

# Study of Detection by Surface Plasmon Resonance Nanobiosensor

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**Abstract**— In this paper, we investigate on the use of Surface Plasmon to provide a powerful detect of a wide diversity of bimolecular interactions. The advantage of the nanobiosensors based on Surface Plasmon Resonance is to monitor molecular interactions in real times and label-free. These nanobiosensors provide a means for identifying these interactions and quantifying their kinetic constants, equilibrium constants and underlying energetic. This identification is released in our work by the study of the dispersion characteristics and the reflectivity of Surface Plasmon waves by using the Transverse Resonance Method.

**Keywords**—Surface Plasmon; Nanobiosensors; Detect; Characteristics dispersion; Reflectivity; Transverse Resonance Method.

## I. INTRODUCTION

The Surface Plasmon has received considerable attention in the last few years, thanks to his behavior which are the coupling of a light wave propagating in the dielectric medium with the collective longitudinal oscillation of the free electrons on the surface of a metal and the exponentially decreasing of propagation on both sides of the interface separating the metal and the dielectric medium and has a transverse magnetic polarization [1]. These behaviors provide the used of Surface Plasmon in many applications. Among the applications, we can quote the Surface Plasmon Resonance nanobiosensor [2] which is the purpose of this paper. This SPR nanobiosensor has enjoyed substantial commercial success. Its major advantage is that it does not require labels. In addition, it allows a real time detection which provides information on the kinetics of the reactions [3].

In our work, we focus to the importance of the Surface mode on the detection of the molecular interactions. In Section 2, we analyze the method of the detection by the dispersion characteristics and by the reflectivity of Surface Plasmon. In Section 3, we discuss the numerical results of the dispersion characteristics of these modes along the considered structure and the reflectivity of Surface Plasmon Resonance. In Section 4, we conclude our work.

## II. ANALYSIS

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The surface Plasmon Resonance nanobiosensor allows to quantify all the perturbations at the metal/ dielectric surface. In order to detect these perturbations, a receptor or “probe” fixed on the surface of the metal layer (gold) is associated with the target [4]. This association produces a variation of the refractive index of the dielectric which induces an angular displacement of the absorption peak and a modification of the dispersion characteristics. This modification allows us to identify the detected target. We have modulated the nanobiosensor by stacking of layers of variable refractive index and thickness (see in Fig. 1). The probe forms by a biological layer characterized by a thickness  $d_2$  and a refractive index  $n_2$  and the target represents a layer of thickness  $d_3$  and a refractive index  $n_3$ .

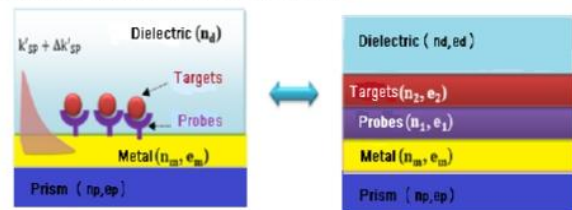


Fig. 1 Equivalent structure to the principle of a perturbation generated by a biomolecular interaction on the surface of a Surface Plasmon biochip

### A. Dispersion Characteristics

To find the dispersion characteristics of this structure, we apply the well-known method which is the Transverse Resonance Method in order to obtain the equivalent circuit, as shown in Fig. 2

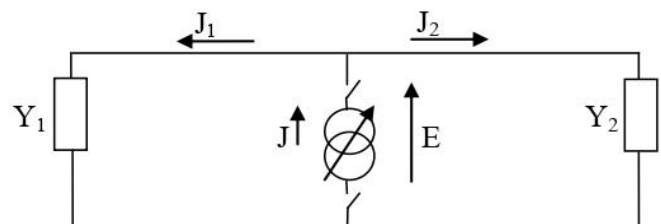


Fig. 2 Equivalent circuit used to illustrate the Transverse Resonance Method

This circuit allows us to find the dispersion equation of Surface Plasmon in this structure.

$$(\bar{Y}_1 + \bar{Y}_2)E = 0$$

1) *Structure without Targets*: The Surface Plasmon propagates along the longitudinal  $z$  direction with propagation constant  $k_z$  with  $k_z$  is real. This mode would be transverse electric (TE) and transverse Magnetic (TM) then the dispersion equation of this mode may be written as follows:

TM

$$-k_{y2}d_2 - \coth^{-1}\left(\frac{k_{y2}}{k_{y1}\varepsilon_2}\right) - \coth^{-1}\left(\frac{\varepsilon_3 k_{y2}}{k_{y3}\varepsilon_2}X\right) = 0$$

$$\text{Where } X = \coth\left(\coth^{-1}\left(\frac{\varepsilon_4 k_{y3}}{k_{y4}\varepsilon_3}\right) + k_{y3}d_3\right)$$

TE

$$-k_{y2}d_2 - \coth^{-1}\left(\frac{k_{y1}}{k_{y2}}\right) - \coth^{-1}\left(\frac{k_{y3}}{k_{y2}}X\right) = 0$$

$$\text{Where } X = \coth\left(\coth^{-1}\left(\frac{k_{y4}}{k_{y3}}\right) + k_{y3}d_3\right)$$

We consider that the dielectric constant of metal (gold) is complex. When the real part of dielectric constant of metal is negative, the wave number  $k_{y1}$ ,  $k_{y2}$  and  $k_{y4}$  are real but the wave number  $k_{y3}$  is purely imaginary. Then, the dispersion equation of Surface Plasmon mode is given by:

$$\begin{cases} -\beta_{y2}d_2 - \coth^{-1}\left(\frac{\beta_{y2}}{\beta_{y1}\varepsilon_2}\right) - \coth^{-1}\left(\frac{\varepsilon_3\beta_{y2}}{\alpha_{y3}\varepsilon_2}X\right) = 0 \\ \beta_{y2}^2 - \beta_{y1}^2 + k_0^2(\varepsilon_2 - 1) = 0 \\ -\alpha_{y3}^2 - \beta_{y2}^2 + k_0^2(\varepsilon_3 - \varepsilon_2) = 0 \\ -\alpha_{y3}^2 - \beta_{y4}^2 + k_0^2(\varepsilon_3 - \varepsilon_4) = 0 \\ \beta_z^2 - \beta_{y1}^2 + k_0^2 = 0 \end{cases}$$

$$\text{Where } X = \cot\left(\cot^{-1}\left(\frac{\varepsilon_4\alpha_{y3}}{\beta_{y4}\varepsilon_3}\right) - i\alpha_{y3}d_3\right)$$

These equations of system can be solved for the unknowns  $\beta_{y1}$ ,  $\beta_{y2}$ ,  $\alpha_{y3}$  and  $\beta_{y4}$  using an iterative procedure to arrive at a final set of solutions from some initial trial values. This procedure is released by “fsolve” in MATLAB.

2) *Structure with Targets*: In this section, we model the detection of a target by an insertion of a layer above the layer modulated for probe. This layer is characterized by a refractive index which differs from one target to another. In this structure, the dispersion equation of TM and TE Surface Plasmon is written as:

TM

$$-k_{y2}d_2 - \coth^{-1}\left(\frac{k_{y2}}{k_{y1}\varepsilon_2}\right) - \coth^{-1}\left(\frac{\varepsilon_3 k_{y2}}{k_{y3}\varepsilon_2}X\right) = 0$$

Where

$$X = \coth\left(\coth^{-1}\left(\frac{\varepsilon_4 k_{y3}}{k_{y4}\varepsilon_3}\right)\coth\left(\coth^{-1}\left(\frac{\varepsilon_3 k_{y4}}{k_{y5}\varepsilon_4}\right) + k_{y4}d_4\right) + k_{y3}d_3\right)$$

TE

$$-k_{y2}d_2 - \coth^{-1}\left(\frac{k_{y1}}{k_{y2}}\right) - \coth^{-1}\left(\frac{k_{y3}}{k_{y2}}X\right) = 0$$

Where

$$X = \coth\left(\coth^{-1}\left(\frac{k_{y4}}{k_{y3}}\right)\coth\left(\coth^{-1}\left(\frac{k_{y5}}{k_{y4}}\right) + k_{y4}d_4\right) + k_{y3}d_3\right)$$

The Surface Plasmon mode is characterized by the wavenumber  $k_{y1}$ ,  $k_{y2}$ ,  $k_{y3}$ ,  $k_{y4}$  and  $k_{y5}$  are real. It is represented by the solution of the system:

TM

$$\begin{cases} \beta_{y2}d_2 + \coth^{-1}\left(\frac{\beta_{y2}}{\beta_{y1}\varepsilon_2}\right) + \coth^{-1}\left(\frac{\varepsilon_3\beta_{y2}}{\beta_{y3}\varepsilon_2}X\right) = 0 \\ \beta_{y2}^2 - \beta_{y1}^2 + k_0^2(\varepsilon_2 - 1) = 0 \\ \beta_{y3}^2 - \beta_{y2}^2 + k_0^2(\varepsilon_3 - \varepsilon_2) = 0 \\ \beta_{y3}^2 - \beta_{y4}^2 + k_0^2(\varepsilon_3 - \varepsilon_4) = 0 \\ \beta_{y4}^2 - \beta_{y5}^2 + k_0^2(\varepsilon_4 - \varepsilon_5) = 0 \\ \beta_z^2 - \beta_{y1}^2 + k_0^2 = 0 \end{cases}$$

Where

$$X = \coth\left(\coth^{-1}\left(\frac{\varepsilon_4\beta_{y2}}{\beta_{y4}\varepsilon_3}\coth\left(\coth^{-1}\left(\frac{\varepsilon_5\beta_{y4}}{\beta_{y5}\varepsilon_4}\right) - \beta_{y4}d_4\right) + \beta_{y3}d_3\right)\right)$$

### B. Reflectivity

Where light is shined through the prism and onto the gold surface at wavelength and angle near the Surface Plasmon Resonance condition, the optical reflectivity of the gold changes very sensitively with the presence of biomolecular interaction on the gold surface. To find this reflectivity, we models our structure without and with target layer by an equivalent circuit.

where

#### 1) *Structure without Targets*:

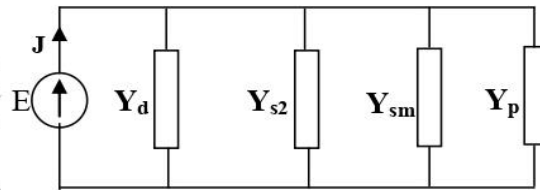


Fig. 3 Equivalent circuit for the structure without target

This circuit allows us to find the reflectivity expression of Surface Plasmon.

$$\Gamma = \frac{Y_{eq} - Y_p}{Y_{eq} + Y_p}$$

Where  $Y_{eq}$  is the equivalent admittance and it equal to the sum of the admittance of dielectric layer  $Y_d$ , the surface admittance of probe layer  $Y_{s2}$  and the surface admittance of metal layer  $Y_{sm}$ . Then

$$Y_{eq} = Y_d + Y_{s2} + Y_{sm}$$

$$\text{Whith } Y_{si} = j\omega\varepsilon_i\delta_i \text{ and } Y_d = \frac{(k_0 \cos\theta \sin\theta)^2 - k_0^2 \varepsilon_0}{j\omega\mu_0}$$

The admittance of prism layer equal to

$$Y_p = \frac{(k_0 \cos\theta \sin\theta)^2 - k_0^2 \varepsilon_p}{j\omega\mu_0}$$



## 2) Structure with Targets:

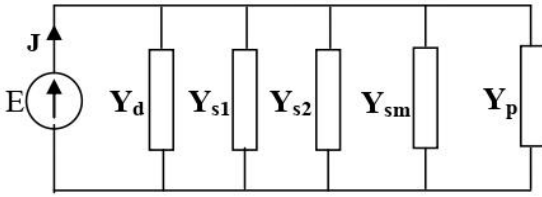


Fig. 4 Equivalent circuit for the structure with target

The same reflectivity expression of Surface Plasmon as the case without target layer  $\Gamma = \frac{Y_{eq} - Y_p}{Y_{eq} + Y_p}$ , but the equivalent admittance  $Y_{eq}$  is different from that of structure without target. Then, it equal to

$$Y_{eq} = Y_d + Y_{s1} + Y_{s2} + Y_{sm}$$

With  $Y_{s1}$  is the surface admittance of target layer.

## III. RESULTS AND DISCUSSION

In the numerical study, we consider that our structure is composed of a gold metal characterized by a permittivity corresponding to the drude model.

$$\epsilon_m(\omega) = 1 - \frac{\omega_p^2}{\omega^2 + i\omega\omega_t}$$

Where  $\omega_p = 13.71 \times 10^{15} \text{ rad s}^{-1}$  and

$$\omega_t = 4.05 \times 10^{13} \text{ rad s}^{-1}.$$

In Fig. 5, we illustrate the permittivity depending on the frequency of  $f = 4.8 \text{ GHz}$  to  $f = 6.2 \text{ GHz}$ . The permittivity becomes positive above  $f = 6 \text{ GHz}$  and it becomes negative below. This frequency is the critical frequency.

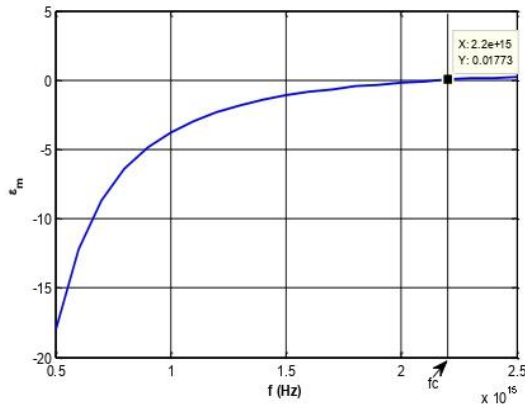


Fig. 5 Relative permittivity  $\epsilon_r$  as a fonction of frequency  $f$

Fig. 6 presents the dispersion curves of TM Surface Plasmon before and after target detection, for a thin layer of gold of thickness 50nm based on a prism SF10 and in contact with a dielectric medium (air). We consider in our case that

the target is DNA strand forming a biological layer with a refractive index of 1.48 and a thickness of 100nm.

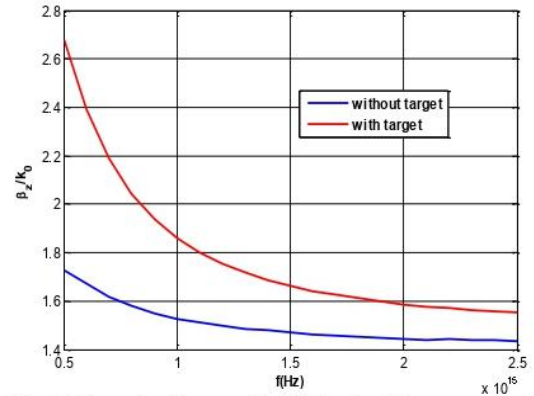


Fig. 6 Dispersion diagrams for TM Surface Plasmon mode of our structure without and with the layer illustrated the target.

The blue curve gives the normalized propagation constant before a disruption and the red curve corresponds to the normalized propagation constant after the disruption. An increase in the propagation constant is visualized. This increase due to the immobilization of the target molecules on the metal surface.

In Fig. 7, we present the dispersion curves of TM Surface Plasmon mode for the detection of different targets: DNA strand ( $n_2=1.48$ ), HIV virus ( $n_2=1.5$ ), EBV DNA ( $n_2=1.718$ ).

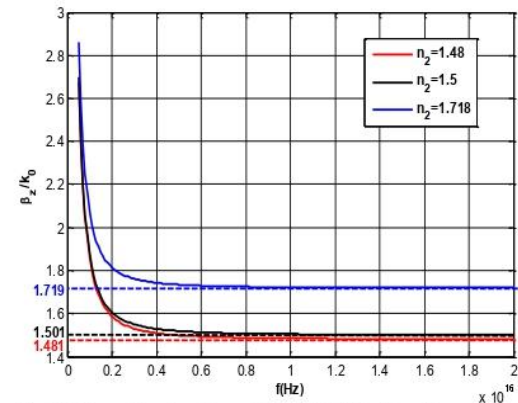


Fig. 7 Normalized real constant of TM Surface Plasmon mode propagating at surface of metal versus frequency for different targets detected

According to the dispersion curves, we can identify the detected target as shown in Fig. 7 Each dispersion curve has a saturation threshold which has a value equal to the refractive index of a target. For example, the saturation threshold of the blue curve is equal to 1.719 which is equal to the refractive index of the EBV virus.

In Fig. 8, we present the reflectivity of Surface Plasmon before and after the detection of target. We consider in our case two targets to be detected which are strands of DNA ( $n_2=1.48$ ,  $d_2=100 \text{ nm}$ ) and EBV DNA ( $n_2=1.718$ ,  $d_2=100 \text{ nm}$ ).

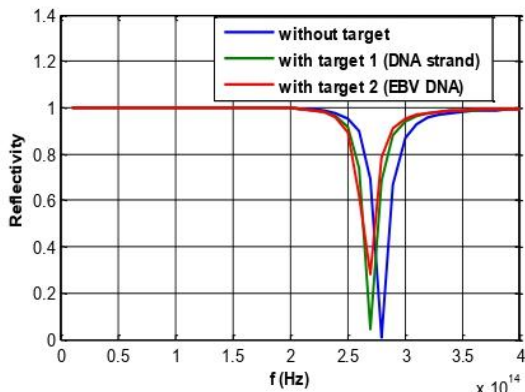


Fig. 8 Spectral shift caused by the deposition of target layer on the metal surface

The blue curve gives the reflectivity before a perturbation and the curves in green and red correspond to the reflectivity after the detection of target. At a fixed angular  $\theta = 53.47^\circ$ , a spectral offset of the order of  $0.1 \times 10^{14} \text{ Hz}$  between the blue curve and the green curve which represents the reflectivity after detection of DNA strand is displayed. According to reflectivity, we can distinguish between detected targets as shown in Fig. 8. The red curve which represents the

reflectivity after detection of EBV DNA is different at the green curve.

#### IV. CONCLUSIONS

The main objective of our work is to study the importance of Surface Plasmon for the detection of the molecular interactions by two methods the dispersion characteristics and the reflectivity. According to the numerical results, we have shown the precision of these methods to detect the biomolecular interactions of a biochip composed of a metal film and based on the Kretschman configuration.

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