Walk Trajectory Generator using Previous Control

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Abstract—A new control approach on the trajectory generator of humanoids robots using predictive control and based on the model of the cart-table, this proposed cart-table model is used to simplify the walking model. On of the purpose of this new control approach based on predictive ZMP model, in the quadratic form, is to make walking smoother and more efficient and avoids the fall down of the robot.

Keywords— **Walking robot, Cart table model, ZMP model. Quadratic program.**

I. INTRODUCTION

The controls of humanoid walking robot present a common problem in robotics. This type of control involves a physical interaction between an articulated system and its environment. This close relationship is actually a common set of fundamental problems such as the planning and implementation of robust stable dynamic movements.

The scientific research on the generation of walking humanoid robot is currently a very active area in robotics. The stability during the walking motion is an essential fact that preventing the robot from falling and causing damage to humans or himself.
This work tries to solve the problem of generating a stable walking of humanoid robots.
The proposed control systems in the literature are based on the use of reference trajectories to be followed in real time. The pattern generator is very important in the humanoid run command. Several types of pattern generators were then proposed. However, those which ensures a priori stability are often based on the following stabilization criteria, the center of mass (COM) [1-2] is the average location of all the masses of robot link, it is generally used as a static stability criterion and the moment Zero Point (ZMP) [1-2-5] is the junction between the vertical reaction forces and soil, which is the dynamic stability criterion.
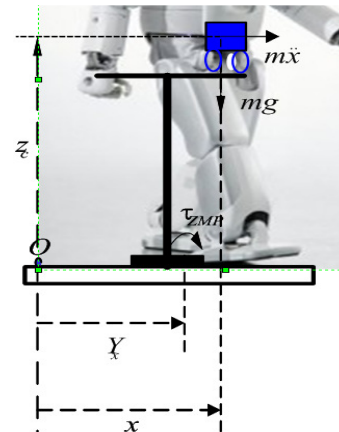

Fig.1 Walking robot is modeled by a cart moving on a no mass table.

m : The mass
Y_x : The position of the COP (ZMP)
x : The horizontal position of COM (center of mass)
$\ddot{x}$: The horizontal acceleration
z_c : The altitude
g: The norm of the gravity

The first part of this paper presents the ZMP model, the second part presents the previous pattern generation based on ZMP control in order to generate a dynamically stable motions through a cart table approximation for the dynamics of the Mass Center of a humanoid robot. The third part present the stability analysis of the walking robot and the forth part present a quadratic limitation of the previous ZMP.

II. ZMP MODEL

An easy way to comply with the conference paper formatting requirements is to use this document as a template and simply type your text into it.
This simplified model proves an extreme efficiency in the context of walking pattern generation.
The equation of motion is represented by:

$$\tau_{ZMP} = -mg(x - p_x) + m\ddot{x}\,z_c = 0 \qquad (1)$$

Where

$$Y_x = x - \frac{z_c}{g}\ddot{x} \qquad (2)$$

Where x is the horizontal position of the CoM, $\ddot{x}$ is the horizontal acceleration, zc is the altitude and g is the norm of the gravity force. This approximation naturally decouples both the forward and the lateral motions of the robot in the analysis of its CoP. Introduced for the first time in the reference [5] the ZMP Preview Control scheme is proposed to generate a trajectory for the CoM of a humanoid robot under the constraint that the footsteps are fixed and impossible to be change [6-7].The constraint is therefore that the trajectory of the CoP given by equation (2) always stays within the convex hull of these fixed footprints. An additional simply assumption is that the altitude zc of the CoM be constant.

Then we deduce Y_x = x which means that the ZMP is at the same position as the center of mass. In this case, the static balance is achieved when the center of mass, hence the ZMP, lies inside the support polygon.

III. PREVIOUS PATTERN GENERATOR WALK

As $\ddot{x}$ increases, the ZMP moves further away from the center of mass. Above a certain level, the ZMP in equation (2) will lie outside of the support polygon. Since the physical ZMP by definition al way falls inside the support polygon. This condition means that the robot, or the cart-table for that matter, cannot hold a plan contact with the floor. So the system is unstable.

A two-dimensional simplified model has been chosen in order to work more rapidly on the control law. It is constituted of a cart moving on top of a table: it is the cart-table. By controlling the force applied on the cart, it is possible to make the table move. The two edges of the foot of the table can be seen as the two feet of the biped robot and the CoM of the cart table is considered smaller to the mass of the biped robot. The cart-table has already been used as a linearized inverse pendulum to model a biped robot during the single support phase [2].

Kajita et al. [1] propose, for this pattern generation method, considering the jerk of the center of mass:

$$\frac{d\ddot{x}}{dt} = u_x \qquad (3)$$

With the introduction of this new quantity, the equation of movement becomes (only the resulting equations along x

direction, the analogy for y direction is completely straight forward).

$$\frac{d}{dt}\begin{bmatrix} x \\ \dot{x} \\ \ddot{x} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ \dot{x} \\ \ddot{x} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u_x$$

$$Y_x = \begin{bmatrix} 1 & 0 & \dfrac{z_c}{g} \end{bmatrix}\begin{bmatrix} x \\ \dot{x} \\ \ddot{x} \end{bmatrix} \qquad (4)$$

The second equation is obtained from previous results in the cart-table model of the ZMP. This is a classic dynamic system with Y_x as known constant (predefined desired ZMP). Discretizing the equation for a time period T we obtain

$$\begin{cases} x(k+1) = Ax(k) + Bu(k) \\ Y(k) = Cx(k) \end{cases} \qquad (5)$$

Where

$$\begin{cases} A = \begin{bmatrix} 1 & T & \dfrac{T^2}{2} \\ 0 & 1 & T \\ 0 & 0 & 1 \end{bmatrix}; \; B = \begin{bmatrix} \dfrac{T^3}{6} \\ \dfrac{T^2}{2} \\ T \end{bmatrix} \\[2em] C = \begin{bmatrix} 1 & 0 & -\dfrac{z_c}{g} \end{bmatrix} \end{cases} \qquad (6)$$

Where $x = \begin{bmatrix} x & \dot{x} & \ddot{x} \end{bmatrix}^T$ a control variable, u is the jerk $u = \dddot{x}$

The idea behind the ZMP Preview Control scheme proposed in the reference [11] is therefore to minimize this jerk while maintaining a position Y of the CoP as close as possible to some prescribed reference positions Yref . So that the output of the system follows as close as possible to the ZMP target, considering the problem of minimizing the performance index:

$$J = \sum_{j=1}^{\infty} \{(Y - Y_{ref})^T Q(Y - Y_{ref}) + \dddot{x}^T R\, \dddot{x}\} \qquad (7)$$

IV. WALKING STABIL

With the linear system (6), we can describe np values of the system output, The recursive relation (3) can be iterated np

times and combined with np versions of the relation (4) in order to relate at once np values of the jerk $\dddot{x}$ of the CoM with np values of the position Y of the CoP:

$$\underbrace{\begin{bmatrix} y(k+1|k) \\ y(k+2|k) \\ \vdots \\ y(k+np|k) \end{bmatrix}}_{Y_{k+1}} = \underbrace{\begin{bmatrix} CA \\ CA^2 \\ \vdots \\ CA^{np} \end{bmatrix}}_{F} x(k) +$$

$$\underbrace{\begin{bmatrix} CB & 0 & \cdots & 0 \\ CAB & CB & \cdots & 0 \\ & & \vdots & \\ CA^{np-1}B & CA^{np-2}B & \cdots & CA^{np-nc}B \end{bmatrix}}_{\Phi} \underbrace{\begin{bmatrix} u(k) \\ u(k+1) \\ \vdots \\ u(k+nc-1) \end{bmatrix}}_{\dddot{x}} \quad (8)$$

$$Y_{k+1} = F \hat{x}_k + \phi \dddot{x}_k \quad (9)$$

The Quadratic program of equation (7) can be solved analytically by the equation (10):

$$\dddot{X}_k = -(\phi^T \phi + \frac{R}{Q} I_{N \times N})^{-1} \phi^T (F \hat{x}_k - Y^{ref}_k) \quad (10)$$

Where IN×N is an identity, and we can propose:

$$w = -(\phi^T \phi + \frac{R}{Q} I_{N \times N})^{-1}$$

$$v = \phi (F \hat{x}_k - Y_{ref})$$

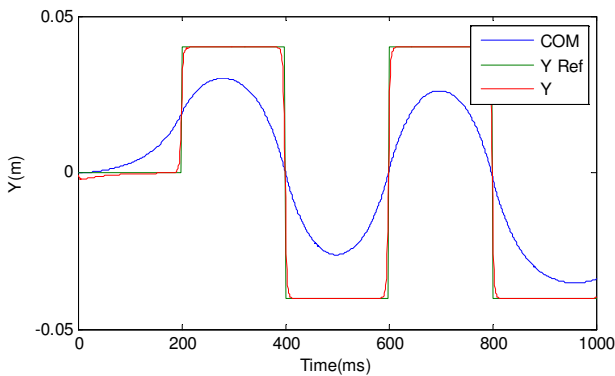Then we can simplify the equation (10):

$$\dddot{X}_k = -w^{-1}v \quad (11)$$

Fig.2 scenario 1: ZMP output and COM response follow the proposed ZMPref
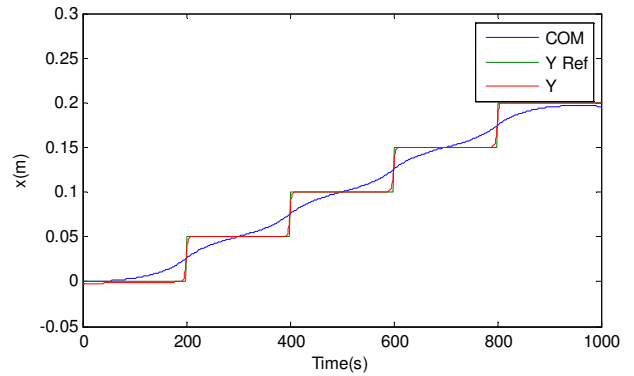

Fig.3 scenario 2: ZMP output and COM response follow the proposed ZMPref

V. THE LIMITS OF THE ZMP POSITION

The important limitation is the reference trajectory of the ZMP model. The role of this proposed quadratic limitation is to make sure that the ZMP is at the center of the convex hull.

$$Fx_k + \phi \dddot{x}_k \leq Y_K^{MAX}$$

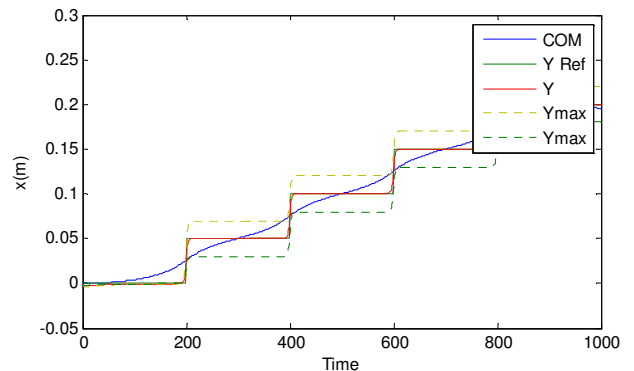$$-Fx_k - \phi \dddot{x}_k \leq -Y_K^{Min} \quad (12)$$

Fig.4 ZMP limitation of ZMPref

VI. CONCLUSION

This paper present a new control theory to generate a smoothly walk for humanoid robot. The application and the simulation result of the ZMP predictive control apply for the control of a nonlinear cart table model present a good performance. ZMP follow the COM trajectory response. The COM move before the ZMP reference value changes so as to minimize global ZMP error

REFERENCES

[1] Kajita. S,Hirukawa. H ,Humanoid Robot,Ohmsha Ltd, Tokyo,Japon.(2005) .

[2] Kajita .S, Kanehiro. F,Kaneko. K Yokoi. K, wa. H. H, The 3DLinear Inverted Pendulum Model: A simple modeling for a biped walking pattern generation,IEE,(2001)..

[3] Clarke, D. W., Mohtadi, C., Tuffs, P. S. Generalized Predictive Control – Part II. Extensions and Interpretations", Automatica, V ol 23, No.2, pp. 149-160. (1987b).

[4] Garcia, C. E., Prett, D. M., Morari, M. Model Predictive Control: Theory and Practice – a Survey, Automatica, Vol 25, No. 3, pp. 335-348. (1989).

[5] Camacho, E. F., Bordons, C. Model Predictive Control, Springer Verlag, London Limited.(2007).

[6] Garcia, C. E., and A. M.Morshedi. Quadratic programming solution of dynamic matrix control (QDMC), Chem. Eng. Comm., 46, 73-87.(1986).

[7] Touati . N, Chemori. A, Predictive control for the stabilization of a fast mechatronic system : from simulation to real-time experiments, The 6th IFAC Symposium on Mechatronic Systems (Mechatronics 2013), Hangzhou : China (2013).

[8] Touati,N.Chemori. Predictive control for the stabilization of a fast mechatronic system: from simulation to real-time experiments, The 6th IFAC Symposium on Mechatronic Systems (Mechatronics 2013), Hangzhou: China (2013).

[9] S. Kajita,H.Hirukawa, Humanoid Robot Ohmsha Ltd 2005, Tokyo,Japon.

[10] R D Kokate, L.M.Waghmare. Research and Application of predictive control based on state feedback theory in power plant control system",

Volume 1 – No. 26, International Journal of Computer Applications. (2010).

[11] Mrabet. A , Cu. J, Generalized predictive control for single link exible joint robot. IJ-STA volume n1 (2009).

[12] Wang W, A Direct Adaptive Generalized Predictive Control Algorithm with Guaranteed

Stability, in Int. J. of Adaptive control and Signal Processing (1994).