International Conference on Green Energy and Environmental Engineering(GEEE-2017)
International Journal of Scientific Research & Engineering Technology - IJSET
Vol.5 pp.71-75

Modified Extend Least-Squares Algorithm for Fault Detection and Diagnosis

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***Abstract*— Modified extend least-squares parameter estimation algorithm is derived for the special case of ARMAX model identification algorithm. In this method, identification algorithm transform into two sub problems with smaller sizes is used. System identification model and noise identification model. The proposed algorithm has high computational efficiency because the dimensions of its covariance matrices become small. Also, residual generation and evaluation are computed. In simulation, the estimation error of standard recursive least algorithm and two stage extended least squares algorithm are calculated and plotted. the figure shows effectiveness of the proposed algorithm. Next, parameters is estimated using of modified extended least squares algorithm to show the accuracy of the method. Finally, residual is generated and threshold is designed to detect the fault. The figures indicate that the proposed algorithm is sensitive to the faults.**

***Keywords*— Parameter estimation; Identification algorithm; Extended least square estimation; Residual generation**

I. INTRODUCTION

In systems and control theory, models generally contain a number of parameters which are unknown or roughly known. A complete knowledge of these parameters is critical to describe and analyse the dynamics of real-world systems. Also, advanced control and diagnosis algorithms for modern industrial, automotive, and aerospace systems require the accurate knowledge of system parameters. Any control or diagnosis algorithm with poor parameter estimates will have poor performance and could also become unstable. Online parameter-estimation schemes allow these algorithms to have accurate parameter estimates even when subjected to perturbations. Several methods have been used previously to solve the problem of parameter estimation. Adaptive estimations using Kalman filters, recursive least squares and sliding-mode estimators are among the frequently used techniques [1]. Parameter Estimation Techniques is one class of the methods of model-based fault detection. A model-based fault detection scheme consists of two main stages: residual generation and residual evaluation. In most practical cases, the process parameters are not known at all, or they are not known exactly enough. Then, they can be determined by means of parameter estimation methods, measuring input and output signals, $u(t)$ and $y(t)$, if the basic structure of the model is known . This approach is based on the assumption that the faults are reflected in the physical system parameters and the basic idea is that the parameters of the actual process are estimated on-line using well-known parameter estimations methods. The results are thus compared with the parameters of the reference model, obtained initially under fault free assumptions. Any discrepancy can indicate that a fault may have occurred [2]. However, the presence of modeling uncertainties, disturbances, and noise is inevitable. Now, instead of setting deviation of residual from zero as indicator of faults, a threshold which cares for the effect of modeling uncertainties, disturbances, and noise should be selected and if the residual exceeds the selected threshold, it gives an indication of the presence of faults. Selection of threshold is important for a fault detection system. If threshold is selected too low, it will result in false alarms, i.e. some of disturbances will cause the residual to cross the threshold and result in an alarm. If the threshold is selected too high, small faults will not be detected [3]. In this paper, the focus is put on the study of the modified extended recursive least squares algorithm. This paper is organized as follows. In Section 2, model identification of the autoregressive model is described. Section 3, modified extended recursive least squares algorithms (M-ELS) is derived. Residual generation and evaluation are discussed in Section 4. while the simulation results is presented in section 5. Section 6 concludes the paper.

II. MODEL IDENTIFICATION OF THE AUTOREGRESSIVE MODEL

The basic step in identification procedure is the choice of suitable type of the model. General linear model takes the following form, called (a special case of ARMAX model):

$$A(z^{-1})y(k) = B(z^{-1})u(k) + D(z^{-1})n(k) \qquad (1)$$

where

$$A(z^{-1}) = 1 + a_1 z^{-1} + a_2 z^{-2} + \ldots + a_n z^{-n}$$

$$B(z^{-1}) = b_1 z^{-1} + b_2 z^{-2} + \ldots + b_n z^{-n}$$

$$D(z^{-1}) = 1 + d_1 z^{-1} + d_2 z^{-2} + \ldots + d_n z^{-n} \qquad (2)$$

are shift operators polynomials is introduced as $z^{-i}y(k) = y(k-i)$, Where $y(k), u(k), n(k)$ are the sequences of system output, measurable input and stochastic input, or noise, respectively, while the constants a_i, b_j, c_i and d_i represent system parameters [4].

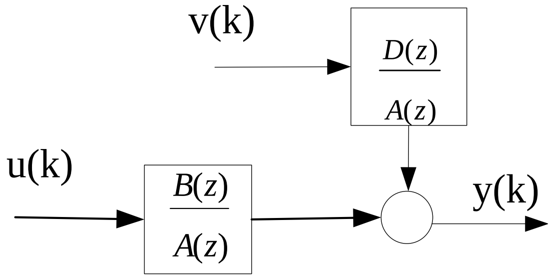

Fig. 1 Model structure for ARMAX system

All linear models can be derived from general linear model by simplification. For special cases of the system in (2), many approaches can estimate their parameters. For example, when (*C*=*D*=1), the system in (2) reduces to an equation error model, or called ARX model (Auto-Regressive model).

$$A(z^{-1})y(k) = B(z^{-1})u(k) + n(k) \quad (3)$$

The recursive parameter estimation algorithms are based on the data analysis of the input and output signals from the process to be identified. This method can be used for parameter estimate of ARX model. The algorithm can be written in following form: Consider linear, time-invariant, discrete-time system, which can be represented by

$$y(k) = -\sum_{i=1}^{n} a_k y(k-i) + \sum_{i=1}^{n} b_k u(k-i) + n(k) \quad (4)$$

Equation (4) can be written in a linear regression form

$$y(k) = Z^T(k)\theta + n(k) \quad (5)$$

where

$$\theta^T = [a_1..a_n, b_1..b_n] \quad (6)$$

Represents vector of constant system parameters and

$$Z^T(k) = [-y(k-1)...-y(k-n) \quad u(k-1)..u(k-n)] \quad (7)$$

Represents a vector of input and output measurable samples (the regression vector) and the residual $v(k)$ is introduced as[5].

$$n(k) = y(k) - Z^T(k)\theta \quad (8)$$

In many practical cases, it is necessary that parameter estimation takes place concurrently with the system's operation. This parameter estimation problem is called on-line identification and its methodology usually leads to a recursive procedure for every new measurement (or data entry). For this reason, it is also called recursive least-squares estimate (RLS) or recursive identification. The proposed recursive algorithm is given by the following theorem. Suppose that $\hat{\theta}(k)$ is the estimate of the parameters of the nth order system for k data entries. Then, the estimate of the parameter vector $\hat{\Theta}(k+1)$ for $(k+1)$ data entries, with $(k = 1,2,3,...)$ is given by the expression [6].

$$\hat{\theta}(k) = \hat{\theta}(k-1) + \gamma(k)[y(k) - Z^T(k)\hat{\theta}(k-1)] \quad (9)$$

The correcting vector is given by

$$\gamma(k) = P(k)Z(k) = [Z^T(k)P(k-1)Z(k)+1]^{-1}Z^T(k)P(k-1) \quad (10)$$

And the matrix $P(k)$ is calculated from the recursive formula

$$P(k) = [I - \gamma(k)Z^T(k)]P(k-1) \quad (11)$$

With initial conditions

$$P(0) = \alpha I \quad and \quad \theta(0) = 0 \quad (12)$$

III. MODIFIED-EXTEND LEAST-SQUARES PARAMETER ESTIMATION ALGORITHM (M-ELS)

To derive the special case of ARMAX model identification algorithm represented by (1), the decomposition technique that transform the original identification problem into two sub problems with smaller sizes is used .First sub problems is system identification model and second sub problems is noise identification model. A System identification model is

$$y_s(k) = -\sum_{i=1}^{n} a_k y(k-i) + \sum_{i=1}^{n} b_k u(k-i) \quad (13)$$

Equation (13) can be written in a linear regression form

$$y_s(k) = Z_s^T(k)\theta_s \quad (14)$$

Where , $\theta_s = [a_1, a_2,..a_n, b_1, b_2,...,b_n]^T$ is vector of constant system parameters and the regression vector or information vector is

$$Z_s^T(k) = [-y(k-1)...-y(k-n), u(k-1)..u(k-n)].$$

The noise identification model is

$$v(k) = D(z^{-1})n(k) \quad \text{or} \quad (15)$$

$$v(k) = \sum_{i=1}^{n} d_k n(k-i) + n(k) \quad (16)$$

the linear regression form is

$$v(k) = Z_n^T(k)\theta_n + n(k) \quad (17)$$

where, $Z_n^T(k) = [n(k-1)..n(k-n)]$ and $\theta_n = [d_1, d_2...,d_n]^T$.

by substituting (15) and (17) into (2), equation (2) can be written as [4].

$$y(k) = Z_s^T\theta_s + Z_n^T(k)\theta_n + n(k) = Z^T\theta + n(k) \quad (18)$$

where , $Z^T = \lfloor Z_s^T \quad Z_n^T \rfloor$ and $\theta = \begin{bmatrix}\theta_s \\ \theta_n\end{bmatrix}$

Because the information vector $Z_n(k)$ in $Z^T(k)$ on the right-hand side of equation (18) contains immeasurable noise terms $n(k-i)$. the standard recursive least squares algorithm as (9) cannot generate the estimate of the parameter vector θ. The solution is to replace these immeasurable variables $n(k-i)$

in $Z_n(k)$) of $Z^T(k)$ with their estimates $\hat{n}(k-i)$, respectively, and define [7].

$$\hat{Z}_n^T(k)=[\hat{n}(k-1)..\hat{n}(k-n)] \quad (19)$$

Then $\hat{Z}^T=\left[Z_s^T \quad \hat{Z}_n^T\right]$, and after system parameters $\hat{\theta}(z)$ be estimated, the estimate $\hat{v}(k)$ can be computed by

$$\hat{v}(k)=y(k)-Z_s^T(k)\hat{\theta}_s(k) \quad (20)$$

The goal is to apply the data filtering technique and to develop a new recursive least squares for estimating the system parameters. If we use the rational fraction $\frac{1}{D(z)}$, the recursive least squares algorithm can be applied. Because $\frac{1}{D(z)}$ is unknown, its estimate $\frac{1}{\hat{D}(z)}$ is used to filter the input–output data [8]. For the model in (2), the filtered inputs $u_f(k)$, the filtered output $y_f(k)$ and the filtered regression vector $Z_f(k)$ are defined as

$$u_f(k)=\frac{1}{D(z)}u(k), \qquad y_f(k)=\frac{1}{D(z)}y(k) \quad (21)$$

$$Z_f^T(k)=[-y_f(k-1) \quad -y_f(k-2)...-y_f(k-n), \\ u_f(k-1) \quad u_f(k-2)..u_f(k-n)] \quad (22)$$

Dividing both sides of (2) by $D(z)$ gives

$$A(z)\frac{1}{D(z)}y(k)=B(z)\frac{1}{D(z)}u(k)+n(k) \quad (23)$$

It can be written as

$$A(z)y_f(k)=B(z)u_f(k)+n(k) \quad (24)$$

This model is (ARX model) and can be rewritten in a linear regression form,

$$y_f(k)=[1-A(z)]y_f(k)+B(z)u_f(k)+n(k) \\ =Z_f^T\theta+n(k) \quad (25)$$

Now, we can compute the estimates $\hat{\theta}_s$ and $\hat{\theta}_n$ of θ_s and θ_n

$$\hat{\theta}_s(k)=\hat{\theta}_s(k-1)+\gamma_f(k)[y_f(k)-Z_f^T(k)\hat{\theta}_s(k-1)] \quad (26)$$

The correcting vector is given by

$$\gamma_f(k)=P_f(k)Z_f(k)= \\ [Z_f^T(k)P_f(k-1)Z_f(k)+1]^{-1}Z_f^T(k)P_f(k-1) \quad (27)$$

And the matrix $P_f(k)$ is calculated from the recursive formula

$$P_f(k)=[I-\gamma_f(k)Z_f^T(k)]P_f(k-1) \quad (28)$$

With initial conditions

$$P_f(0)=\alpha I \quad and \quad \theta(0)=0 \quad (29)$$

With α large ($\alpha=100,\ldots,1000$)

$$\hat{\theta}_n(k)=\hat{\theta}_n(k-1)+\gamma_n(k)[v_n(k)-Z_n^T(k)\hat{\theta}_n(k-1)] \quad (30)$$

$$\gamma_n(k)=P_n(k)Z_n(k)= \\ [Z_n^T(k)P_n(k-1)Z_n(k)+1]^{-1}Z_n^T(k)P_n(k-1) \quad (31)$$

$$P_n(k)=[I-\gamma_n(k)Z_n^T(k)]P_n(k-1) \quad (32)$$

$u_f(k)$, $y_f(k)$, $Z_f(k)$ and $Z_n(k)$ are unknown because of the polynomials $D(z)$ are unknown. So it is impossible to implement the algorithm in (26)–(32) .

The derivation of modified extended recursive least squares (M-ELS) identification algorithms is to replace the unknown variables with their estimate to as

$$v(k)=A(z)y(k)-B(z)u(k)=y(t)-Z_s^T(k)\theta_s \quad (33)$$

Substituting (17) into (33),we get

$$y(k)=Z_s^T(k)\theta_s+v(k)=Z^T(k)\theta+n(k) \quad (34)$$

Replacing θ_s on equation (33) with its estimate $\hat{\theta}_s(k-1)$ and $\hat{v}(k)=y(t)-Z_s^T(k)\hat{\theta}_s(k-1)$. Also the estimate $\hat{v}(k)$ is $\hat{n}(k)=\hat{v}(k)-\hat{Z}_n^T(k)\hat{\theta}_n(k-1)$. The parameter estimation of the noise model is $\hat{\theta}_n(k)=\left[\hat{d}_1(k),\hat{d}_2(k)...,\hat{d}_{nd}(k)\right]^T$. Then $\hat{u}_f(k)=\frac{1}{\hat{D}(z)}u(k)$, $\hat{y}_f(k)=\frac{1}{\hat{D}(z)}y(k)$. Now, the computation of $\hat{u}_f(k)$ and $\hat{y}_f(k)$ are

$$\hat{u}_f(k)=-\hat{d}_1(k)\hat{u}_f(k-1)-\hat{d}_2(k)\hat{u}_f(k-2)- \\ ..\hat{d}_n(k)\hat{u}_f(k-n)+u(k) \quad (35)$$

$$\hat{y}_f(k)=-\hat{d}_1(k)\hat{y}_f(k-1)-\hat{d}_2(k)\hat{y}_f(k-2)- \\ ..\hat{d}_n(k)\hat{y}_f(k-n)+y(k) \quad (36)$$

and

$$Z_f^T(k)=[-\hat{y}_f(k-1),-\hat{y}_f(k-2),\ldots,-\hat{y}_f(k-n) \\ ,\hat{u}_f(k-1),\hat{u}_f(k-2),\ldots,\hat{u}_f(k-n)] \quad (37)$$

Finally, when replacing $Z_f(k)$ with $\hat{Z}_f(k)$, $Z_n(k)$ with $\hat{Z}_n(k)$, $y_f(k)$ with $\hat{y}_f(k)$, and $v(k)$ with $\hat{v}(k)$,we get modified extended recursive least squares (M-ELS) algorithm [8].

$$\hat{\theta}_s(k)=\hat{\theta}_s(k-1)+\gamma_f(k)[\hat{y}_f(k)-\hat{Z}_f^T(k)\hat{\theta}_s(k-1)] \quad (38)$$

The correcting vector is given by

$$\gamma_f(k)=P_f(k)\hat{Z}_f(k)= \\ [\hat{Z}_f^T(k)P_f(k-1)\hat{Z}_f(k)+1]^{-1}\hat{Z}_f^T(k)P_f(k-1) \quad (39)$$

And the matrix $P_f(k)$ is calculated from the recursive formula

$$P_f(k)=[I-\gamma_f(k)\hat{Z}_f^T(k)]P_f(k-1) \quad (40)$$

$$\hat{\theta}_n(k)=\hat{\theta}_n(k-1)+\gamma_n(k)[\hat{v}_n(k)-\hat{Z}_n^T(k)\hat{\theta}_n(k-1)] \quad (41)$$

$$\gamma_n(k) = P_n(k)\hat{Z}_n(k) =$$

$$[\hat{Z}_n^T(k)P_n(k-1)\hat{Z}_n(k)+1]^{-1}\hat{Z}_n^T(k)P_n(k-1) \quad (42)$$

$$P_n(k) = [I - \gamma_n(k)\hat{Z}_n^T(k)]P_n(k-1) \quad (43)$$

$$\hat{v}(k) = y(t) - Z_s^T(k)\hat{\theta}_s(k-1) \quad (44)$$

$$\hat{\theta}_s(k) = \left[\hat{a}_1(k), \hat{a}_2(k), ..\hat{a}_n(k), \hat{b}_1(k), \hat{b}_2(k), \ldots, \hat{b}_n(k)\right]^T \quad (45)$$

$$\hat{\theta}_n(k) = \left[\hat{d}_1(k), \hat{d}_2(k), \ldots, \hat{d}_n(k)\right]^T \quad (46)$$

IV. Residual Generation and Evaluation

In order to detect and isolate faults in a system we need to look for some fault symptoms. The most common fault symptom that is used for fault detection and isolation is residual. The common procedure for fault detection and isolation using residuals is made of two main steps: residual generation, and residual evaluation [9].

A. *Residual Generation*

the residual quantities which are computed as differences between the measured output $y(k)$ and the corresponding output generated by the mathematical model (estimated output $\hat{y}(k)$). In theory, the residuals must be either zero in a fault free case, to indicate that no fault occurs, or non-zero in the case of a fault. However, in practice, deviations normally exist with different magnitudes[10].

$$e(k) = y(k) - \hat{y}(k) \quad (47)$$

The fault detection system consists of two parts, the first is the generation of the fault detection residual and the second is the evaluation of the residual against a specified threshold[11].

B. *Residual Evaluation (Decision Making)*

The residual evaluation examines symptoms for the likelihood of faults and a decision rule is then applied to determine if any faults have occurred. The residual evaluation may perform a simple threshold test (geometrical methods) on the instantaneous values of the residuals[12]. A common choice of evaluation signal is

$$\|e(t)\| = \sqrt{\int_0^t |e(\tau)|^2 d\tau} \quad (48)$$

where τ is the time window.

The threshold is obtained based on the residual dynamics in a fault-free case. For the evaluation signal in (48), the occupancy of faults can be alarmed if

$\|e(t) > T_r\| \Rightarrow$ a fault is detected

and

$\|e(t) < T_r\| \Rightarrow$ no fault is detected.

T_r is the threshold [11].

V. Simulation Results

In order to show the performance of the proposed algorithm, consider the following example

$$A(z^{-1}) = 1 + 0.2z^{-1} + 0.85z^{-2}$$

$$B(z^{-1}) = -0.8z^{-1} + 0.55z^{-2}$$

$$D(z^{-1}) = 1 - 0.3z^{-1}$$

$$\hat{\theta}_s(k) = [0.2, 0.85, -0.8, 0.55]^T$$

$$\hat{\theta}_n(k) = [-0.3]^T$$

The sequence $u(k)$ is generated as a white sequence with a Gaussian distribution of zero mean and unit variance, while the disturbance $n(k)$ is generated as a white noise sequence with zero mean and variance $\sigma^2 = 0.2$. The system parameters were estimated using modified extended recursive least squares algorithm. Fig. 2 shows estimated parameters. Data from n =0 to n =2500 has been used.

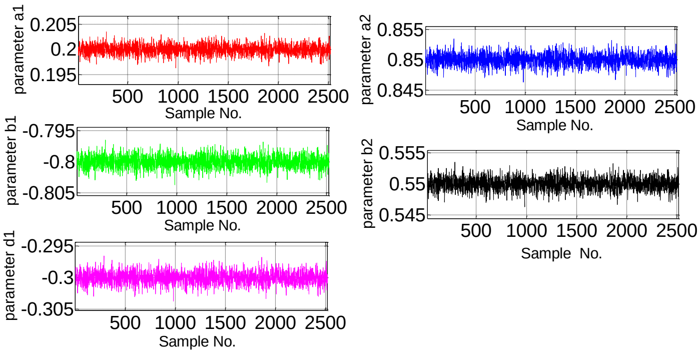

Fig. 2 Estimated parameters for (M-ELS) algorithm

For more clarification, window (frame) of the 100 samples between (1700-1800) is taken. The results shows that the proposed M-ELS algorithm give accurate parameter estimates.

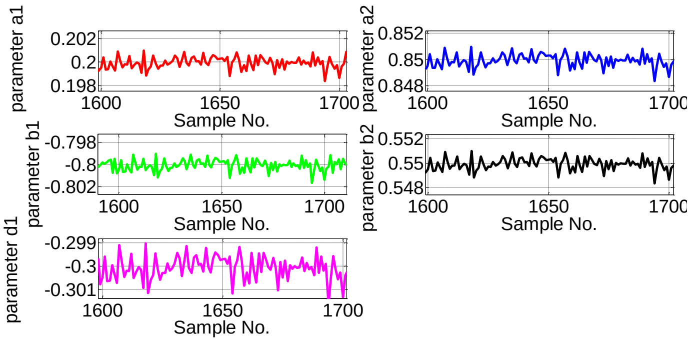

Fig. 3 Estimated parameters for (M-ELS) algorithm over a period between (1700-1800) second

The estimation error is introduced as a measure of the algorithm's effectiveness.

$$\delta := \left\|\hat{\theta} - \theta\right\| / \|\theta\| \quad (49)$$

Fig. 4 shows the estimation errors versus time of standard recursive least algorithm and modified extended least squares algorithm. the figure shows that the parameter estimation errors become smaller and smaller with the time increasing. Also, the estimation errors of the M-ELS algorithm are smaller than those of the RLS algorithm, which means that

ME-RLS parameter estimation have higher accuracy than the RLS parameter estimation.

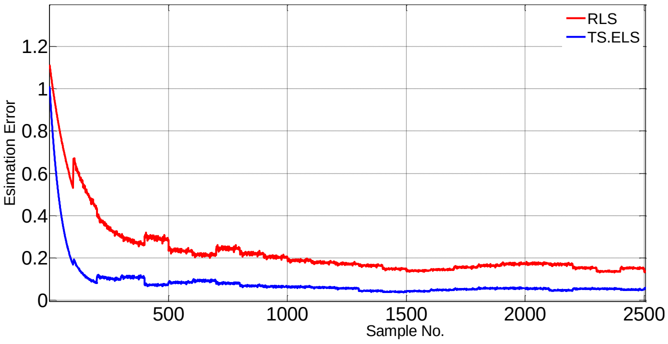

Fig. 4 The estimation errors δ versus n $(\sigma^2 = 0.2)$

Fig. 5 shows the residual and the threshold in free fault situation.

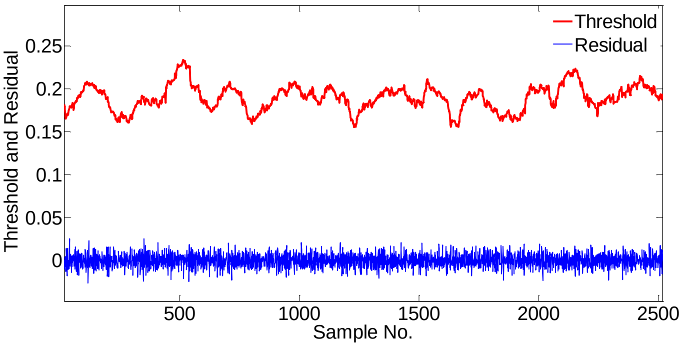

Fig. 5 Threshold design and residual when there is no fault

Fig. 6 shows the residual and the threshold in faulty situation. As shown in Fig. 5, the threshold is sensitive to faults .

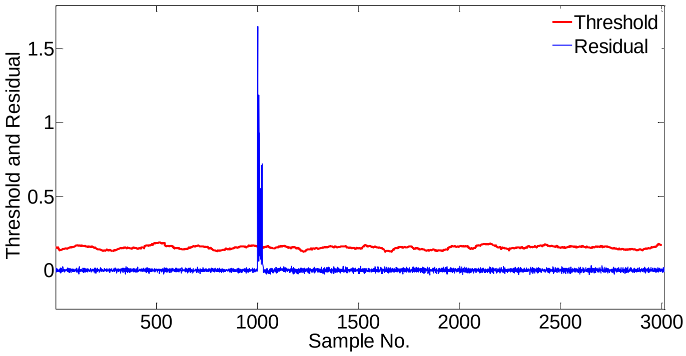

Fig. 6. Threshold design and residual when faults have been detected

VI. Conclusion

Modified extend least-squares parameter estimation algorithm has been derived for a class linear, time-invariant, discrete-time system. The proposed algorithms has less computational load and can give more accurate parameter estimates compared with the conventional recursive least squares algorithm. Also, The proposed algorithm can used to estimate the parameters of linear and non linear systems. Finally, generate a residual and design threshold show the effectiveness of the algorithm.

References

[1] M. Iqbal, A. Bhatti, S. Ayubi, and Q. Khan, “Robust Parameter Estimation of Nonlinear Systems Using Sliding-Mode Differentiator Observer,” IEEE Transaction On Industrial Electronics, Vol. 58, No. 2, Febrary 2011.

[2] M. A. M. A.Shahin, “Fault Diagnosis and Performance Recovery Based on the Dynamic Safety Margin,” PhD thesis, University of Mannheim, 2006.

[3] M. Abid, “ Fault detection in nonlinear systems: An observer-based approach,” PhD thesis, University of Duisburg, July, 2010.

[4] G. Yaoa, R. Ding, “Two-stage least squares based iterative identification algorithm for controlled autoregressive moving average (CARMA) systems,” Computers and Mathematics with Applications,2012, pp. 975–984.

[5] P. NAVRÁTIL and J.IVANKA, “Recursive Estimation Algorithms in Matlab & Simulink Development Environment,” Wseas Transaction On Computers,Vol. 13, 2014.

[6] P.N. Paraskevopoulos, “ Modern Control Engineering,” Marcel Dekker, 2002.

[7] D, Wang and F. Ding, “Input–output data filtering based recursive least squares identification for CARARMA systems,” Digital Signal Processing, 2010, pp. 991–999.

[8] X. Lu, W. Zhou and W. Shi, “Data Filtering Based Recursive Least Squares Algorithm for Two-Input Single-Output Systems with Moving Average Noises,” Journal of Applied Mathematics,Vol. 2014.

[9] M. H. Sobhani and J. Poshtan, “ Fault Detection and Isolation Using Unknown Input Observers With Structured Residual Generation,” International Journal of Instrumentation and Control Systems, Vol.2, No.2, April 2012.

[10] N. Orani, “ Higher-Order Sliding Mode Techniques For Fault Diagnosis,” PhD thesis,Dept. of Electrical and Electronic Engineering, University of Cagliari, March, 2010.

[11] T. Sedighi , P.Phillips and P. d. Foote, “Model-based intermittent fault detection,” 2nd International Through-life Engineering Services Conference, November 05-06, 2013, pp. 68–73.

[12] S. Simani, C. Fantuzzi and R. J. Patton, “ Model-Based Fault Diagnosis in Dynamic Systems Using Identification Techniques, Springer, 2003.

ISSN 2356-5608