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Robust Output Feedback Synchronization of Redundant Manipulators in Task Space with Time-varying Delays

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Abstract—This study focuses on synchronization of kinematically redundant robot manipulators in task space under the effects of parametric uncertainty and time-varying communication delay.Weconsiderasjointand/orend-effectorvelocitiesarenot measurable and to overcome this problem we propose a linear-filter, which has a model-free structure. It is assumed that the position error and linear-filter output signals are shared over a directed communication topology under theeffect of time-varying communicationdelaytoguaranteetheend-effectorsynchronizationbetweenrobotmanipulators.Controlobjectivesareachieved by proposed output-feedback (OFB) robust controller and with the help of a combination of Lyapunov and Krasovskii stability analysis,wereachtoauniformlyultimatelyboundedresult for both tracking and synchronization error signals. Finally, we presentthesimulationresultsonasystemwith3three-linkplanar robot manipulators to validate the feasibility of the suggested controller mechanism forsynchronization.

I. Introduction

Design of general control structures for the use of robot manipulators in industrial applications has been an active and challenging research area in control engineering for many years. In parallel to the recent technological developmentsand appearance of new technologies, the size and scope of industrial applications arealsodrastically changing. Giventhe requirements in this area, it is clear that a single robot manipulator will not suffice for the required tasks such as assembly, welding, painting or transportation.For this reason, itisnecessarytoprovidecoordinationofmultiplerobotmanipulators with the use of communication structures so that they can performa pre-plannedtask in a collaborativemanner.The coordinated control structure to be designed must ensure that the end-effectors of the robot manipulators in operation are synchronized before reaching to a desired trajectory and track thetrajectorysimultaneously.Besides,manipulatortrajectories for specific tasks to be performedare usually designed in task space by using the position that the end-effectors of the robot manipulators are to be located.

Also, redundancy in the manipulator structure, i.e. the structure that the dimension of link position variables n is greater than the dimension of operation space m, has an important role on dexterity [1].

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In literature, most of the recommended control structures needvelocitymeasurementsof joints and/orend-effectors,beside their positions. The velocitysensors used forthis purpose are generally expensive and give noisy measurements, that may affect the system, negatively. Another subject that needs to be discussed aboutsynchronizationis the time delay caused by communication during the information sharing between robot manipulators. There are several proposed controllers assuming that there exist constant and bounded time delays,in literature. Considering a real communication system, it can not be expected that the time delay in the whole system hasthe same value during the operation or the time delay value between each agent is the same [2].

In[3],theleader-followersynchronization wasprovided bythevelocitysignalobtainedfromanvirtualagentwith the assumption of exact model knowledge. In [4] and [5], global asymptotic synchronization was guaranteed by usingan adaptive control structure. Also, in [5], time-varying communication delay and redundancy were taken into consideration.Undertheassumptionofmeasurablejointvelocities in [6] and [7] adaptive based controllers were suggested for directed communication topology with constant time-delay. Recently, Phukan and Mahanta suggested in [8], a full-state feedback sliding mode controller for the synchronization of non-redundant robot manipulators.

In this work, we purposea comprehensivesolution forOFB synchronization of robot manipulators in task space. For the replacementofvelocityerror,inspiredby[9],alinear-filter is designed. With thehelp of thesurrogate signal generatedby linear-filter, trajectory tracking problem of each agent is solved without measuring joint and/or end-effector velocities. Toovercometheparametricuncertaintyofdynamicmodel,we suggested a robust controller. It is assumed that the position error and linear-filter output signals are shared over a directed communicationtopologyundertheeffectoftime-varyingcommunication delay to guarantee the synchronization between robot manipulators. Also, redundancy problem is taken into consideration. Performing a combination of Lyapunov and Krasovskii stability analysis, we reach to a uniformly ultimatelyboundedresultforbothtrackingandsynchronization error signals. Finally, we present the simulation results on a system with 3 three-link planar robot manipulators to validate the feasibility of the suggested robust OFB control structure for synchronization.

II. Notations and Mathematical Prelimineries

The Euler-Lagrange based formulation of dynamic model for n-link, revolute, direct drive robot manipulator is given by the following form [10]

$$M(q)\ddot{q}+C(q,\dot{q})\dot{q}+F_d\dot{q}+G(q)=\tau \quad (1)$$

Due to page limitation, readers are referred to see:

- [10], [11] for the properties of given model and signals in (1),
- [12], [13] for the properties of Jacobian matrix $J(q)$, pseudo-inverse Jacobian matrix $J^+(q)$ and their relation with task-space.

III. Output Feedback Task-Space Synchronization

A. Problem Definition

In this study we focused on the end-effector position synchronization of multiple robot manipulators, while each of them follows the given desired trajectory. To ensure the tracking goal for each robot manipulator in the system with the dynamical model given in (1), the position tracking error $e_i(t)\in\mathbb{R}^m (i\in S, \{1,..,N\})$ is defined as

$$e_i = x_{di} - x_i \quad (2)$$

where $x_i(t)\in\mathbb{R}^m$ stands for actual position and $x_{di}(t)\in\mathbb{R}^m$ symbolizes the desired trajectory given to the robot manipulators.

B. Linear-filter Design

Since we assumed that joint and/or end-effector velocities are not measurable, based on the subsequent stability analysis, we design a linear-filter $r_{fi}\in\mathbb{R}^m$ as

$$r_{fi}= p_i-(k_{1i}+1)e_i \quad (3)$$

where $p_i(t)\in\mathbb{R}^n$ is an auxiliary variable with the dynamic equation

$$\dot{p}_i=-(k_{1i}+1+\alpha)p_i+((k_{1i}+1)^2+\alpha(k_{1i}+1)-(k_{1i}+1)+k_{2i})e_i-e_{fi}. \quad (4)$$

In the given equation, $k_{1i},k_{2i}\in\mathbb{R}^{nxn}$, $\alpha\in\mathbb{R}^1$ are positive constant gains and α is same for all agents. $e_{fi}(t)$ is an auxiliary variable with the following dynamics

$$\dot{e}_{fi}= -k_{3i}e_{fi}+ r_{fi} \quad (5)$$

with a positive constant gain $k_{3i}\in\mathbb{R}^{nxn}$. Dynamics of (3) can be presented as,

$$\dot{r}_{fi}=-\alpha r_{fi}-(k_{1i}+1)J_i(q_i)\eta_i+k_{2i}e_i-e_{fi} \quad (6)$$

where $\eta_i(t)\in\mathbb{R}^n$ is an auxiliary signal as follows

$$\eta_i=J^+(q_i)(r_{fi}+e_i+\dot{x}_{di})+(I_n-J^+(q_i)J_i(q_i))g_i-\dot{q}_i. \quad (7)$$

Given vector function $g(.)\in\mathbb{R}^n$ is the sub-task function, that is chosen according to pre-defined sub-task, e.g. joint limitation, manipulability, obstacle avoidance, etc. If we define sub-task error as $e_N=(I_n-J_i^+(q_i)J_i(q_i))(g_i-\dot{q}_i)$ it is easy to show that $e_N=(I_n-J_i^+(q_i)J_i(q_i))\eta_i$ which means we can regulate e_N as long as we can regulate η_i. For further information the reader is referred to [12], [13]. From this point, for ease of presentation we will use the notation

$$z_i=J_i^+(q_i)(r_{fi}+e_i+\dot{x}_{di})+(I_n-J_i^+(q_i)J_i(q_i))g_i \quad (8)$$

Also we can define the time derivative of position error as $\dot{e}_i=J_i(q_i)\eta_i-r_{fi}-e_i$ and the joint velocity as $\dot{q}_i=z_i-\eta_i$.

C. Error System Development

To obtain the dynamic formulation for error system, we take the time derivative of (7), multiply both sides with $M_i(q_i)$, substitute (6), $\dot{e}_i$ and use $\ddot{q}_i$ from E-L model given in (1), we yield to the open loop dynamics as

$$\begin{aligned}M_i\dot{\eta}_i=&C_i(q_i,\dot{q}_i)\dot{q}_i+F_{di}\dot{q}_i+G_i(q_i)-\tau_i+W_i(q_i)\dot{q}_i\\&+M_i(q_i)J_i^+(q_i)\quad-(\alpha+1)r_{fi}-k_{1i}J_i(q_i)\eta_i\\&+(k_{2i}-1)e_i-e_{fi}+\ddot{x}_{di}\end{aligned} \quad (9)$$

with $W_i(q_i)\in\mathbb{R}^{nxn}$

$$\begin{aligned}W_i(q_i)\dot{q}_i=&M_i(q_i)\quad \dot{J}_i^+(q_i)(r_{fi}+e_i+\dot{x}_{di})\\&+(-\dot{J}_i^+(q_i)J_i(q_i)-J^+(q_i)\dot{J}_i(q_i))g_i\\&+(I_n-J^+(q_i)J_i(q_i))\dot{g}_i.\end{aligned} \quad (10)$$

By using the definition given for $\dot{q}_i$ and Property $C_i(q_i,\varsigma_i)v=C_i(q_i,v)\varsigma_i$, we reformulate the open-loop error dynamics as

$$\begin{aligned}M_i(q_i)\dot{\eta}_i=&-M_i(q_i)J^+(q_i)k_{1i}J_i(q_i)\eta_i-C_i(q_i,\dot{q}_i)\eta_i\\&-F_{di}\eta_i-W_i(q_i)\eta_i-C_i(q_i,\eta_i)z_i-\tau_i+Y_i\vartheta_i\end{aligned} \quad (11)$$

where $Y_i\vartheta_i\in\mathbb{R}^n$ is a linear parameterization of

$$\begin{aligned}Y_i\vartheta_i=&C_i(q_i,z_i)z_i+F_{di}z_i+G_i(q_i)+W_i(q_i)z_i\\&+M_i(q_i)J_i^+(q_i)\quad-(\alpha+1)r_{fi}\\&+(k_{2i}-1)e_i-e_{fi}+\ddot{x}_{di}.\end{aligned} \quad (12)$$

denoting $Y_i(x_{di},\dot{x}_{di},\ddot{x}_{di},x_i,q_i,e_i,e_{fi},r_{fi},g_i)\in\mathbb{R}^{nxr}$ the regression matrix and $\vartheta_i\in\mathbb{R}^r$ system parameters, e.g. mass, intertia, friction coefficients. Based on the subsequent stability analysis, we design the controller as follows

$$\begin{aligned}\tau_i=&Y_i\hat{\vartheta}_i+J_i^T(q_i)-\gamma_i(k_{1i}+1)r_{fi}+\gamma_ik_{2i}e_i\\&+\Gamma\|Y_i\|^2(r_{fi}+e_i)+\sum_{j\in S_i}\delta\tau_{ij\,syn}\end{aligned} \quad (13)$$

with synchronization part τ_{syn}

$$\begin{aligned}\tau_{syn}=&k_{4ij}(e_i(t)-e_j(t-T_{ij}(t)))\\&-(k_{1i}+1)k_{5ij}(r_{fi}(t)-r_{fj}(t-T_{ij}(t)))\end{aligned} \quad (14)$$

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In (13), $\hat{\vartheta}_i$ stands for the constant estimated values of system parameters. First three terms in pharantesis are for tracking problem of each robot manipulator, with the nonlinear damping term $\|Y_i\|^2(r_{fi}+e_i)$. δ_{ij} given in (13), symbolizes the communication among agents in the system, i.e. $\delta_{ij}=1$ if agent i receives information from agent j, otherwise $\delta_{ij}=0$, and $\pounds_i$ is a subset of $\pounds$ containing all agents that send information to agent i. Terms in τ_{syn} ensure the position synchronization as well as the velocity synchronization between agents, respectively. The signals $e_j(t-T_{ij}(t))$ and $r_{fj}(t-T_{ij}(t))$ represent time-varying delayed position error signals and surrogate velocity error signals transferred over communication links. Based on the subsequent Lyapunov-Krasovskii [14] stability analysis, the time-varying delay $T_{ij}(t)$ is a continuously differentiable function and $\dot{T}_{ij}(t)=d_{ij}<1$. Given $\Gamma_i, \gamma_i, k_{2i}, k_{4ij}, k_{5ij} \in \mathbb{R}^{m\times m}$ are the positive and diagonal control gain matrices.

By substituting the control law (13) into (11), we obtain the closed loop dynamics for $\eta_i(t)$ as

$$\begin{aligned} M_i(q_i)\dot{\eta}_i = & -M_i(q_i)J^{+}(q_i)k_{1i}J_i(q_i)\eta_i - C_i(q_i,\dot{q}_i)\eta_i \\ & - F_{di}\eta_i - W_i(q_i)\eta_i - C_i(q_i,\eta_i)z_i + Y_i\tilde{\vartheta}_i \\ & - J_i^T(q_i) - \gamma_i(k_{1i}+1)r_{fi} + \gamma_i k_{2i}e_i \\ & + \Gamma_i\|Y_i\|^2(r_{fi}+e_i) + \sum_{j\in \pounds_i}\delta_{ij}\tau_{syn} \end{aligned} \quad (15)$$

D. Stability Analysis

Theorem 1: Considering a system, that consists of N robot manipulators with the dynamic structure in (1), the filter structure of (3), (4), (5) and the OFB robust controller given by (13), guarantee uniform ultimate boundedness of tracking and synchronization errors in the sense that

$$\|y(t)\| \le \sqrt{\frac{\lambda_{max}}{\lambda_{min}}\|y(0)\|^2 \exp(-Kt) + \frac{\lambda_{max}}{4\Gamma K\lambda_{min}}\|\tilde{\vartheta}\|^2(1-\exp(-Kt))}$$

where

$$y(t) = \begin{bmatrix}\eta_i(t)^T & r_{fi}(t)^T & e_{fi}(t)^T & e_i(t)^T & e_{syn}(t)^T\end{bmatrix}^T$$

and

$$e_{syn}(t) = \begin{bmatrix}\delta_{ij}(e_i(t)-e_j(t-T_{ij}(t)))^T \\ \delta_{ij}(r_{fi}(t)-r_{fj}(t-T_{ij}(t)))^T\end{bmatrix}$$

with

$$\lambda_{1i} = \frac{1}{2}\min\left\{m_{1i}, \lambda_{min}\{\gamma_i\}, \lambda_{min}\{k_{2i}\}, \lambda_{min}\{k_{4ij}\}, \lambda_{min}\{k_{5ij}\}\right\},$$

$$\lambda_{2i} = \frac{1}{2}\max\left\{m_{2i}, \lambda_{max}\{\gamma_i\}, \lambda_{max}\{k_{2i}\}, \lambda_{max}\{k_{4ij}\}, \lambda_{max}\{k_{5ij}\}\right\}. \quad (16)$$

In (16), m_{1i} and m_{2i} symbolize the lower and upper bounds of $\|M_i(q_i)\|$, respectively. All closed-loop signals remain bounded, as long as the positive gains satisfy following conditions

$$\alpha = \frac{k_{6ij}\lambda_{min}\{k_{5ij}\}}{\lambda^2_{max}\{k_{2i}\}} + \frac{\lambda^2_{max}\{k_{1j}+1\}(1-d_{ij})}{4\lambda_{min}\{k_{5ij}\}} + \frac{\lambda^2_{max}\{k_{2i}\}}{\lambda_{min}\{k_{5ij}\}} + \frac{\lambda^2_{max}\{k_{2j}\}(1-d_{ij})}{4\lambda_{min}\{k_{5ij}\}d^2_{ij}} + \frac{\lambda^2_{max}\{k_{5ij}\}(1-d_{ij})}{4\lambda_{min}\{k_{5ij}\}}$$

$$\lambda_{min}\{k_{1i}\} > \zeta_{C_{2i}}\|z_i(0)\| + \Gamma_i(\|Y_i\|^2+\|Y_i\|^4) + \sum_{i\in S_l}\delta\theta_{li1li}$$

$$\gamma_i = \frac{k_{7i}}{\alpha} + \frac{\lambda^2_{max}\{k_{4ij}\}}{4\alpha} + \frac{\Gamma_i}{4\alpha}$$

$$k_{2i} = \frac{k_{9i}}{\gamma_i} + \frac{\lambda^2_{max}\{k_{5ij}\}}{4\gamma_i} + \frac{\Gamma_i}{4\gamma_i}, \quad k_{3i} = \frac{k_{8i}}{\gamma_i} + \frac{\lambda^2_{max}\{k_{5ij}\}}{4\gamma_i}$$

$$\lambda_{min}\{k_{4ij}\} > 1 + \frac{1-d_{ij}}{4} + \frac{(1-d_{ij})}{4d^2_{ij}} + \frac{1}{4(1-d_{ij})}$$

$$k_{6ij} > 1 + \frac{1}{4(1-d_{ij})}, \quad k_{7i} > \sum_{i\in S_l}\delta\theta_{li3li}d^2_{li}$$

$$k_{8i} > \sum_{i\in S_l}\delta_{li}I_{n\times n}, \quad k_{9i} > \sum_{i\in S_l}\delta_{li}\theta_{2li}d^2_{li}$$

where $\theta_{1ij} = j^2_{2j}(\lambda^2_{max}\{k_{4ij}\} + \lambda^2_{max}\{k_{5ij}\})$, $\theta_{2ij} = \lambda^2_{max}\{k_{4ij}\} + \lambda^2_{max}\{k_{5ij}\}$, $\theta_{3ij} = \lambda^2_{max}\{k_{4ij}\} + \alpha^2\lambda^2_{max}\{k_{5ij}\}$ with the upper bound of $\|J_j(q_j)\| \ge j_{2j}$ and the subindice l stands for other agents in the system, that receive information from agent i.

TABLE I: Robot Parameters and Controller Gains

Parameters&Gains	Robot1	Robot2	Robot3
p_1	0.4752[$kg.m^2$]	0.32[$kg.m^2$]	0.5666[$kg.m^2$]
p_2	0.12[$kg.m^2$]	0.0819[$kg.m^2$]	0.1594[$kg.m^2$]
p_3	0.108[$kg.m^2$]	0.0725[$kg.m^2$]	0.1442[$kg.m^2$]
β_1	1.2684[$kg.m^2$]	0.8528[$kg.m^2$]	1.4993[$kg.m^2$]
β_2	0.3884[$kg.m^2$]	0.2587[$kg.m^2$]	0.485[$kg.m^2$]
β_3	0.045[$kg.m^2$]	0.0304[$kg.m^2$]	0.0626[$kg.m^2$]
f_{d1}	5.3[$Nm.s$]	4.6[$Nm.s$]	4.8[$Nm.s$]
f_{d2}	2.4[$Nm.s$]	1.9[$Nm.s$]	1.6[$Nm.s$]
f_{d3}	1.1[$Nm.s$]	0.8[$Nm.s$]	1.2[$Nm.s$]
l_1	0.2[m]	0.3[m]	0.25[m]
l_2	0.25[m]	0.3[m]	0.35[m]
l_3	0.25[m]	0.3[m]	0.25[m]
k_{1i}	0.5,0.5	0.5,0.5	0.5,0.5
k_{2i}	2,2	2,2	2,2
k_{3i}	50,50	50,50	50,50
k_{4ij}	35,35	35,35	35,35
k_{5ij}	60,60	60,60	60,60
γ_i	3,3	3,3	3,3
Γ_i	0.01,0.01	0.01,0.01	0.01,0.01
α	15	15	15

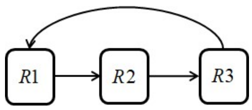

Fig.1: Communication Topology.

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Proof1: To begin the stability analysis, we define a non- negative function as

$$
\begin{aligned}
V_1(t)=&\sum_{i\in\Box}V_{1i}(t)\\
=&\frac{1}{2}\sum_{i\in\Box}\Big(\eta_i^TM_i\eta_i+e_i^T\gamma_ik_{2i}e_i+e_{f_i}^T\gamma_ie_{f_i}\\
&+r_{fi}^T\gamma_ir_{fi}\\
&+\sum_{j\in\Box_i}\delta_{ij}(e_i(t)-e_j(t-T_{ij}(t)))^T\\
&\times k_{4ij}(e_i(t)-e_j(t-T_{ij}(t)))\\
&+\sum_{j\in\Box_i}\delta_{ij}(e_{f_i}(t)-e_{f_j}(t-T_{ij}(t)))^T\\
&\times k_{5ij}(r_{fi}(t)-r_{fj}(t-T_{ij}(t)))\Big)
\end{aligned}
\quad (17)
$$

that can be upper and lower bounded as $\lambda_{1i}\|y_i\|^2\le V_{1i}\le\lambda_{2i}\|y_i\|^2$. Differentiating (17) with respect to time and substituting (6),(5),$\dot e_i$,(15) with some algebraic manipulations we obtain

$$
\begin{aligned}
\dot V_1(t)\le&\sum_{i=1}^{N}\Big(-\big[m_{1i}j_{1i}^{+}+\lambda_{min}\{k_{1i}\}\big]\|\eta_i\|^2\\
&+\big[\|z_i(t)+\Gamma_i(\|Y_i\|^2+\|Y_i\|^4)\big]\|\eta_i\|^2\\
&+\frac{1}{4\Gamma_i}\|\tilde\vartheta_i\|^2-k_{7i}\|r_{fi}\|^2-k_{8i}\|e_{fi}\|^2-k_{9i}\|e_i\|^2\\
&-\sum_{j\in\Box_i}\delta_{ij}\Big[\lambda_{min}\{k_{4ij}\}-1+\frac{1-d_{ij}}{4}+\frac{(1-d_{ij})}{4d_{ij}^2}\\
&+\frac{1}{4(1-d_{ij})}\Big]\|e_i(t)-e_j(t-T_{ij}(t))\|^2\\
&-\sum_{j\in\Box_i}\delta_{ij}\Big[k_6-(1+\frac{1}{4(1-d_{ij})})\Big]\\
&\times\|r_{fi}(t)-r_{fj}(t-T_{ij}(t))\|^2\\
&+\sum_{j\in\Box_i}\delta_{ij}(1-d_{ij})\theta_{1ij}\|\eta_j(t-T_{ij}(t))\|^2\\
&+\sum_{j\in\Box_i}\delta_{ij}d_{ij}^2(1-d_{ij})\theta_{2ij}\|e_j(t-T_{ij}(t))\|^2\\
&+\sum_{j\in\Box_i}\delta_{ij}(1-d_{ij})\|e_{fj}(t-T_{ij}(t))\|^2\\
&+\sum_{j\in\Box_i}\delta_{ij}d_{ij}^2(1-d_{ij})\theta_{3ij}\|r_{fj}(t-T_{ij}(t))\|^2\Big)
\end{aligned}
\quad (18)
$$

where j_{1i} and j_{1i}^{+} are lower bounds of $\|J_i(q_i)\|$ and $\|J_i^{+}(q_i)\|$, respectively. To analyze the stability of last four terms indicating time-delay, we define a Lyapunov-Krasovskii functional as

$$
\begin{aligned}
V_2(t)=&\sum_{i\in\Box}\Big(V_{1i}(t)+\sum_{j\in\Box_i}\delta_{ij}\theta_{1ij}\int_{t-T_{ij}}^{t}\eta_j^T(\omega)\eta_j(\omega)d\omega\\
&+\sum_{j\in\Box_i}\delta_{ij}\theta_{2ij}d_{ij}^2\int_{t-T_{ij}}^{t}e_j^T(\omega)e_j(\omega)d\omega\\
&+\sum_{j\in\Box_i}\delta_{ij}\theta_{3ij}d_{ij}^2\int_{t-T_{ij}}^{t}r_{fj}^T(\omega)r_{fj}(\omega)d\omega\\
&+\sum_{j\in\Box_i}\int_{t-T_{ij}}^{t}e_{fj}^T(\omega)e_{fj}(\omega)d\omega\Big)
\end{aligned}
\quad (19)
$$

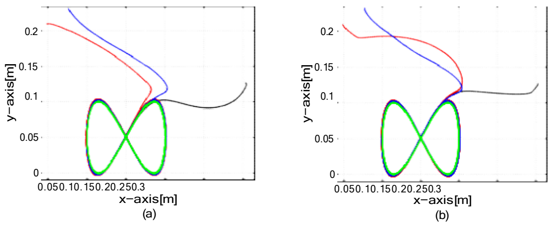

Fig.2: End-effector trajectories (a) Without synchronization (b) With synchronization.

that can be bounded similar as V_{1i} and we can write the upper bound for $\dot V_2(t)$ as

$$
\begin{aligned}
\dot V_2(t)\le&\sum_{i=1}^{N}\Big(-\big[m_{1i}j_{1i}^{+}+\lambda_{min}\{k_{1i}\}\big]\|\eta_i\|^2\\
&+\big[\|z_i(t)+\Gamma_i(\|Y_i\|^2+\|Y_i\|^4)+\sum_{i\in S_l}\delta\theta_{1li}\big]\|\eta_i\|^2\\
&+\frac{1}{4\Gamma_i}\|\tilde\vartheta_i\|^2-\big[k_{7i}-\sum_{i\in S_l}\delta\theta_{3li}d_{li}^2\big]\|r_{fi}\|^2\\
&-\big[k_{8i}-\delta_{li}I_{n\times n}\big]\|e_{fi}\|^2\\
&-\big[k_{9i}-\sum_{i\in S_l}\delta_{li}\theta_{2li}d_{li}^2\big]\|e_i\|^2\\
&-\sum_{j\in S_i}\delta\Big[\lambda_{min}\{k_{4ij}\}-1+\frac{1-d_{ij}}{4}+\frac{(1-d_{ij})}{4d_{ij}^2}\\
&+\frac{1}{4(1-d_{ij})}\Big]\|e_i(t)-e_j(t-T_{ij}(t))\|^2\\
&-\sum_{j\in S_i}\delta_{ij}\Big[k_{6ij}-(1+\frac{1}{4(1-d_{ij})})\Big]\\
&\times\|r_{fi}(t)-r_{fj}(t-T_{ij}(t))\|^2\Big).
\end{aligned}
\quad (20)
$$

For the general system, we can reform (20) as

$$\dot V_2(t)\le-K\|y\|^2+\frac{1}{4\Gamma}\|\tilde\vartheta\|^2 \quad (21)$$

and we can solve the differential inequality as [15]

$$\|y(t)\|\le\sqrt{\frac{\lambda_{max}}{\lambda_{min}}\|y(0)\|^2exp(-Kt)+\frac{\lambda_{max}}{4\Gamma K\lambda_{min}}\|\tilde\vartheta\|^2(1-exp(-Kt))}. \quad (22)$$

From (22), we can conclude that $y(t)$ is bounded as given in Theorem 1 (i.e. $\eta_i, r_{fi}, e_i, e_{fi}, e_{syn}\in L_\infty$). Under the assumption that desired trajectory x_{di} and its derivatives, sub-task function $g(t)$ and $\dot g(t)$ are all bounded, by employing standard signal chasing arguments, we can say that all signals remain bounded.

IV. Numerical Studies

The suggested filter-based robust synchronization scheme was tested in Simulink™ of Matlab™, using three 3-link planar

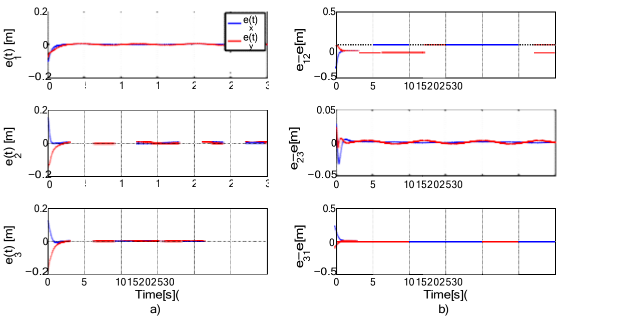

Fig. 3: (a) Position errors of each robots (b) Synchronization errors.

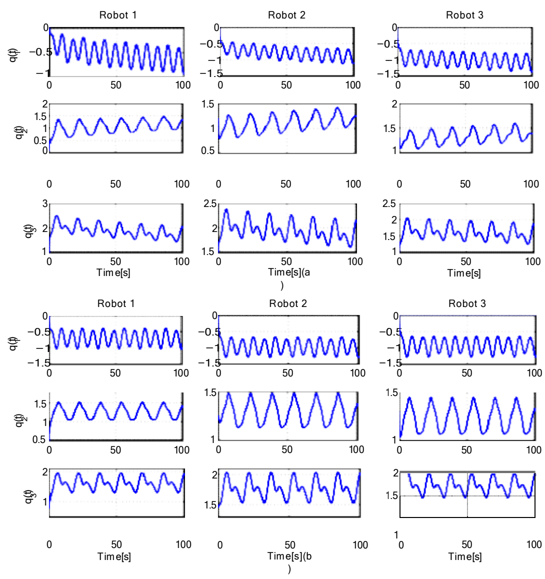

Fig. 4: Joint positions (a) Without subtask function (b) With subtask function.

robotmanipulatorswiththedynamicmodelpresentedin [16] and parameters given in Table 1. It was assumed that linklengthsareknownandtheotherparametervalueswere estimatedtobe20%,30%and15%forRobot1,2and 3incorrect,respectively.Time-delayvaluesoncommunication graph were taken as $T_{1,2}= 0.1 + 0.06sin(t), T_{2,3}= 0.12+0.05sin(0.5t), T_{3,1}= 0.15+0.14sin(0.3t)$. The desired trajectory of robot manipulators was defined as $x_{di}(t)=[0.6+0.1cos(t) 0.9- 0.1sin(t)]^T[m]$.Thesubtaskfunction$g(t)$was selected for all robots as $g(t) =- 2(q_3- q_2+ 0.5q_1)[1- 1 1]^T$as given in [13], to obtain the optimum linkconfigurationisgivenby $(q3- 0.5q2)=0.5(q2- q1)$.

End-effector positions of robot manipulator are presentedinFigure2(a)and(b).Itisclearthatundertheeffect of synchronization, end-effectors of robot manipulators meet before they tracktothegiven desiredtrajectory. Itcanbe seenfromtheFigure3thattrackingerrorsofeachrobot and synchronization error between them stay in a bound around zero. Finally, Figure 4 shows that sub-task function$g(t)$ensurestheoptimumlinkconfigurationofeachrobot manipulatorduringsynchronizationandtracking.

V. Conclusion

In this work, we presented a complete solution for cooperative end-effector position synchronization of robot manipulators under the effects of time-varying delay and parametric uncertainties. We proposed a filter-based OFB robust control scheme to achieve aforementioned control objectives without velocitymeasurement.Theproposedstructureensuresthesynchronizationunderadirectedcommunicationnetwork,with auniformlyulimatelyboundedtrackingandsynchronization errors. Future work will be on extending this result to global asymptotic synchronization.

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