Iterative learning traking control for a class of linear discrete-time switched systems

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Abstract— **In this paper, we propose a new numerical approach for the iterative learning control of linear switched system .It is assumed that the considered switched systems are operated during a finite time interval repetitively, and then the iterative learning control scheme can be introduced .The objective of learning law is to guarantee the asymptotic convergence of the output error between the desired output and the actual output for the entire time interval through the iterative learning process. All the results are presented in terms of linear matrix inequalities (LMIs). A numerical simulation example is established shown the effectiveness of the proposed method.**

Keywords—: Switched systems; Iterative learning control (ILC); Hinf control; asymptotic stability ;Linear matrix inequality.

I. INTRODUCTION

The stability analysis of switched linear systems is one of the active over the past research due to their wide applications in many areas such (the automotive industry, aircraft and air-traffic control…) [1]. A switched system is a hybrid dynamical system, which consists of a family of continuous-time or discrete-time subsystems and a rule that orchestrates the switching between them [2]. However, the stabilization problem of discrete switched systems with classic control law has been studied in [3], [4], [5], and [6]. Specifying the asymptotic stability for arbitrary switching systems over time systems is an especially attracting problem in this paper. Compared with the different results for synthesis issue by applying the classic control laws, relatively few efforts are made for designing a controller to achieve the strong stability for switched repetitive systems. This paper is addressed for the design feedback controllers (ILC) such that the system is asymptotically stable [7], [8]. Motivated by human learning, the basic idea of (ILC) is to use information from previous executions of the task in order to improve performance from pass-to-pass in the sense that the tracking error is sequentially reduced, and guaranteed the asymptotic stability of system. Control objectives can be achieved iteratively through updating the control input in the iteration domain. However, to the best of our knowledge, the tracking error problem of discrete-time switched linear systems operating in a repetitive manner shows good results in practice, especially for the asymptotic stability problem with (ILC) control of switched systems, which motivates our present study. In this paper, the problem of iterative learning control for a class of linear discrete- time switched system is studied. The obtained formulation by applying ILC control is transformed into a synthesis problem of a special 2D repetitive switched system [9], [12]. However, the basic control problem like stability and stabilization can be relatively easily extended to involve robustness analysis and more control performance specifications using norms H_∞ [5], [10]. In particular, the main contribution of the paper is to provide the LMI characterization for asymptotic stability and then for solving the tracking error problem of linear discrete-time switched systems operating in a repetitive manner. In this context, the ILC tracking problem has been transformed into a stability analysis concept of 2D repetitive switched systems. In this sense, a necessary condition ensuring the stability under switching has been developed. This new formulation can be viewed as an interpretation of the H_∞ problem in the case of 2D repetitive switched systems.

This paper is organized as follows: In section 2, based on the problem formulation and the idea for transform the switched systems in to repetitive switched systems. In section 3, the control law design (ILC) by constructing a sequence of control inputs to a discrete switched system produces 2D repetitive system, and sufficient conditions for the existence of a stabilizing controller (ILC) are derived in terms of a set of matrix inequalities. In section 4, Numerical examples are presented to illustrate the feasibility and the effectiveness of the proposed design algorithms in this paper. Finally, the conclusion of this paper is given in Section 5.

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II. PROBLEM FORMULATION

Consider the following discrete linear switched system described by the following form:

$$\begin{cases} x(t+1) = A_{\alpha(t)}x(t) + Bu(t) \\ y(t) = Cx(t) \end{cases} \quad x(0) = 0, t \geq 0 \qquad (1)$$

Where $x(t) \in IR^n$ is the state vector in IR^n , $u(t) \in IR^m$ is the control vector in IR^m , $y(t) \in IR^m$ is the output vector $\alpha(t): \{1,2,..\} \rightarrow M: \{1,2,..\}$ is a random switching rule, this means that the matrices (A_α, B, C) are allowed to take values, at an arbitrary discrete time, in the finite set $\{A_1, \dots A_n\}$.

Note that $\alpha(t)$ is an arbitrary switching rule during the finite time interval $[0,T]$, we can assume the arbitrary switching rule is given as:

$$\alpha(t): \begin{cases} 1, t \in [0, t_1] \\ 2, t \in [t_1, t_2] \\ \vdots \\ m, t \in [t_{m-1}, T] \end{cases} \qquad (2)$$

Where $\alpha(t)$ denotes the switching sequence between three subsystem.

In the sequel, if k is defined as the switching number, Thus (1) is operated during a finite time interval repetitively, the system can be describes as:

$$\begin{cases} x_k(t+1) = A_k x_k(t) + Bu_k(t) \\ y_k(t) = Cx_k(t) \end{cases} \quad x(0) = 0, t \geq 0 \qquad (3)$$

Where, $x_k(t)$ and $u_k(t)$ are respectively the state and the control vector of system in switching k. $t = 0, \dots T$, $k \in IN$.

According to the principle of ILC, there are two independent dynamic processes: system time t and learning iteration k during learning (process/ each mode) [8], [9], [10]. Every variable can be expressed as a 2-D repetitive switched function, such as $x_k(t)$, which represents $x(t)$ in the *kth*-learning iteration (*kth* switching) [9]. This philosophy proposed to tackle the ILC problem for 2-D repetitive switched systems, with variable initial conditions.

Basic assumption for the system is given as follow:

Assumption: the desired trajectory $y_d(t)$ is iteration invariant.

III. MAIN RESULTS

In this section, we analyze the stability of switched system with three repetitive subsystems for the sake of convenience.

A. *Stability analysis for linear switched system using ILC control*

In this work, to solve the formulated problem we use the learning control described by the following updating ILC law:

$$u_{k+1}(t) = u_k(t) + K_1^p \eta_{k+1}(t+1) + K_2^p e_k(t+1) \qquad (4)$$

And

$$\eta_{k+1}(t+1) = x_{k+1}(t) - x_k(t) \qquad (5)$$

Where, $e_k(t) = y_d(t) - y_k(t)$ is output tracking error, η_{k+1} denotes the state vector computed the cycle direction, K_1^p and K_2^p are gains appropriately dimensioned matrices to be designed, with p is number of subsystem.

Then clearly (4) and (5) can be written as, for a discrete switched system:

$$\eta_{k+1}(t+1) = x_{k+1}(t) - x_k(t)$$
$$= A_{k+1}x_{k+1}(t-1) + Bu_{k+1}(t-1) - A_k x_k(t-1) - Bu_k(t-1)$$
$$= (A_{k+1} + BK_1^p)\eta_{k+1}(t) + (A_{k+1} - A_k)x_{k+1}(t-1) + BK_2^p e_k(t) \qquad (6)$$

And

$$e_{k+1}(t) = y_d(t) - y_{k+1}(t)$$
$$= y_d(t) - C(\eta_{k+1}(t+1) + x_k(t))$$
$$= e_k(t) - C\eta_{k+1}(t+1) \qquad (7)$$
$$= -C(A_{k+1} + BK_1^p)\eta_{k+1}(t) - C(A_{k+1} - A_k)x_{k+1}(t-1) + (I - CBK_2^p)e_k(t)$$

$$x_{k+1}(t-1) = \eta_{k+1}(t) - x_k(t-1) \qquad (8)$$

The obtained system in the closed-loop is given by the following form:

$$\begin{cases} \eta_{k+1}(t+1) = (A_{k+1} + BK_1^p)\eta_{k+1}(t) + [BK_2^p \quad (A_{k+1} - A_k)] \begin{bmatrix} e_k(t) \\ x_k(t-1) \end{bmatrix} \\ \begin{bmatrix} e_{k+1}(t) \\ x_{k+1}(t-1) \end{bmatrix} = \begin{bmatrix} -C(A_{k+1} + BK_1^p) \\ 1 \end{bmatrix} \eta_{k+1}(t) + \begin{bmatrix} (I - CBK_2^p) & -C(A_{k+1} - A_k) \\ 0 & 1 \end{bmatrix} \begin{bmatrix} e_k(t) \\ x_k(t-1) \end{bmatrix} \end{cases} \qquad (9)$$

The state-space model (9) is that of a discrete linear repetitive process of the form defined by the pass output and state vectors $e_{k+1}(t), x_{k+1}(t-1)$ and $\eta_{k+1}(t+1)$, respectively.

The main contribution results in transforming the tracking error ILC problem into stability analysis concept of 2D repetitive switched systems.

B. *H_∞ objectif problem*

In the sequel we can rewrite the augmented system (9) as in [11].

Let $G_{ibf}(t)$ is the transfer from $\begin{bmatrix} e_k(t) \\ x_k(t-1) \end{bmatrix}$ to $\begin{bmatrix} e_{k+1}(t) \\ x_{k+1}(t-1) \end{bmatrix}$:

$$G_{ibf} = \begin{bmatrix} A_{bf} & B_{bfi} \\ C_{bfi} & D_{bfi} \end{bmatrix} = \qquad (10)$$

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$$\begin{bmatrix} \overbrace{(A_{k+1}+BK_1^P)}^{A_{bf}} & \left[\overbrace{BK_2^P}^{B_{bf1}} \quad \overbrace{(A_{k+1}-A_k)}^{B_{bf2}}\right] \\ \underbrace{\left[-C(A_{k+1}+BK_1^P) \quad 1\right]}_{[C_{bf1}\quad C_{bf2}]^T} & \underbrace{\left[\begin{matrix}(I-CBK_2^P) & -C(A_{k+1}-A_k) \\ 0 & 1\end{matrix}\right]}_{[D_{bf1}\quad D_{bf2}]} \end{bmatrix}$$

We consider the matrix G_{ibf} of the closed loop system (9), with the gain K^P and seek to minimize the norms H_∞ of the closed loop systems guaranteeing the convergence of error to zero. The objective problem is defined as follows:

$$\left\|G_{1_{e_{k+1}/e_k}}\right\|_\infty < \gamma \qquad (11)$$

C. LMI approach to H_∞ synthesis

First, we consider the increased system (9) , if $\eta_{k+1}(t+1)$is the input signal and $e_{k+1}(t)$ is the output signals the iterative learning control law can guarantee the asymptotic convergence of the output error between the desired output and the actual output for the entire time interval through the iterative learning process.

We have:

$$\begin{cases} \eta_{k+1}(t+1) = (A_{k+1}+BK_1^P)\eta_{k+1}(t) + BK_2^P e_k(t) \\ e_{k+1}(t) = -C(A_{k+1}+BK_1^P)\eta_{k+1}(t) + (I-CBK_2^P)e_k(t) \end{cases} \qquad (12)$$

The design objectives for finding K_1^P , $K_2^P$is to minimize the H_∞ norm of the closed-loop transfer function $G_{ibf}(t)$ for $e_{k+1}(t)$ to $e_k(t)$ i.e. Satisfies:

$$\left\|G_{1bf_{e_{k+1}/e_k}}\right\|_\infty < \gamma \qquad (13)$$

Theorem 2 [5]: Applying the theorem given in [5] to the closed-loop system (12), we see that our desired H_∞ controller exists if and only if there is symmetric positive matrix S_i , positive matrix G_i, N_1^P, K_2^P, with appropriate dimensions, and a scalar $\alpha\in\,]-1,1[$, such that we obtained the LMI (14) are feasible.

$$\begin{bmatrix} -\gamma^2 I & * & * & * \\ BK_2^P & \alpha^2 S_i - S_j & * & A_{k+1}G_i + BN_1^P + \alpha G_i - \alpha S_i \\ (I-CBK_2^P) & 0 & -I & -CA_{k+1}G_i - CBN_1^P \\ 0 & * & * & S_i - (G_i - G_i^T) \end{bmatrix} < 0 \qquad (14)$$

In this case K_1^P are given by $K_1^P = N_1^P(G_i)^{-1}$

Proof: Appling theorem in [5] with,

$$A_{ibf} = (A_{k+1}+BK_1^P),\; B_{1bf} = BK_2^P$$
$$C_{1bf} = -C(A_{k+1}+BK_1^P),\; D_{1bf} = (I-CBK_2^P)$$

We obtained the following LMI condition (14).

IV. Numerical Example

In this section, a numerical example is given. Let us consider the linear discrete-time switched system (3), composed with three subsystems:

$$A_1 = \begin{bmatrix} -0.98 & 9.8 \\ 1.225 & -1.95 \end{bmatrix},\; A_2 = \begin{bmatrix} -1.65 & 6.6 \\ 0 & -1.95 \end{bmatrix}$$
$$A_3 = \begin{bmatrix} 8 & 0 \\ 1.6 & -0.8 \end{bmatrix},\; B = \begin{bmatrix} 0 \\ 1 \end{bmatrix},\; C = \begin{bmatrix} -0.1 & 1 \end{bmatrix}$$

And the desired trajectory,

$$y_d(t) = \sin(8(T_j-1)/25)\,,\; j = \{1,\dots,n\}$$

The initial conditions $x_k(0)=0$, and the control law $u_k(t)$ are given in (4).

In this case, we assume the switching sequence is:

$$\alpha(t) : \begin{cases} 1, t\in[0,5] \\ 2, t\in[6,11] \\ 3, t\in[12,17] \end{cases}$$

Appling the of Theorem 2, and by varying values of γ and α , the corresponding control law gains are obtained.

TABLE I. COMPARISON BETWEEN CONTROL LAW GAINS FOR DIFFERENT VALUE OF , γ AND α.

$\gamma = 0.85, \alpha = 0.95$	$\gamma = 0.01, \alpha = 0.01$
$K_1^1 = [-1.317 \quad 2.9314]$, $K_2^1 = 0.8233$	$K_1^1 = [-1.323 \quad 2.94]$ $K_2^1 = 1.0001$
$K_1^2 = [-0.161 \quad 2.568]$ $K_2^2 = -0.2735$	$K_1^2 = [-0.165 \quad 2.64]$ $K_2^2 = -7.354e^{-4}$
$K_1^3 = [-0.163 \quad 2.644]$, $K_2^3 = 0.5563$	$K_1^3 = [-0.165 \quad 2.64]$, $K_2^3 = 1$

Thus, the resulting ILC process can be guaranteed with its tracking error converging to zero along the iteration and the time. Figure 1 show the time evolution of the reference trajectory $y_d(t)$ and the output $y_k(t)$ at different value γ, α .It is worth noting that the given conditions in Theorem 2 are obtained by using the lyapunov approach over the time switching and the relaxed parameters α and γ, play a key role to guarantee the asymptotic convergence of the error

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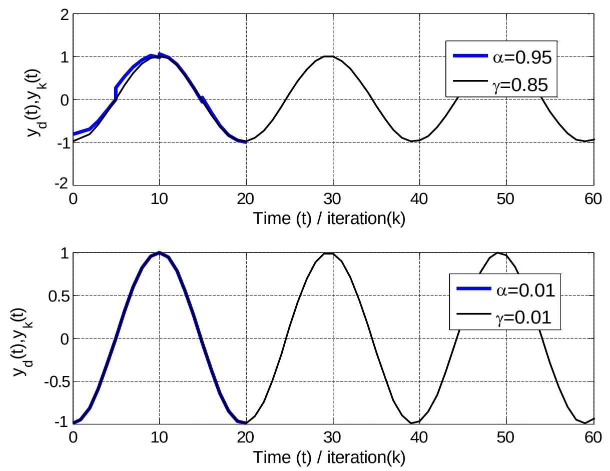

Figure1. The time and iteration evolution of the desired trajectory $y_d(t)$ and the output $y_k(t)$

Figure.2 shows the time and iteration evolution of the output error $e_k(t) = y_d(t) - y_k(t)$. As shown in this figure, the tracking error converging to zero along the time/iteration and more accurate as the iteration number increases.

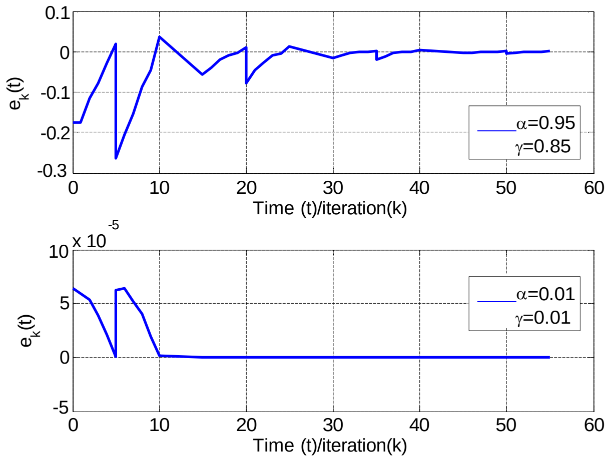

Figure2. Decreasing of the output tracking error as iteration number increased, for different value of γ and α

The previous results show the effectiveness of the repetitive control ILC compared to other conventional control [4]. The (ILC) control achieves the asymptotically stability of the studied closed-loop switched system, and guaranteed a small value of γ.

V. CONCLUSION

In this paper, the problem of ILC-tracking control for a class of discrete-time linear switched systems under arbitrary switching signals in time domain has been investigated. The control law design (ILC) by constructing a sequence of control inputs to a discrete switched system produces 2D switched repetitive system. This can guarantee the asymptotic stability of the closed loop switched system and the convergence error along the time and the pass. Necessary conditions for the existence of such controller are formulated in terms of a set of LMI. A numerical example is given to illustrate the effectiveness of the proposed method.

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