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Imprecise assessment of systems performance using generalized stochastic Petri nets

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***Abstract*—In this article, we address the problem of imprecision in assessing the systems performance using Petri nets. The elementary probabilities usually considered in Petri nets are replaced by interval. It allows experts to express their uncertainty concerning the basic parameters of systems and to assess the impact of this uncertainty on the systems unavailability. We show how the imprecision induces significant changes on the systems performance. The proposed method ensures the relevance of the results.**

I. Introduction

The performance evaluation of the industrial systems must be proven by using adopted models. Various techniques nevertheless are recommended in the appendices of the safety standard without however excluding any relevant method of probabilistic calculation. Between the quoted methods, one finds the faults tree, the reliability diagram blocks as well as the Petri nets. The performance evaluation must be obtained by quantitative methods. In this context, Petri nets are good formal models to represent different states that can take and its characteristic parameters can take [1], [2]. For instance, it is possible to model different failure modes of the components, repair operation and common cause failure. In system unavailability studies, probabilities are often considered precise and perfectly known. Real problems are not easily captured with a precise knowledge of the probabilities involved [3]. This problem of imperfect knowledge about the probability values is known and handled in various ways. Probability interval is a simple and attractive representation of imprecision [4]. The problem of precision is considered by other authors using imprecise probabilities [3] or fuzzy numbers [5], [6].

In this work, we propose to use work of Kozine [4] within the framework of evaluation of the systems unavailability by modeling the imprecision on the characteristic parameters knowledge of the components. The second section of the article is devoted to the study of the systems unavailability using stochastic Petri nets. The third section sticks to the modeling of the knowledge imprecision of the system parameters as intervals, and the integration of the imprecise parameters in the stochastic Petri nets for the unavailability evaluation. The last section is devoted to the study of a practicable case relating to the industry of process.

II. Unavailability Modeling

We consider the case of a system that, once broken, can be repaired. The behavior of a repairable system over time is therefore determined not only by the way in which it fails, but also by the way in which it is repaired, and we can consider the life of a system as an alternation between two states: Up (system is functioning) and Down (system is not functioning and it is under repair). It is clear that if a system is subject to failures and repairs. In this case the dependability of the system is characterized by the availability function $A(t)$. The reliability function is defined as the probability that the system is Up at time t.

$$A(t) = Prob\{at\ time\ t,\ state = \mathbf{Up}\} \tag{1}$$

The unavailability $U(t)$ is instead the probability that the system is Down at time t.

$$U(t) = 1 - A(t) = Prob\{at\ time\ t,\ state = \mathbf{Down}\} \tag{2}$$

Assuming that we are able to characterize the failure distribution of a component λ and its repair distribution μ, we have seen how to predict its unavailability. But if a system is a complex aggregate of components, it may be difficult to associate directly to the system a failure and repair distribution. We show how to compute the performance (unavailability) of a repairable system.

The safety system unavailability must be quantitatively proven using suitable models. No particular model is recommended neither in IEC 61508 nor in IEC 61511, nevertheless some of the well known models are cited in their appendices. Among these models, one finds faults tree [7], [8], reliability block diagram [9] as well as Petri nets [5], [2].

The use of generalized stochastic Petri nets allows to take into account the occurrence of faults and their influence on the system behavior [10], [11].

A. Generalized Stochastic Petri Nets

The assessment is associated to the computation of the systems unavailability on demand [12]. In this context, stochastic Petri nets are good formal models of all the system states considering all the events met (failure, maintenance, etc) and all the studied parameters (failure rate, CCF factor, repair

rate, etc) [2]. They bring a relevant modeling suitable for the behavior of the systems studied and are preferred for this work [13], [11]. The Generalized stochastic Petri nets have two different classes of transitions. Timed transitions : which a random exponentially distributed firing delays are associated to transition, immediate transitions : which fire in zero time with priority over timed transitions [10].

A Generalized stochastic Petri nets is an eight-tuple $(P;T;Pre;Post;M_0;V_1;V_2;W)$, where:

- $P = \{p_1, p_2, ..., p_k\}$, is a finite set of places, (drawn as circle).
- $T = \{t_1, t_2, ..., t_l\}$, is a finite set of transition, (drawn as bars).
- *Pre* is the pre-incidence function and defines weighted arcs between places and transitions.
- *Post* is the post-incidence function, which defines weights of arcs from transitions to places.
- $M_0 = (m_{p_{1_0}}, m_{p_{2_0}}, ..., m_{p_{k_0}})$: $M_0 \in \mathbb{N}^+$, M_0 is is the initial marking of place $p \in P$ and defines the number of tokens in the place p.
- $V_1 \subseteq T$ is the set of timed transition, $V_1 \neq \emptyset$
- $V_2 \subset T$ is the set of immediate transitions $V_1 \cap V_2 = \emptyset$, $V_1 \cup V_2 = T$
- $W = (\omega_1, \omega_2, ..., \omega_l)$ is an array whose entry $\omega_i \in \mathbb{R}^+$
  - If t_i is a timed transition, w_i is the parameter of the negative exponential probability distribution function of the transition firing delay.
  - If t_i is a is an immediate transition, w_i is a weight used for the computation of firing probabilities of immediate transitions [11].

Let M_i be a marking of the petri net N, S is a vector having the some dimension of the vector T representing the sequence of fineable transition. If is a firing sequence from M_i and that M_j is reachable from M_i, one notes $M_i[S\langle M_j$ *or* $M_i[\rightarrow M_j$, the new marking is defined by :

$$M_j = M_i + (Post - Pre).S \tag{3}$$

The probability distribution function of the sojourn time in a marking M_i corresponds to the probability distribution function of the minimum among the firing times of the transitions enabled in the same marking; it thus follows that the probability that a given transition $t_k \in E(M_i))$,fires (first) in marking M_i has the expression :

$$P\{t_k/M_i\} = \frac{\omega_k(M_i)}{\sum\limits_{j:T_j \in E(M_i)} \omega_j(M_i)} \tag{4}$$

with ω_k the firing rate of t_k, and $E_j(M_i)$ the set of transitions whose firings bring the net from marking M_i to marking M_j.

The system unavailability is computed as the sum of all P_j where j represents the state probabilities where the system is Down [2].

III. Unavailability Imprecise Modeling

When safety system feedback data is weak and handled probabilities may seem weakly credible, referring to the uncertainty principle (what is precise is more uncertain). The uncertainty on a parameter can be represented in several ways. A probabilistic view based on the Monte Carlo sampling led to the modelling of uncertain parameters by a uniform distribution on the set of values the parameters can take. In our particular case, it can be considered in Petri nets. Another simple representation of imprecision is obtained by interval valued probabilities [6] where no assumption is made about the distribution.

A. Intervals

The imprecision can be represented very easily and suitably by using intervals without making any assumption on the distribution of probabilities. The interval bounds are those used for the distribution. There is no need for a Monte Carlo sampling but it is relevant to base the calculations on the interval theory [14]. By definition, an interval is a closed and bounded set of real numbers. If x indicates a bounded real variable, then the interval $[x]$ to which it belongs to is defined by :

$$[x] = [x_L, x_R] = \{x \in R / x_L \leq x \leq x_R\} \tag{5}$$

where x_L and x_R are real numbers representing respectively the lower and upper bounds of x. The intervals calculation is frequently used to model the imprecision on system parameters. Uncertainties are then represented as interval valued probabilities and the performances calculation is equivalent to a worse case and better case calculation. The interest of this method lies in its simplicity. Nevertheless, interval arithmetic suffers from subdistributivity property when variables are repeated in the model. Thus, the resulting imprecision is more pessimistic (conservative) than necessary.

B. Interval Generalized Stochastic Petri Nets

We mentioned that our knowledge of the characteristic parameters values is imperfect. We model the imprecision of these rates by value intervals as previously defined.

The failure rate is represented by an interval $[\lambda]$ (cf. equation 5). $[\lambda]$ represents the interval of values which can be taken by λ and is bounded by two values $[\lambda_L, \lambda_R]$. the repair rate μ is also modeled by an interval $[\mu]$. The imprecise characteristic parameters $[\lambda]$ and $[\mu]$ integrate directly the Petri nets transition of the studied system (cf. eq. 4).

In order to compute all $[P\{t_k/M_i\}]$, the interval bounds have to be determined. For that, the following equations should be solved :

$$\begin{cases} P_L\{t_k/M_i\} = min(\frac{\omega_k(m)}{\sum\limits_{j:T_j \in E(M_i)} \omega_j(m)}) \\ P_R\{t_k/M_i\} = max(\frac{\omega_k(m)}{\sum\limits_{j:T_j \in E(M_i)} \omega_j(m)}) \end{cases} \tag{6}$$

Our goal is to compute the system performance starting from its imprecise characteristic parameters such as the failure rate and the repair rate, by using the imprecise Petri nets. The system unavailability can be computed by the combination of probability of failure of all sub-system providing the set of safety function.

C. Assumptions

In order to assess the system unavailability, we made the following assumptions:

- All components have constant failure rates λ, *i.e.* the times to failure are exponentially distributed.
- No other types of dependency between components are relevant.
- Common cause failures (CCF) are not considered.

IV. Application: Study of a Safety System

The system given in figure 1 has been studied in [2] and is used for application of the proposed approach. The system is composed of three subsystems. Pressure transmitter (PT), logic solver (LS) and final control element (FC). Upon detection of either high temperature or pressure, the safety system cuts the reactor supply off in order to prevent a runaway reaction. Each subsystem is parallel redundant. The field instrumentation comprises both the pressure and temperature sensor/transmitter subsystems, and the final control element subsystem (basically composed of a set of actuator and shutdown valve). The three

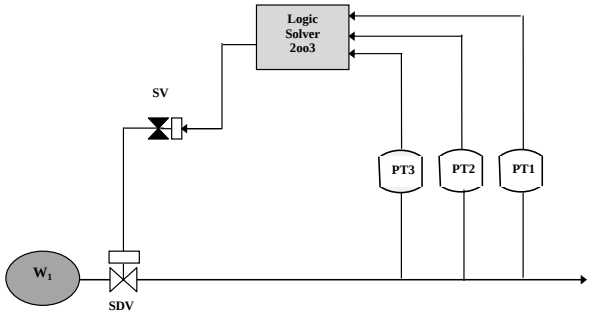

Figure 1: Safety Instrumented System (SIS)

subsystems are to be modelled in a logical series structure. The reliability block diagram of the SIS is given in Figure 2. The

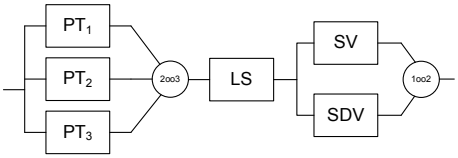

Figure 2: Safety system Reliability Block-Diagram

subsystems of a SIS are illustrated in Figure 2. Each subsystem may have one or more voted groups of channels. A channel is a structure of one or more elements and can independently perform a channel safety function. Using the stochastic Petri nets method proposed in this paper, the system unavailability is determined according to the characteristic parameters of components. The reliability parameters of the SIS components are given in Table II.

The equivalent Petri nets of the studied safety system is given in figure 3.

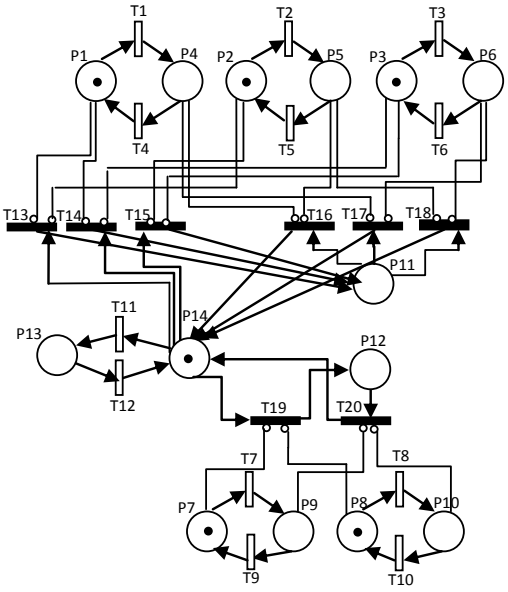

Figure 3: Safety system Petri nets

Table I: Places, transitions and their firing rates used in the model

Places	Interpretation	Transitions	Interpretation
p1	Working state of PT1	T1	Failure rate of PT_1, λ_{PT}
p2	Working state of PT2	T2	Failure rate of PT_2, λ_{PT}
p3	Working state of PT3	T3	Failure rate of PT_3, λ_{PT}
p4	Failed PT1	T4	Repair rate of PT_1, μ_{PT}
p5	Failed PT2	T5	Repair rate of PT_2, μ_{PT}
p6	Failed PT3	T6	Repair rate of PT_3, μ_{PT}
p7	Working state of SV	T7	Failure rate of SV, λ_{SV}
p8	Working state of SDV	T8	Failure rate of SDV, λ_{SDV}
p9	Failed SV	T9	Repair rate of SV, μ_{SV}
p10	Failed SDV	T10	Repair rate of SDV, μ_{SDV}
p11	Failed of actuator layer	T11	Failure rate of LS, λ_{LS}
p12	Failed of sensor layer	T12	Repair rate of LS, μ_{LS}
p13	Failed of logic unit layer	T13→T20	Immediate transition
p14	Working state of LS and SIS		

Using the interval generalized stochastic Petri net method the system performance is determined according to the characteristic parameters of components modeled by interval. The characteristic parameters of the system components are given in table II. The failure rate λ_i as well as the repair rate μ_i of each subset of components are described by intervals provided by experts. Considering only the imprecision of λ_i and μ_i, we can evaluate the influence of the safety system performance.

Table II: Numerical data

System Components	$\lambda(\times 10^{-4}/h)$	$\mu(\times 10^{-2}/h)$
PT_i	[4.30, 6.00]	[1.67, 3.12]
LogicSover	[1.20, 3.10]	[2.10, 4.15]
SDV	[7.26, 9.05]	[8.33, 12.5]
SV	[7.26, 9.05]	[8.33, 12.5]

The system unavailability U_{Sys} can be computed by the com-

bination of probability of failure of all sub-system providing the safety function according to Eq.7. The interval generalized stochastic Petri nets approach allows to simplify the model of the system. It is expressed by the following formulas under the assumption of rare events:

$$U_{Sys}(t) = U_{Sens}(t) + U_{LS}(t) + U_{Act}(t) \tag{7}$$

In this study, we assume that each subsystem is tested independently of each other at its own frequency. Thus, the problem complexity does not increase since each subsystem can be studied independently. Figure 4 shows the system failure probability value. This probability of failure is equal to the asymptotic system unavailability computed by the method of stochastic Petri nets from imprecise characteristic parameters, modeled by value intervals. The distribution of system

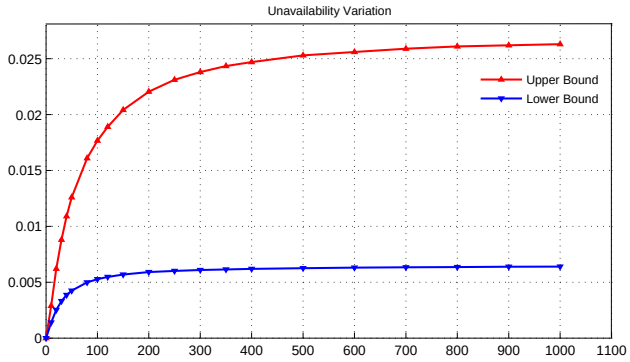

Figure 4: System Unavailability Variation

unavailability is bounded by upper and lower values, according to the interval defined by the characteristic parameters, thanks to monotonic inclusion property of the reliability function of the system. The resulting unavailability is the interval given in Figure 4. This failure probability varies from 0.6402×10^{-3} to 2.643×10^{-3}. As shown in Figure 4, the uncertainty on the failure and repair rates lead to a possible change in the performance level of system, whereas an uncertain but precise value would have provided only one performance of safety system. If a performance classification without ambiguity is desired, it is then necessary to change either the set of components or the system architecture (level of redundancy) or increase our knowledge on the characteristic parameters to reduce its uncertainty.

V. Conclusion

In this article, a approach by intervals in stochastic Petri nets to assess the safety system performance has been proposed. This approach uses value intervals to represent the uncertainty on the probability of failure of the safety system components. Failure and repair rates are considered in this study. The complex nature of these failures makes their quantification more difficult and more uncertain. The proposed approach allows the analysis of the influence of imperfect knowledge of several factors to the imprecision of the SIS unavailability. It clearly shows that characteristic parameters are influencing the results. So, the analysis simultaneously provides an assessment and a sensitivity analysis at the same time. The obtained interval value of the unavailability shows that the imprecision due to imperfect knowledge could involve variations concerning the level of the performance of the Safety system. These variations can put the decision maker in a risky situation that asks for a dedicated strategy to reduce the qualification uncertainty in order to reduce the legal responsibility of the decision maker.

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