A. Alsaedi and T. Hayat,“ Recursive Algorithms for Multivariable Output-Error-Like ARMA Systems,” Mathematics, vol. 7, pp. 558, 2019.
[4] G.Yao and R.Ding, “Two-stage least squares based iterative identification algorithm for controlled autoregressive moving average (CARMA) systems,” Computers and Mathematics with Applications, vol. 63, pp. 975–984, 2012.
[5] M. Arwini and N. Shashoa,“ Extended Three-Stage Recursive Least Squares Identification Algorithm for multiple-input single-output CARARMA Systems,” 2021 IEEE International IOT, Electronics and Mechatronics Conference (IEMTRONICS). Toronto, Canada, 2021.
[6] T. Söderström and P. Stoica, System Identification, Prentice Hall International (UK), NJ, USA: Prentice-Hall, 1989.
[7] S. Arshad, S. Qamar, T. Jabbar and A. Malik, “Parameter Estimation of a DC Motor using Ordinary Least Squares and Recursive Least Squares Algorithms,” International Conference on Frontiers of Information Technology, Pakistan 2010.
[8] Y. JI AND L. LV, “ M-Decomposed Least Squares and Recursive Least Squares Identification Algorithms for Large-Scale Systems,” IEEE Access, vol.9, PP(99),2021.

ISSN: 2961-6611

[9] N. A. A. Shashoa, I. N. Jleta, A. A. Agll, and A. S. Elmezughi, " A Naive Bayes Classifier for Fault Detection and Classification Using Dimension Reduction Technique ", 2024 5th International Conference in Electronic Engineering, Information Technology & Education (EEITE), Chania, Greece, 29-31 May 2024.

[10] N. A. A. Shashoa, I. N. Jleta, A. A. Agll, and A. S. Elmezughi, "Four-Stage Recursive Least Squares Algorithm for CARARMA Systems", In 2022 International Congress on Human-Computer Interaction, Optimization and Robotic Applications (HORA), pp. 1-5. IEEE, 2022.

[11] W. Saheri, N. Shashoa, and A. Abougarair, "Recursive Least Squares Algorithm for MISO CARAMA Systems Utilizing Data Filtering", 2021 IEEE 1st International Maghreb Meeting of the Conference on Sciences and Techniques of Automatic Control and Computer Engineering MI-STA, May 2021.

[12] M. Han, S. Zhang, M. Xu, T. Qiu, and N. Wang," Multivariate Chaotic Time Series Online Prediction Based on Improved Kernel Recursive Least Squares Algorithm," IEEE Transactions on Cybernetics, 49 (4), pp. 1160-1172, 2018.

ISSN: 2961-6611