Stability of a Linear Discrete System With Time-Delay via Lyapunov-Krasovskii Functional

Bemri H'mida[1], Mezlini Sahbi[2] ,Soudani Dhaou[3]

Tunis El Manar University
Laboratory of Research on Automatic (LARA)
National Engineers School of Tunis
BP 37, Belvedere 1002 Tunis. Tunisia.

bemrihmida@gmail.com
mezlini.sahbi@yahoo.fr
dhaou.soudani@enit.rnu.tn

***Abstract* — This paper focuses on the problem of asymptotic stability analysis of discrete time-delay systems. Based on the methods, delay-dependent stability and delay-independent stability condition is derived. The stability criterion is expressed in the form of linear matrix inequalities (LMI), which can be easily solved using numerical standard software such as MATLAB. The problem is solved by applying a Lyapunov functional that has enabled us to obtain new results with a great accuracy in programming, such that they are reliable and accurate. An illustrative numerical example is provided to show the advantage of the proposed stability condition and the reliability of the results.**

Keywords -- asymptotic stability; discrete systems; delay systems; Lyapunov functional; Lyapunov-Krasovskii method; linear matrix inequalities (LMI) .

I. Introduction

Due to the development in the field of microelectronics analog controllers are yielding their places to digital computers. Indeed, and given the importance of these control systems, we are using methods and numerical models to analyze and / or to control industrial processes.

Two types of representations are available to model a continuous or discrete dynamic system namely the external representation that uses input-output relations (transfer function) or the internal representation (matrix) of dynamic system which is based on the concept of state. To implement such a control structure and ensure the desired objectives, a modeling in the generally requires discrete-time analog systems is needed.

Digital control of physical systems requires, usually the development of discrete models. Several modeling strategies, developed in the literature reflecting a meaningful description of dynamical systems to be studied led to mathematical tools leading generally to linear or non-linear models with or without delays whose behavior may be more or less close to the real system. These models are described by relations between input variables and output variables that can be modified by inputs considered as secondary (disturbances) that always exist in practice.

The initial modeling of a discrete time-delays system often leads to writing a recurrent equation between different terms of the input and output sequences. This formulation of the recurrent equation is well suited for numerical calculation. This is the form in which these algorithms are digital control methods. The system is fully defined and the recurrent equation can be solved if the initial conditions are specified.

The analysis of the stability of delay systems has been conducted in the literature by numerous fundamental researches that depend on the type of systems considered and the scope. There are many methods studying the stability of linear discrete time-delay systems. These stability criteria can be classified into two main categories namely the frequency criterion using the notion of the characteristic equations and the time criterion based on Lyapunov theory.

The stability results for the existing time-delay systems can be classified into two types: independent delay stability [1,2] and dependent-delay stability [3, 4]. Despite the above mentioned importance, less attention has been given to discrete-time delays systems [5, 8].

The use of Lyapunov methods for stability analysis of delay systems has always been the most interest subject in this research topic [9,10,16,17]. Recently, in [6, 8,11] the Lyapunov-Krasovskii functional that have been introduced to

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the derivative of the state in time includes terms which depend not only on the present but also on the past states of the delay system. This extension (modification) allows the robustness analysis of the delay systems. In [12] the method of Lyapunov-Krasovskii for systems with discrete delay processing descriptor model was considered, whereas in [13] is discussed method of Lyapunov-Krasovskii neutral nonlinear discreet time delay systems.

In this work, an extension of the Lyapunov-Krasovskii method to recurrent delay equation is developed, which describes a discrete delay linear system. Our main objective is to develop a new simple theory of the stability of discrete autonomous systems counterpart to delay the method of Lyapunov-Krasovskii for the suggested systems in [14, 19, 20]. The recurrent relation presented in this work in simple mathematical forms in [12, 13, 15] can be applied to autonomous discrete delay systems.

This paper is organized as follows. In Section II, we present our notation and preliminary. Then, in Section III, we develop the asymptotic stability theorem admitting Lyapunov-Krasovskii functional for discrete delay systems. In section IV, a theoretical application of asymptotic stability type Lyapunov - Krasovskii that can be applied to certain types of discrete time delay systems, the stability condition is obtained in the form of linear matrix inequality. In addition, an example is given to illustrate the obtained results.

II. Formulation of The Problem and Some Preliminary

R	Real vector space.
$F=(f_{ij})\in R^{n*n}$	Real matrix.
F^T	Transpose of the matrix F.
$F>0$	Positive definite matrix.
$F\geq 0$	Positive semi-definite matrix.
$\lambda(F)$	Eigenvalue of the matrix F.
$\sigma(F)=\|F\|$	Singular value of the matrix.
$\|F\|=\sqrt{\lambda_{\max}(F^T F)}$	Euclidean norm of the matrix F.

Fact 1. For any positive scalar α, and for any two vectors x and y, we present the following inequality:

$$x^T y + y^T x \leq \alpha x^T x + \alpha^{-1} y^T y \quad (1)$$

Note that : $V_\delta = \{ x \in R^n : \|x\| < \delta \}$ (2)

Lemma 1. [7] The zero solution of the difference system is asymptotically stable if there exists a positive definite $V(x(k)) : R^n \to R^+$ knowing that there is a $\rho > 0$ as:

$$\Delta V(x(k)) = V(x(k+1)) - V(x(k)) \leq -\rho \, \|x(k)\|^2 \quad (3)$$

The above inequality is true throughout the linear resolution of the discrete system. If the above condition is valid for all $x(k) \in V_\delta$, the zero solution of the difference system is locally asymptotically stable.

Lemma 2. [7] For any constant symmetric matrix: $M \in R^{n*n}, M = M^T > 0, \beta$ scalar as $\beta \in Z^+ / \{0\}$, and the vector function $W : [0, \beta] \to R^n$, we have the following inequality:

$$\left(\sum_{i=0}^{\gamma-1} w(i)\right)^T \times M \times \left(\sum_{i=0}^{\gamma-1} w(i)\right) \leq \beta \sum_{i=0}^{\gamma-1} \left(w(i)^T \times M \times w(i)\right) \quad (4)$$

III. STABILITY CRITERION

Roughly speaking, the stability of a system is its ability to resist any unknown small influences. Since in reality disturbances are always encountered, stability is an important property of any control system, delayed or non-delayed.

In this section, we give a stability condition for linear discrete delay dynamic systems. Lyapunov functional are defined, in autonomous system, by the following recurrent equation:

$$x(k+1) = Ax(k) + Bx(k-q) \quad (5)$$

with $x(\theta) = \psi(\theta)$, $\theta \in \{-q, -q+1, \ldots, 0\}$ an initial state of the associated function.

$x(k) \in R^n$ is the state at time k.

$A_i \in R^{n*n}$ are constant matrices of appropriate size.

$q = 1, 2, \ldots$ is a positive integer representing the time delay existing in the system.

Whether $V : R^n \to R$ in such a way that $V(x)$ is bounded for all $\|x\|$ is bounded.

A. *Delay-dependent stability*

This group includes exact algebraic stability criteria depending on the delay and on the system constants and stability criteria which yield an upper bound of the admissible delay.

Using the stated theorem in the following and previously stated lemmas we can determine the asymptotic stability of the linear discrete system that is presented in equation (5).

Theorem 1.

The discrete time-delay system (5) is asymptotically stable for any delay $q > 0$, if there exist symmetric positive definite matrices $P = P^T > 0$, $G = G^T > 0$ and $W = W^T > 0$ satisfying the following matrix inequalities:

$$\psi_0 = \begin{bmatrix} (1,1) & 0 & 0 \\ 0 & (2,2) & 0 \\ 0 & 0 & (3,3) \end{bmatrix} < 0 \quad (6)$$

Such as:

$$(1,1) = A^T PA + \alpha\, A^T P^2 A + q\, G + W - P \quad (7)$$

$$(2,2) = B^T PB + \alpha^{-1} B^T B - W \quad (8)$$

$$(3,3) = -q\, G \quad (9)$$

Evidence. Consider the Lyapunov function defined as follows:

$$V(y(k)) = V_1(y(k)) + V_2(y(k)) + V_3(y(k)) \quad (10)$$

Where:

$$V_1(y(k)) = x^T(k) \times P \times x(k) \quad (11)$$

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$$V_2(y(k)) = \sum_{i=k-q}^{k-1} (q-k+i) \times x^T(i) \times G \times x(i) \tag{12}$$

$$V_3(y(k)) = \sum_{i=k-q}^{k-1} x^T(i) \times W \times x(i) \tag{13}$$

$$y(k) = [x(k),\, x(k-q)] \tag{14}$$

With $P = P^T > 0$, $G = G^T > 0$ and $W = W^T > 0$ is symmetric positive definite solutions of (6) and $y(k) = [x(k),\, x(k-q)]$.

Then the difference of $V(y(k))$ along the path of the solution (3) is given by:

$$\Delta V(y(k)) = \Delta V_1(y(k)) + \Delta V_2(y(k)) + \Delta V_3(y(k)) \tag{15}$$

With:

$$\Delta V_1(y(k)) = V_1(x(k+1)) - V_1(x(k)) \tag{16}$$

From (16) we can write:

$$\begin{aligned}\Delta V_1(y(k)) &= V_1(x(k+1)) - V_1(x(k)) \\ &= [Ax(k) + Bx(k-q)]^T P [Ax(k) + Bx(k-q)] - x^T(k)Px(k) \\ &= x^T(k)[A^T PA - P]x(k) + x^T(k)A^T PBx(k-q) + \\ &+ x^T(k-q)B^T PAx(k) + x^T(k-q)B^T PBx(k-q)\end{aligned} \tag{17}$$

$$\begin{aligned}\Delta V_2(y(k)) &= V_2(x(k+1)) - V_2(x(k)) \\ &= \Delta\left[\sum_{i=k-q}^{k-1} (q-k+i)\, x^T(i)Gx(i)\right] \\ &= qx^T(k)Gx(k) - \sum_{i=k-q}^{k-1} x^T(i)Gx(i)\end{aligned} \tag{18}$$

And:

$$\begin{aligned}\Delta V_3(y(k)) &= V_3(x(k+1)) - V_3(x(k)) \\ &= \Delta\left[\sum_{i=k-q}^{k-1} x^T(i)\, W\, x(i)\right] \\ &= x^T(k)\, W\, x(k) - x^T(k-q)\, W\, x(k-q)\end{aligned} \tag{19}$$

Applying the Fact 1 in equation (17), the following inequality is obtained:

$$\begin{aligned}&x^T(k)A^T PBx(k-q) + x^T(k-q)B^T PAx(k) \le \\ &\alpha x^T(k)A^T P^2 Ax(k) + \alpha^{-1} x^T(k-q)B^T Bx(k-q)\end{aligned} \tag{20}$$

and therefore :

$$\begin{aligned}\Delta V_1(y(k)) \le\ & x^T(k)[A^T PA + \alpha A^T P^2 A - P]x(k) + \\ &+ x^T(k-q)[B^T PB + \alpha^{-1} B^T B]x(k-q)\end{aligned} \tag{21}$$

Thus the expression (15) of $\Delta V(y(k))$ is rewritten as follows:

$$\begin{aligned}\Delta V(y(k)) \le\ & x^T(k)[A^T PA + \alpha A^T P^2 A - P]x(k) + \\ &+ x^T(k-q)[B^T PB + \alpha^{-1} B^T B]x(k-q) + qx^T(k)Gx(k) - \\ &- \sum_{i=k-q}^{k-1} x^T(i)Gx(i) + x^T(k)\, W\, x(k) - x^T(k-q)\, W\, x(k-q)\end{aligned} \tag{22}$$

Which is equivalent to:

$$\begin{aligned}\Delta V(y(k)) \le\ & x^T(k)[A^T PA + \alpha A^T P^2 A + qG + W - P]x(k) + \\ &+ x^T(k-q)[B^T PB + \alpha^{-1} B^T B - W]x(k-q) - \\ &- \sum_{i=k-q}^{k-1} x^T(i)Gx(i)\end{aligned} \tag{23}$$

By using Lemma 2, we obtain the following inequality:

$$\left(\frac{1}{q}\sum_{i=k-q}^{k-1} x(i)\right)^T qG \left(\frac{1}{q}\sum_{i=k-q}^{k-1} x(i)\right) \le \sum_{i=k-q}^{k-1} x^T(i)Gx(i) \tag{24}$$

It follows that:

$$\begin{aligned}\Delta V(y(k)) \le\ & x^T(k)[A^T PA + \alpha A^T P^2 A - P]x(k) + \\ &+ x^T(k-q)[B^T PB + \alpha^{-1} B^T B]x(k-q) + \\ &+ qx^T(k)Gx(k) - \sum_{i=k-q}^{k-1} x^T(i)Gx(i) + \\ &+ x^T(k)\, W\, x(k) - x^T(k-q)\, W\, x(k-q)\end{aligned} \tag{25}$$

From Fact 1 we get the following expression:

$$\begin{aligned}\Delta V(y(k)) \le\ & x^T(k)[A^T PA + \alpha A^T P^2 A + qG + W - P]x(k) + \\ &+ x^T(k-q)[B^T PB + \alpha^{-1} B^T B - W]x(k-q) - \\ &- \sum_{i=k-q}^{k-1} x^T(i)Gx(i)\end{aligned} \tag{26}$$

Using Lemma 2, equation (26) will be rewritten as follows:

$$\Delta V(y(k)) \le \left[\begin{aligned}&x^T(k)[A^T PA + \alpha A^T P^2 A + qG + W - P]x(k) + \\ &+ x^T(k-q)[B^T PB + \alpha^{-1} B^T B - W]x(k-q) - \\ &- \left(\frac{1}{q}\sum_{i=k-q}^{k-1} x(i)\right)^T qG \left(\frac{1}{q}\sum_{i=k-q}^{k-1} x(i)\right)\end{aligned}\right] = \Lambda \tag{27}$$

Then:

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$$\Lambda = x^T(k)\left[A^T PA + \alpha A^T P^2 A + qG + W - P\right]x(k) + x^T(k-q)\left[B^T PB + \alpha^{-1} B^T B - W\right]x(k-q) - \left[\frac{1}{q}\sum_{i=k-q}^{k-1} x(i)\right]^T qG\left[\frac{1}{q}\sum_{i=k-q}^{k-1} x(i)\right]$$

$$= \left[x^T(k),\, x^T(k-q),\, \left(\frac{1}{q}\sum_{i=k-q}^{k-1} x(i)\right)^T\right] \times \begin{bmatrix} (1,1) & 0 & 0 \\ 0 & (2,2) & 0 \\ 0 & 0 & (3,3) \end{bmatrix} \times \begin{bmatrix} x(k) \\ x(k-q) \\ \frac{1}{q}\sum_{i=k-q}^{k-1} x(i) \end{bmatrix} \quad (28)$$

$$= y^T(k) \times \psi_0 \times y(k)$$

With:

$$y(k) = \begin{bmatrix} x(k) \\ x(k-q) \\ \frac{1}{q}\sum_{i=k-q}^{k-1} x(i) \end{bmatrix} \quad (29)$$

Where:

$$\Delta V(y(k)) \le y^T(k) \times \psi_0 \times y(k) \quad (30)$$

Thus the condition (6) is satisfied, then $\Delta V(y(k)) < 0$, $\forall$ x(k) ≠0 which allowed us to conclude that the system defined in equation (5) is asymptotically stable.

Finally we conclude that $\Delta v(y(k))$ is negative definite; namely, there is a number $\rho > 0$ such that $\Delta v(y(k)) \le -\rho \ \|y(k)\|^2$ and, consequently, the asymptotic stability of the system follows immediately from Lemma 1.

B. Delay-independent stability

Delay-independent stability criteria are very useful, since in reality it is difficult to estimate the delays, especially if those delays are time-varying and/or state-dependent.

Theorem 2:

The discrete time-delay system (5) is asymptotically stable, if there exist symmetric positive definite $N = N^T > 0$ and $S = S^T > 0$ such that following linear matrix inequality (LMI) hold:

$$\psi_1 = \begin{bmatrix} N-S & 0 & A^T S \\ 0 & -N & B^T S \\ A^T S & B^T S & -S \end{bmatrix} < 0 \quad . \quad (31)$$

Proof. Let the Lyapunov functional be:

$$V(x(k)) = x^T(k)Sx(k) + \sum_{j=1}^{q} x^T(k-j)Nx(k-j) \quad (32)$$

$N = N^T > 0$ and $S = S^T > 0$.

The forward difference along the solutions of system (5) is:

$$\Delta V(y(k)) = \left[Ax(k) + Bx(k-q)\right]^T S\left[Ax(k) + Bx(k-q)\right] - x^T(k)Sx(k) + x^T(k)Nx(k) - x^T(k-q)Nx(k-q)$$
$$= \begin{bmatrix} x(k) \\ x(k-q) \end{bmatrix}^T \begin{bmatrix} A^T SA - S + N & A^T SB \\ A^T SB & B^T SB - N \end{bmatrix} \begin{bmatrix} x(k) \\ x(k-q) \end{bmatrix} \quad (33)$$

If the following equation is satisfied:

$$\begin{bmatrix} A^T SA - S + N & A^T SB \\ A^T SB & B^T SB - N \end{bmatrix} < 0 \quad (34)$$

Then

$$\begin{bmatrix} A^T SA - S + N & A^T SB \\ A^T SB & B^T SB - N \end{bmatrix} = \begin{bmatrix} N-S & 0 \\ 0 & -N \end{bmatrix} + \begin{bmatrix} A^T SA & A^T SB \\ A^T SB & B^T SB \end{bmatrix} = \begin{bmatrix} N-S & 0 \\ 0 & -N \end{bmatrix} + \begin{bmatrix} A^T \\ B^T \end{bmatrix} S\left[A \quad B\right] < 0 \quad (35)$$

Using Schur complement [18], it is easy to see that the condition (31)is equivalent to :

$$\begin{bmatrix} N-S & 0 & A^T S \\ 0 & -N & B^T S \\ A^T S & B^T S & -S^{-1} \end{bmatrix} < 0 \quad (36)$$

Note that the condition (36) is not LMI condition due to the existence of the term $-S^{-1}$. Pre and post multiply (36) with dig {I, I, S } we obtain LMI condition (31).

Thus the condition (31) is satisfied, then $\Delta V(y(k)) < 0$, $\forall\, x(k) \neq 0$ Which allowed us to conclude that the system defined in equation (5) is asymptotically stable.

Finally we conclude that $\Delta v(y(k))$ is negative definite; namely, there is a number $\beta > 0$ such that $\Delta v(y(k)) \le -\beta \ \|y(k)\|^2$ and, consequently, the asymptotic stability of the system follows immediately from Theorem 2.

IV. Numerical Example

Consider the linear discrete time delay system autonomous defined by the following equation:

$$x(k+1) = \begin{bmatrix} 0.1 & 0.02 \\ 0.1 & -0.15 \end{bmatrix} x(k) + \begin{bmatrix} 0.1 & 0.01 \\ 0.2 & 0.2 \end{bmatrix} x(k-1) \quad (37)$$

With:

$$A = \begin{bmatrix} 0.1 & 0.02 \\ 0.1 & -0.15 \end{bmatrix}, \quad B = \begin{bmatrix} 0.1 & 0.01 \\ 0.2 & 0.2 \end{bmatrix}$$

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A- Applying Theorem 1 to the equation defined in system (37) and through the relationship (6) .Matrices P, W and G symmetric positive definite which satisfy the sufficient condition for stability is obtained:

$$P = \begin{bmatrix} 3.2162 & 0.0172 \\ 0.0172 & 3.1592 \end{bmatrix},$$

$$G = \begin{bmatrix} 1.0696 & -0.0055 \\ -0.0055 & 1.0555 \end{bmatrix} \quad \text{and} \quad W = \begin{bmatrix} 1.1628 & 0.0712 \\ 0.0712 & 1.1295 \end{bmatrix}$$

B-Applying Theorem 2 to the equation defined in system (37) and through the relationship (31).Matrices N and S symmetric positive definite which satisfy the sufficient condition for stability is obtained:

$$N = \begin{bmatrix} 0.1158 & 1.2007 \\ 1.2007 & 1.0573 \end{bmatrix} \quad \text{and} \quad S = \begin{bmatrix} 0.6163 & 1.6801 \\ 1.6801 & 1.0583 \end{bmatrix}$$

V. CONCLUSION

In this paper, a sufficient condition has been derived which ensures the asymptotic stability of discrete systems with time delays. This condition is derived using an approach based on the direct method of Lyapunov. It was presented in terms of LMI. It is demonstrated that these results can be applied in practice in an efficient manner. It has been shown that these results are less restrictive than some of those in the current literature. The numerical calculations are carried out to illustrate the results.

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AUTHORS' INFORMATION

Bemri H'mida was born in Tunisia, in January1979.He received the scientific degree in mathematics, physics and computer science within the Faculty of Sciences of Tunis (FST) in 2003.He received the degree of MA within the Faculty of Sciences of Tunis (FST) in 2005.He received the certificate of Aptitude Teaching of Second Degree (CAPES) in 2008.He has been working as a teacher in 2 Mars1934 secondary school in Siliana since 2008. He received the degree of Master of Research in automatic and signals processing (ATS) within the National Engineering School of Tunis (ENIT) in 2013. His research interests include modeling and control systems sampled to delays.

Mezlini Sahbi was born in Tunisia, in March 1972.He received the scientific degree in mathematics and physics preparatory cycle of the National School of Engineers of Gabes (ENIG) in 2000.He received the engineering degree in electrical engineering in the National School of Engineers of Monastir(ENIM) in 2003. He received the degree of Master of research in automatic and signal processing (ATS) within the National Engineering School of Tunis (ENIT) in 2005. His research interests include the command by internal model of discrete system.

Dhaou Soudani was born in Tunisia, in July 1954. He received the Master degree in Electrical and Electronic Engineering and the "Diplôme des Etudes Approfondies" in Automatic Control from the "Ecole Normale Supérieure de l'Enseignement Technique" Tunisia in 1982 and 1984, respectively. He obtained both the Doctorat in 1997, and the "Habilitation Universitaire" in 2007 in Electrical Engineering from the "Ecole Nationale d'Ingénieurs de Tunis" (ENIT) Tunisia. He is currently a professor in Automatic Control and a member of the unit research "Laboratoire de Recherche en Automatique" (LA.R.A).

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