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Cu and SiO2 Nanoparticules Behavior in a Kerosene based flow through Convergent/Divergent Channels.

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***Abstract*— This paper is devoted to study the impact of nanofluids in a convergent/ divergent flow. The chosen nanoparitcules for this study are Cu and SiO_2 nanoparticules with kerosene as the considered base fluid.. The study employs a dual approach, utilizing analytical methods (specifically, the Duan-Rach method) and numerical solutions (implemented through the Runge-Kutta scheme). The results are obtained, presented, and subjected to a comparative analysis. The discussion focuses on the dimensionless velocity and friction analysis, examining their behavior concerning the pertinent parameters in the research. It was also deduced that the flow's speed varied from the base fluid flow differently for every nanofluid, which means that every nanofluid has its own special effect on the flow.**

***Keywords*— C/D channel, Nanofluid, Skin friction, DRA, Velocity**

Nomenclature and Abbreviations:

ADM	-Adomian Decomposition Method	J-HF	-Jeffery-Hamel flow
C/D	-Convergent/Divergent	K	-Velocity
Cf	-Skin friction coefficient	Re	-Reynolds number
DRA	-Duan Rach Approach	α	-Angle between two plates
DTM	-Differential Transformation Method	ϕ	-Solid volume fraction
$F(\psi)$	-Dimensionless velocity	θ	-Angular coordinate
HAM	-Homotopy Analysis Method	ρ	-Density
HPM	-Homotopy Perturbation Method	μ	-Dynamic viscosity

I. Introduction

Over the few past years, studying fluid behavior across various geometries has captured the researchers' full attention. A geometry that has been prominently explored in recent years is the non-parallel channel, which can be either convergent or divergent. The flow inside these channels is known as the Jeffrey-Hamel flow, a phenomenon initially formulated by George Barker Jeffrey [1] and Georg Hamel [2]. This flow is characterized as the movement of an incompressible, viscous and two-dimensional fluid through a non-parallel walls channel, which may take the form of either convergence or divergence. This versatile flow pattern is observed in diverse applications such as

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pipe sections, gas compressors, chemical vapor deposition reactors, river flows, and even in biological systems like blood circulation, where arteries and capillaries are interconnected [3]. The broad spectrum of fields and applications in which this type of flow occurs underscores the significance of its thorough investigation, as evidenced by the extensive research conducted by numerous scholars.

In the early 1990s, Professor Choi Sus [4] introduced the term "nanofluid" for the first time. He defined a nanofluid as the mixture of submicronic solid particles that he named “nanoparticles” ranging in size from 1 to 100 nanometers, combined with a liquid known as the base fluid. Ethylene glycol, oil or water are some exemples of the most used base fluids. By adding nanoparticles to a normal fluid , the performance of its heat transfer get enhanced greatly by increasing the coefficient of the convective heat transfer and the thermal conductivity of the mixture [5]. Nanofluids also demonstrate a wide range of properties, that includes magnetic properties, electrical, some optical and mechanical attributes, making them widely applicable not only in the engineering of heat transfer systems but also in diverse fields such as biomedicine, aerospace, electromechanics, and energy [6].A plethora of studies have focused on nanofluids in the context of flow inside convergent/divergent (C/D) channels. For instance, in the paper by Moradi et al. [7], they studied the impact of introducing Alumina (Al2O3), Copper (Cu), and Titania (TiO2) nanoparticles into a water-based fluid on Jeffrey-Hamel flow (J-HF). They used the differential transformation method (DTM) to find an analytical solution, which they then compared with the results they obtained numerically using the Runge-Kutta (RK) method, demonstrating excellent agreement. It is also worth mentioning that they found that, compared to Titania and Copper, the skin friction (Cf) for Alumina (Al_2O_3) was higher. In 2016, Sari et al. [8] made an investigation on the heat transfer of Cu-water nanofluid in a C/D channel. They concluded that, within non-parallel surfaces, thermal transference is greatly enhanced with the introduction of these nanoparticles. The paper published by Ullah [9] delved into the study of a hybrid nanofluid obtained by mixing Silver (Ag) and Titanium dioxide (TiO_2) nanoparticles into blood as the chosen base fluid. The flow passes through a C/D channel with shrinkable/stretchable walls, incorporating couple stress for drug delivery applications. The analysis highlighted the important role of the couple stress parameter in blood flow analysis, and it was concluded that hybrid nanofluids greatly improve heat transfer performance. The analysis conducted by Usman et al. [10] presented the characteristics of a hybrid nanofluid composed of cobalt ferrite ($CoFe_2O_4$) and copper (Cu) nanoparticles in a water-based fluid passing through a sheet that stretches exponentially, considering the thermal slip factors and the velocity . The study concluded that the temperature of the hybridized flow was enhanced by the Darcy-Forchheimer medium, the geometric parameters of the section, and the non-slip condition.In 2023, Alqahtani et al. [11] studied the mass and energy transfer in a Casson hybrid nanofluid (a mixture of Cu and Al2O3 nanoparticles suspended in blood). It was found that the energy transfer rate of the hybrid nanofluids was reduced due to the effects of the Darcy-Forchheimer medium and the suction parameters.The work of Habiyaremye [12] examines the effects of low-intensity magnetism on the hydromagnetics of Ag-water nanoparticles in C/D channels. The study finds that improved energy distribution increases temperature and reduces the nanofluids concentration, which facilitates the heat transfer of the nanoparticles. In the same year, Vinutha et al. [13] investigated the flow of nanofluid between C/D channels. Their study focused on the impact of pollutant concentration, nanoparticle diameter, and the interfacial layer on thermal conductivity. The differential equations describing this flow were solved using the Galerkin finite element method (GFEM). The results show that changes in Schmidt number and external source variation had different impacts in C/D channels. The study by Wachira et al. [14] investigated the flow of non-Newtonian nanofluids through linearly stretching C/D conduits with a chemical reaction. The study highlights the promising potential of adding nanoparticles, such as copper, which enhance heat transfer, reduce wear and tear, and boost cooling rates, making it ideal for applications that require cooling machinery in very high-temperature environments.

The fact that the problem governing the flow through a C/D channel does not have an exact analytical solution is a well-known fact in the research world. This has pushed various researchers to develop and test new approximate methods to solve the governing equations associated with this flow, which gave birth to numerous mathematical methods. Some of the most used approaches are the (HPm) Homotopy Perturbation Method [15,16], (HAm) Homotopy Analysis Method [17,18], (DTm) the Differential Transform Method [19,20], and (ADM) the Adomian Decomposition Method. George Adomian introduced the ADM in 1986 [21]. Compared to other analytical approaches, the ADM approximates analytically a vast range of nonlinear equations without resorting to perturbation, linearization, or discretization methods. Duan and Rach [22] presented, in 2011, a modified version of the ADM that allows for finding the solution to a broad range of multi-point and multi-order nonlinear BVPs (Boundary Value Problems) in a simpler way. Dib et al. [23] used the DRA (Duan-Rach Approach) to find solutions

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to the classical Jeffrey-Hamel flow. Using the DRA helped them obtain the solutions without evaluating the second derivative at the starting point, which usually requires additional numerical methods. The obtained results were compared to those found by the numerical RK method and the HAM, demonstrating perfect agreement and proving the high accuracy of the DRA approach.

In aircraft, kerosene is typically transferred from fuel tanks to the engines using a system of valves, pumps, filters, and pipes. Although converging/diverging channels are generally not the primary design feature of these systems, in certain parts of the fuel transfer system, converging channels can be found where the cross-sectional area of the fuel lines decreases as the fuel approaches critical components like fuel injectors. Generally, fuel lines or main transfer channels are wider and may diverge slightly to prevent cavitation (where pressure drops too low) or turbulence as fuel moves through larger pipes.This paper aims to investigate how the introduction of two specific nanofluids (Copper, Cu, and Silica, SiO_2) with kerosene serving as the base fluid affects the flow through convergent/divergent (C/D) channels. Particular emphasis is placed on skin friction, and the velocity profile within these channels is scrutinized. The analysis includes important parameters such as the Re (Reynolds number), α (the opening angle) and the SVF (solid volume fraction of the nanofluid) (ϕ). The analytical solutions for the nonlinear differential equations governing this flow are obtained using the modified Adomian method (DRA), and these analytical outcomes are then compared with results obtained using the Runge-Kutta scheme (RK).

II. MATHEMATICAL FORMULATION OF THE HYDRODYNAMICAL PROBLEM

This study examines the flow coming from a source at the intersection of two solid that form an angle 2α in a Kerosene-based nanofluid containing SiO_2 and Cu nanoparticles. It is assumed that the C/D channel has a macro-scale, thus adhering to the no-slip condition at the wall. Additionally, in the context of incompressible laminar flow, the same velocity is shared by both nanoparticles and the base fluid. It is noteworthy that the walls are characterized as forming a convergent channel when α < 0 and a divergent one when α > 0 and. Utilizing plane polar coordinates (r, θ), we assume a purely radial velocity dependent on r and θ, denoted as K(k(r, θ), 0) [24].

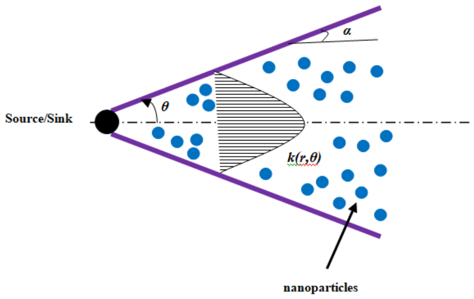

Fig. 1 General geometry of the J-HF problem

The continuity and Navier-Stokes formulas for a nanofluid J-HF through a C/D channel are:

$$\frac{\rho_{nf}}{r}\frac{\partial}{\partial r} r k(r,\theta) = 0. \tag{1}$$

$$k(r,\theta)\frac{\partial k(r,\theta)}{\partial r} = -\frac{1}{\rho_{nf}}\frac{\partial p}{\partial r} + \nu_{nf}\left[\frac{\partial^2 k(r,\theta)}{\partial r^2} + \frac{1}{r}\frac{\partial k(r,\theta)}{\partial r} + \frac{1}{r^2}\frac{\partial^2 k(r,\theta)}{\partial \theta^2} - \frac{k(r,\theta)}{r^2}\right]. \tag{2}$$

$$-\frac{1}{\rho_{nf} r}\frac{\partial p}{\partial \theta} + \frac{2\nu_{nf}}{r^2}\frac{\partial k(r,\theta)}{\partial \theta} = 0. \tag{3}$$

The specified boundary conditions are :

At the centerline of the channel: $\frac{\partial k(r,\theta)}{\partial \theta} = 0.$

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At the walls of the channel: $k(r,\theta) = 0$.

Where p represents the fluid pressure, ρ_{nf} represents the density of the nanofluid, and ν_{nf} is its the coefficient of kinematic viscosity. The latter two parameters are expressed by the following definitions [25-26]:

$$\mu_{nf} = \frac{\mu_f}{(1-\phi)^{2.5}}; \quad \rho_{nf} = (1-\phi)\rho_f + \phi\rho_s. \tag{4}$$

Here, ϕ represents the SVF of the nanofluid, ρ_s is the density of the solid nanoparticle, ρ_f defines the density of the chosen base fluid, and μf denotes its viscosity. The expression for this viscosity has been provided in Reference [25]. From Equation (1) we have:

Using dimensionless parameters

$$f(\theta) = rk(r,\theta). \tag{5}$$

Using dimensionless parameters

$$F(\psi) = \frac{f(\theta)}{f_{max}} \quad ; \quad \psi = \frac{\theta}{\alpha}. \tag{6}$$

By eliminating the pressure (P) between Equation (2) and (3), we find a third-order ODE (Ordinary Differential Equation) for the normalized function profile $F(\psi)$:

$$F'''(\psi) + 2\alpha Re\left[(1-\phi)^{2.5}\left(1-\phi+\phi\frac{\rho_s}{\rho_f}\right)\right]F(\psi)F'(\psi) + 4\alpha^2 F'(\psi) = 0. \tag{7}$$

The boundary conditions are expressed as :

$$F(0) = 1\,; \qquad F(1) = 0; \qquad F'(0) = 0\,. \tag{8}$$

And the Reynolds number is defined as:

$$Re = \frac{K_{max} r \rho_f \alpha}{\mu_f} \;; \quad (\alpha > 0,\ K_{max} > 0\,,\ \textit{for a divergent channel}) \tag{9}$$

$$(\alpha < 0,\ K_{max} < 0,\ \textit{for a convergent channel})$$

The expression of skin friction coefficient is expressed as[27]:

$$C_f = \frac{\tau_w}{\rho_f K_{max}^2}. \tag{10}$$

$$\tau_w = \mu_{nf}\left(\frac{1}{r}\frac{\partial k(r,\theta)}{\partial \theta}\right) \tag{11}$$

Employing Equations (16),(21) and (22) will get when $\eta = 1$:

$$C_f = \frac{1}{Re(1-\phi)^{2.5} f'(1)} \tag{12}$$

III. SOLUTION BY APPLICATION OF THE DRA

From Equation (7), we have:

$$F'''(\psi) + 2\alpha Re\left[(1-\phi)^{2.5}\left(1-\phi+\phi\frac{\rho_s}{\rho_f}\right)\right]F(\psi)F'(\psi) + 4\alpha^2 F'(\psi) = 0. \tag{13}$$

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The inverse linear operator is defined as:

$$L^{-1}(.) = \int_0^{\psi}\int_0^{\psi}\int_0^{\psi}(.)d\psi d\psi d\psi \tag{14}$$

Applying Equation (14) on Equation (13) and after using the boundary conditions of Equation (8) on it, we have:

$$F(\psi) = F(0) + F'(0)\psi + F''(0)\frac{\psi^2}{2} + L^{-1}\big(NF(\psi)\big). \tag{15}$$

With:

$$NF(\psi) = -2\alpha Re\left[(1-\phi)^{2.5}\left(1-\phi+\phi\frac{\rho_s}{\rho_f}\right)\right]F(\psi)F'(\psi) - 4\alpha^2F'(\psi) \tag{16}$$

By putting ψ = 1 in Equation (15), we obtain:

$$F''(0) = -2\times(L_1^{-1}NF(\psi) - 1) \tag{17}$$

Where:

$$L_1^{-1}(.) = [L^{-1}(.)]_{\psi=1} = \int_0^{1}\int_0^{\psi}\int_0^{\psi}(.)d\psi d\psi d\psi. \tag{18}$$

By Substituting Equation (17) into Equation (15), we find:

$$F(\psi) = 1 - \psi^2 + L^{-1}[NF(\psi)] - \psi^2L_1^{-1}[NF(\psi)]. \tag{19}$$

Finally, the modified recursive scheme is :

$$F_0(\psi) = 1 - \psi^2. \tag{20}$$

$$F_{n+1}(\psi) = 1 - \psi^2 + L^{-1}A_n(\psi) - \psi^2L_1^{-1}A_n(\psi) \quad ; n \geq 0 \tag{21}$$

Where the An(ψ) are the Adomian polynomials, which can be found using the formula:

$$A_n(\psi) = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[N\sum_{i=0}^{n}\lambda^i F_i(\psi)\right]. \tag{22}$$

By applying Equation (22), we find the following terms of the Adomian polynomials:

$$A_0(\psi) = -2\alpha Re\left[(1-\phi)^{2.5}\left(1-\phi+\phi\frac{\rho_s}{\rho_f}\right)\right]F_0(\psi)F_0'(\psi) - 4\alpha^2F_0'(\psi)$$

$$A_1(\psi) = -2\alpha Re\left[(1-\phi)^{2.5}\left(1-\phi+\phi\frac{\rho_s}{\rho_f}\right)\right]f_1(\psi)f_0'(\psi) - 2\alpha Re\left[(1-\phi)^{2.5}\left(1-\phi+\phi\frac{\rho_s}{\rho_f}\right)\right]f_0(\psi)f_1(\psi) - \ldots \tag{23}$$

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To determine Fn(ψ), we have:

$$F_0(\psi) = 1 - \psi^2.$$

$$F_1(\psi) = -\left(0.1333333333\alpha Re\left[(1-\phi)^{2.5}\left(1-\phi+\phi\frac{\rho_s}{\rho_f}\right)\right] + 0.3333333333\alpha^2\right)\psi^2 - 0.0333333333\alpha Re\left[(1-\phi)^{2.5}\left(1-\phi+\phi\frac{\rho_s}{\rho_f}\right)\right]\psi^6 + \cdots. \quad (24)$$

The functions F2(ψ), F3(ψ), F4(ψ)...etc,can be found using the same steps from Equation (21). For convenience, we only represent the first terms of Fn(ψ). We finally obtain:

$$F_n(\psi) = -\psi^2 + 1 - \left(0.1333333333\alpha Re\left[(1-\phi)^{2.5}\left(1-\phi+\phi\frac{\rho_s}{\rho_f}\right)\right] + 0.3333333333\alpha^2\right)\psi^2 - 0.0333333333\alpha Re\left[(1-\phi)^{2.5}\left(1-\phi+\phi\frac{\rho_s}{\rho_f}\right)\right]\psi^6 + \cdots. \quad (25)$$

It is worth mentioning that the more in the number of solution terms n increases The higher the accuracy Fn(ψ).

IV. RESULTS AND DISCUSSION

In this research, we applied the DRA to obtain an analytical solution for the nonlinear differential problem described by Equation (13). The resulting analytical solution was then compared with numerically obtained solutions using the Runge-Kutta technique (RK). The study also investigates the influence of pertinent physical parameters on the velocity profile and skin friction for both SiO_2-kerosene and Cu-kerosene nanofluids. Table I provides the thermophysical properties of the nanoparticles and the base fluid used in this study.

TABLE I
KEROSENE AND NANOPARTICULES THERMOPHYSICAL PROPERTIES

Material	Density(Kg.m−3)
Kerosene	780
Cu	8933
SiO2	2220

Table II offers a comparative assessment between the DRA and the RK method in a scenario with α = 3, Re = 50 and φ = 0.1. A detailed examination of the error bars underscores the superior accuracy demonstrated by the DRA method in this specific analysis.

TABLE II
COMPARISON OF DRA AND NUMERICAL SOLUTIONS OF F(Ψ) FOR Φ = 0.1, RE = 50 AND ALPHA = 3

ψ	DRA	RK	ERR(%)
0	1	1	0
0.1	0.9941726	0.9941723	$3.02*10^{-7}$
0.2	0.9762107	0.9762102	$5.12*10^{-7}$
0.3	0.9446544	0.9446541	$3.18*10^{-7}$
0.4	0.8970075	0.8970069	$6.69*10^{-7}$
0.5	0.8296588	0.8296582	$7.23*10^{-7}$
0.6	0.7378336	0.7378331	$6.78*10^{-7}$
0.7	0.6156407	0.6156401	$9.75*10^{-7}$
0.8	0.456342	0.4563417	$6.57*10^{-7}$
0.9	0.2530381	0.2530377	$1.58*10^{-6}$
1	0	0	0

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Fig. 2 and Fig. 3 illustrate the impact of the SVF (solid volume fraction) on the velocity of the Cu-Kerosene and SiO_2-Kerosene nanofluids when α = 3 and Re = 50. Notably, there is an inverse relationship between the flow's speed in the C/D channels. For the Cu-Kerosene nanofluid, it is found that as the SVF (ϕ) increases, the nanofluid velocity augments in the convergent channel, while it diminishes in the divergent channel. As for the SiO_2-Kerosene nanofluid, the speed of the flow diminishes in the convergent channel when ϕ is enhanced but augments in the divergent channel. It is also worth mentioning that the impact of the SVF is more evident in the case of copper nanoparticles than in silica nanoparticles. This suggests that adding different nanoparticles impacts the flow differently.

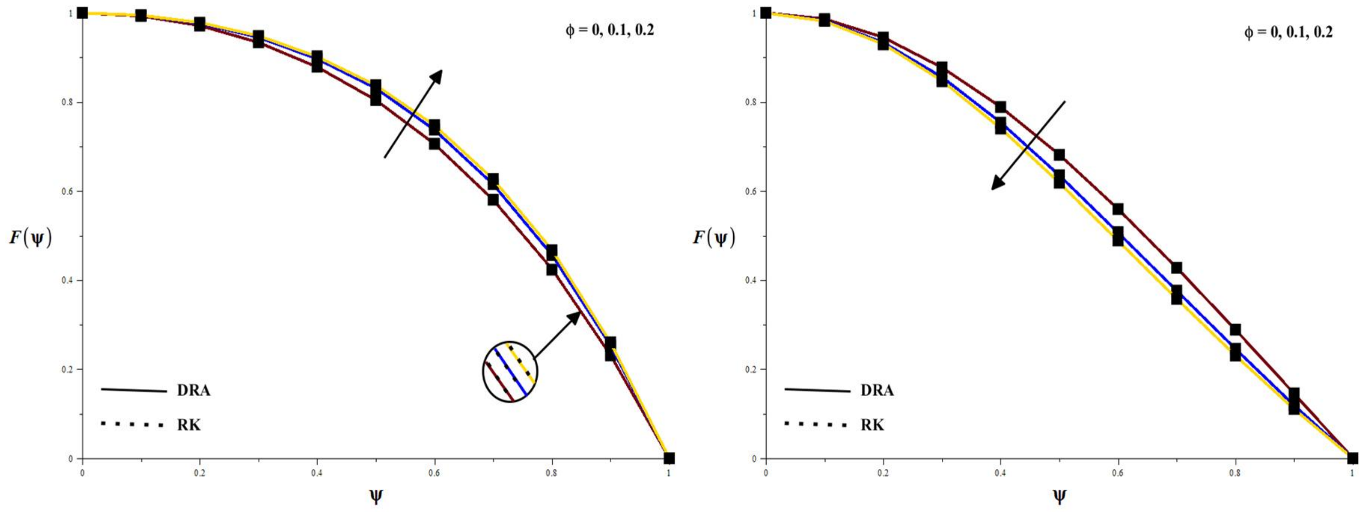

Fig. 2 F(ψ) variation for Cu-Kerosene when α = 3◦ and Re= 50 with different values of ϕ in a C/D channel

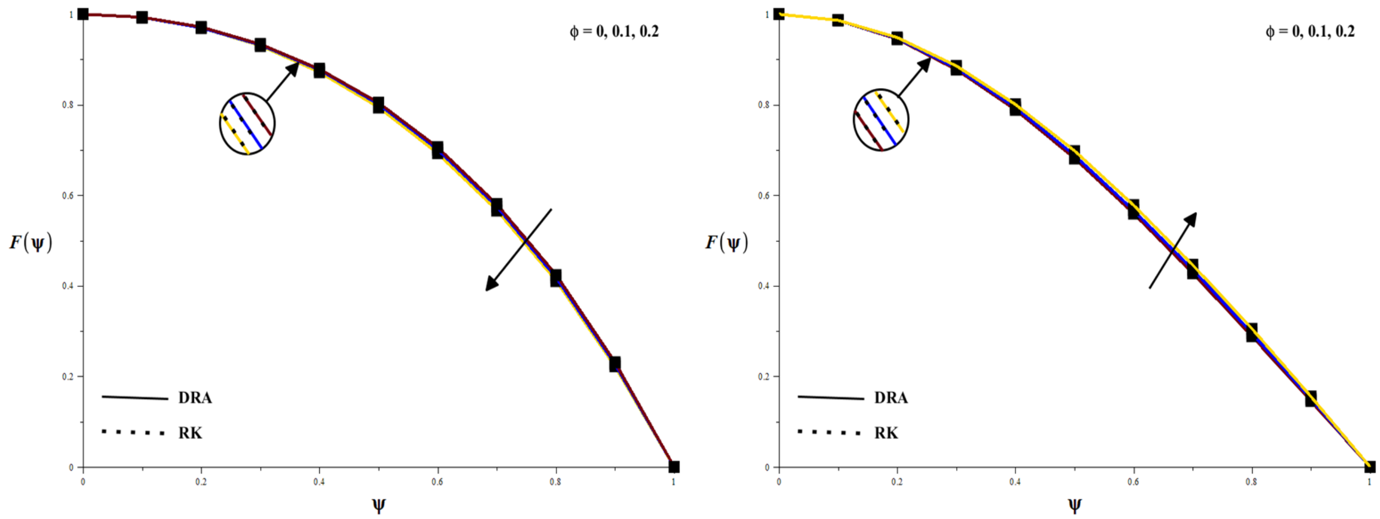

Fig. 3 F(ψ) variation for SiO_2-Kerosene when α = 3◦ and Re= 50 with different values of ϕ in a C/D channel.

The impact of Re on the speed of the flow of the Cu-Kerosene nanofluid when ϕ = 0.1 and α = 3°, is illustrated in Fig. 4. Since the two nanofluids showed similar behavior, it was decided to present the results for only one. The conclusion drawn is that increasing the Reynolds number for both nanofluids results in an enhancement of the flow velocity in the convergent channel and a reduction of the flow speed in the divergent channel.

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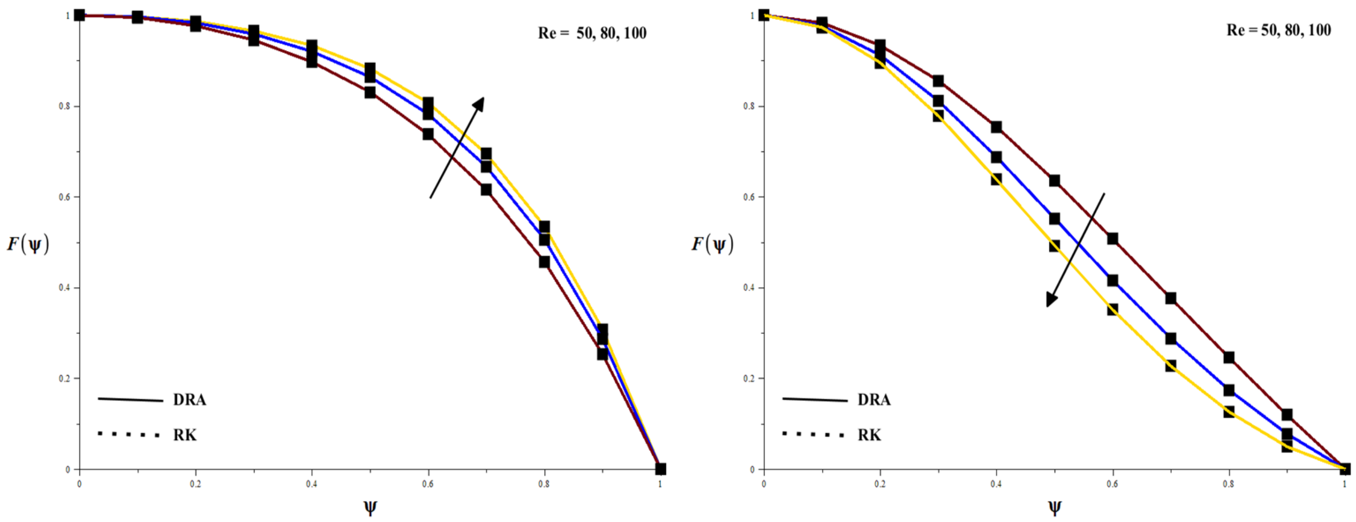

Fig. 4 F(ψ) variation for Cu-Kerosene when ϕ = 0.1 and α = 3◦ with different values of Re in a C/D channel.

Fig. 5 is provided to investigate the impact of the opening angle α on the velocity profile of the SiO_2-Kerosene nanofluid, with other parameters held constant at ϕ = 0.1 and Re = 50. Similar to the Reynolds number, α affects the two chosen nanofluids in the same manner, this is why the results of only one nanofluid are illustrated. It is evident from the graph that as α increases, F(ψ) also expands in a convergent channel, while it drops in a divergent channel. It is noteworthy that the impact of the angle α is more pronounced compared to the effects of the SVF and the Re, making the control of the channel angle a crucial factor in the optimization of system design.

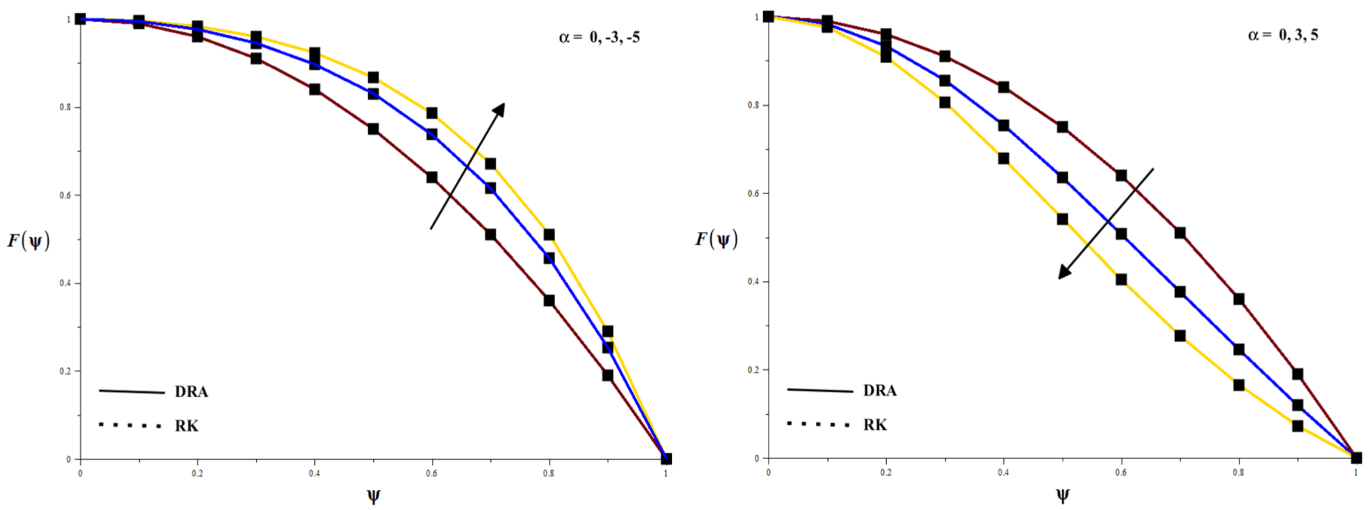

Fig. 5 F(ψ) variation for SiO_2-Kerosene when Re = 50 and α = 3◦ with different values of α in a C/D channel.

The difference between the velocity profiles of the two nanofluids in comparison to the basic Kerosene fluid when α = 3°, Re = 50, and ϕ = 0.1 in a C/D channel is displayed in Fig. 6. It is observed that in a convergent channel, the dimensionless velocity of SiO_2 is smaller than that of the Kerosene, while the speed flow of Cu outranks both, showing a promising enhancement of dynamics to the Kerosene base fluid. In the divergent channel, the results are opposite to those obtained from the convergent channel case; the velocity profile of Cu is lower than that of the base fluid, and the flow speed of SiO_2 is slightly higher than the Kerosene's speed. This shows that the geometry of the system is highly important for the choice of the added nanoparticles, and that both the geometry (whether it is a convergent or divergent channel) and the choice of nanoparticles impact the flow in a different way.

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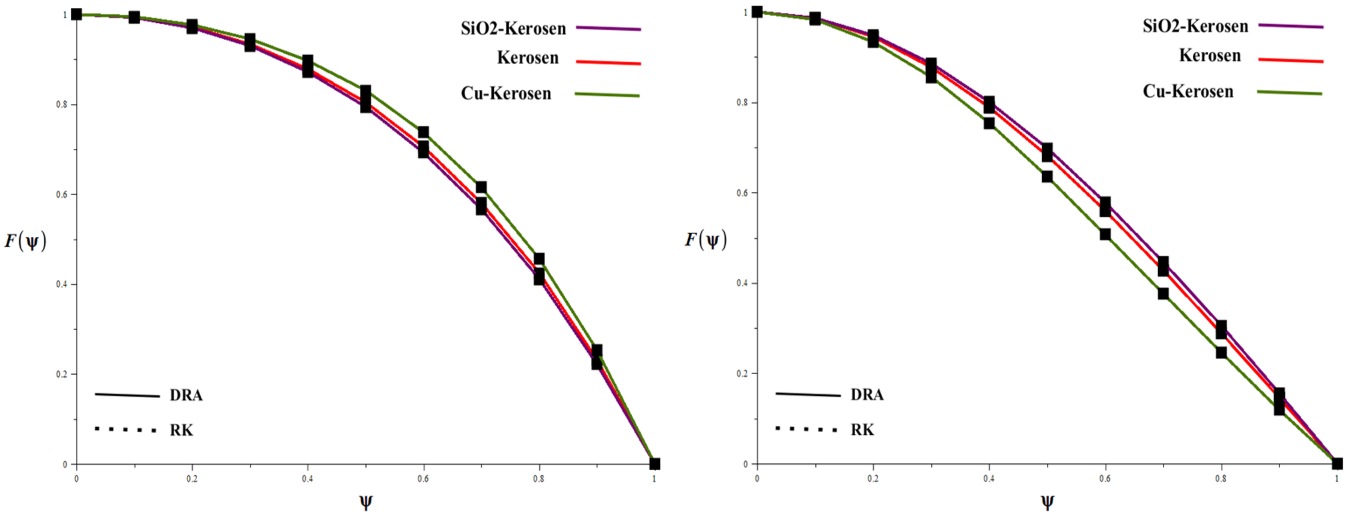

Fig. 6 The variation of F(ψ) for Kerosene, SiO_2-Kerosene and Cu-Kerosene when Re = 50, ϕ = 0.1 and α = 3◦ in a C/D channel.

Tables (3) and (4) present the variation of the Cf (skin friction coefficient) of the Cu-Kerosene and SiO_2-Kerosene nanofluids in a C/D channel when varying Re, ϕ, and α. Observing the tables, several conclusions can be drawn. In all cases, an increase in solid volume fraction enhances the Cf, while an increase in Reynolds number decreases it. It is also noteworthy that the skin friction in both halves of the channel was the same before introducing the nanoparticles, demonstrating that adding the nanoparticles affects the dynamics of the flow. Another observation from the tables is that, for both nanofluids in the divergent channel, increasing the opening angle α leads to an augmentation in Cf, resulting in a decrease in flow velocity. Conversely, enhancing α in a convergent channel decreases Cf, causing an improvement in flow velocity. The two tables also show that the skin friction of the Cu-Kerosene is higher than the Cf of the SiO_2-Kerosene nanofluid in the convergent channel. However, in the divergent channel, the skin friction coefficient of the Cu-Kerosene nanofluid is lower than that of the SiO_2-Kerosene nanofluid. These results validate those obtained by the DRA and RK methods above and further highlight the importance of choosing both the geometry and the nanoparticles added when designing and optimizing fluid transfer systems.

TABLE III

SKIN FRICTION OF THE CU-KEROSENE NANOFLUID THROUGH A C/D CHANNEL.

		C_f for divergent channels			**C_f convergent channel**		
Cu-kerosene	**Re=50**	**α**	**ϕ1 =0**	**ϕ2=0.2**	**α**	**ϕ1=0**	**ϕ2=0.2**
		0	**-0.0100000**	**-0.0174693**	**0**	**-0.0100000**	**-0.0174693**
		2	**-0.0121457**	**-0.0254455**	**-2**	**-0.0085343**	**-0.0134351**
		4	**-0.0155236**	**-0.0465504**	**-4**	**-0.0074793**	**-0.0110571**
	Re=80	**α**	**ϕ1=0**	**ϕ2=0.2**	**α**	**ϕ1=0**	**ϕ2=0.2**
		0	**-0.0062500**	**-0.0109183**	**0**	**-0.0062500**	**-0.0109183**
		2	**-0.0087214**	**-0.021882**	**-2**	**-0.0049122**	**-0.0074211**
		4	**-0.0144220**	**-0.3153849**	**-4**	**-0.0040905**	**-0.005782**

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TABLE IV
SKIN FRICTION OF THE SIO_2-KEROSENE NANOFLUID THROUGH A C/D CHANNEL

		C_f for divergent channels			C_f convergent channel		
SiO2-kerosene	**Re=50**	**α**	**ϕ1=0**	**ϕ2=0.2**	**α**	**ϕ1=0**	**ϕ2=0.2**
		0	**-0.0100000**	**-0.0174693**	**0**	**-0.0100000**	**-0.0174693**
		2	**-0.0121457**	**-0.0202736**	**-2**	**-0.0085343**	**-0.0153919**
		4	**-0.0155236**	**-0.0242263**	**-4**	**-0.0074793**	**-0.0138006**
	Re=80	**α**	**ϕ1=0**	**ϕ2=0.2**	**α**	**ϕ1=0**	**ϕ2=0.2**
		0	**-0.0062500**	**-0.0109183**	**0**	**-0.0062500**	**-0.0109183**
		2	**-0.0087214**	**-0.0140305**	**-2**	**-0.0049122**	**-0.0089892**
		4	**-0.0144220**	**-0.0196998**	**-4**	**-0.0040905**	**-0.007694**

V. CONCLUSIONS

Various fields and industries today employ a wide array of C/D channels in their applications and manufacturing processes. However, aircraft still do not use these geometries extensively. The main objective of this study is to demonstrate the potential of C/D channels, along with nanofluids, in flow dynamics, and to open new avenues for further studies and optimization in the design and manufacture of aircraft fluid transfer systems. Among the many nanoparticles available, Cu and SiO_2 nanoparticles stand out for their cost-effectiveness and simplicity in synthesis, especially compared to counterparts like Ag nanoparticles, which are commonly used in heat transfer applications.

This paper delves into the intricate effects of introducing copper and silica nanoparticles into a kerosene-based fluid within a C/D channel. To comprehensively understand this phenomenon, the governing equations for the flow were carefully formulated and analytically solved using the Duan-Rach approach. The obtained analytical solutions were then rigorously compared to numerical results obtained through the Runge-Kutta method, highlighting the efficacy and remarkable accuracy of the DRA in capturing the complexities of nanofluid flow.

Furthermore, the study goes beyond mere analysis and explores the influence of pivotal parameters such as the solid volume fraction (ϕ), opening angle (α), and Reynolds number (Re) on the velocity profile and skin friction of the nanofluid flow. Through systematic examination, it was revealed that an increase in Re and α leads to a noticeable reduction in flow velocity, primarily attributed to heightened skin friction in the divergent channel. Conversely, nanofluids within a convergent channel experience a substantial enhancement in flow speed with an increase in these parameters.

Additionally, it was noted that the velocities of the different nanofluids analyzed exhibit opposite trends in the two channels. In the divergent channel, the velocity order is Cu-Kerosene nanofluid < Kerosene fluid < SiO_2-Kerosene nanofluid. In the convergent channel, the order is SiO_2-Kerosene < Kerosene < Cu-Kerosene. This shows that each nanoparticle has a distinct impact on the dynamics of the flow, highlighting the intricate interplay between nanoparticle characteristics and fluid dynamics, and providing crucial insights into nanofluid behavior in C/D channels.

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