Stream-flow Prediction in Ergene River Basin via Kalman Filter

Emrecan Ozdogan*, Mohsen M. Vanolya†, Levent Ucun*, Seref Naci Engin*

*Department of Control and Automation Engineering

Yildiz Technical University, Davutpasa, Istanbul

Emails: emrecanozdogan95@gmail.com, lucun@yildiz.edu.tr, nengin@yildiz.edu.tr

†Department of Civil Engineering

Yildiz Technical University, Davutpasa, Istanbul

Email: mvanolya1976@gmail.com

Abstract—This study deals with the implementation of Kalman filter for the prediction of stream-flow in Ergene River Basin. In the study, stream-flow, precipitation and wastewater are chosen as the state variables during the prediction process since these parameters are highly effective on the stream-flow. Effects of precipitation and wastewater are calculated via Soil and Water Assessment Tool (SWAT) model in the study. Covariance matrices are calculated by using real-time data with 5 year length and model performance is tested with short and long-term predictions based on measurements and the accuracy of the proposed method is evaluated with Nash-Sutcliffe efficiency coefficient(NS) and root mean squared error (RMSE).

I. INTRODUCTION

A large number of empirical and analytical models are available for streamflow forecasting that can be classified as short, medium and long-term forecasting models [1], [2]. Linear quadratic estimation (LQE) and Kalman filtering are considered as empirical stochastic models, which combine the dynamics and probability distribution of the measured variables in current state for forecasting future ones [3]. Jens et al. (1985) used Kalman filter for real time operation of surface water flow by forecasting in stochastic space in rainfall-runoff model of Mike 11 hydrodynamic model [4]. They discussed the source of uncertainty and stated that it came from the precipitation that is the input to rainfall-runoff. Ngan (1986) compared autoregressive models with Kalman filter based flow forecasting in his PhD thesis [5]. He showed that Kalman filter had better reliability in flow prediction compared to ARMAX. Jean (2004) used it for groundwater level forecasting as well as rainfall-runoff prediction in Danish Hydraulic Institution (DHI) [6]. Moradkhani et al. (2005), forecasted one-day ahead streamflow of the Leaf River watershed by using a dual state parameter estimation approach based on the Ensemble Kalman Filter (EnKF) and showed that the results are very consistent with the observations [7]. Clark et al. (2008) described an application of the EnKF in which streamflow observations are used to update the states in a distributed hydrological model for extracting the source of uncertainty [8]. In another study similar to their work, Noh et al. (2013) assessed EnKF and particle filter (PF) with another distributed hydrologic model and showed that the Kalman filter model is sensitive for the length of lag time [9]. Rasmussen et al. (2015)

assessed the assimilation of groundwater and streamflow data in integrated hydrologic model in the size of ensemble and localization of Kalman filter [10]. They concluded that the required ensemble size depends heavily on the assimilation of discharge observations and estimation of parameters as well as on the number of observed variables. Deng et al. (2016) used ensemble Kalman filter for identification of temporal variation of hydrologic parameters in a monthly water balance model [11]. They used the filter for Wudinghe basin in China and showed the effectiveness of its detection on storage capacity. Mathematical models involved in streamflow prediction to provide more simplistic solutions considering physical ones require comprehensive geographic and measured data. They chose a few of hundreds of variables that affect the streamflow most and dealt with the error caused by linearization and variable ignorance. For this purpose, Kalman Filters are used [12]. They achieved promising results. Later, regression models and Artificial Neural Networks are added to the methods with their own approach to the problem and successful predictions [13], [14], [15]. Today, numerous different methods are used to predict streamflow or enhance the ones that are already being used such as Chaos Theory to improve prediction length of Kalman Filter [16]. Another recent addition to this study area is wavelets, by adding periodic knowledge to the model, they increase the accuracy of it [17], [18].

Kalman filter is first proposed by R.E. Kalman [19]. This method takes observation errors and disturbances into account, minimizes the modelling errors and its convergence is guaranteed. Because of these features, Kalman filter is commonly used in, but not limited with, aircraft position estimation and control systems [20], [21]. Chemical processes are other study areas that prediction accuracy of Kalman filter is frequently exploited [22]. Also, increasing awareness of global warming is attracting more attention every year to prediction and management of water resources [12]. In some cases, Kalman Filter's accuracy outperforms other prediction methods [23].

SWAT is used in many studies with the help of its wide access to environmental data such as soil moisture, snow cover fraction, streamflow and many more. In its cooperation with Kalman Filter, generally SWAT is the predicting part and Kalman Filter is a tool that prepares inputs to the model by

© Copyright 2020 ISSN: 2356-5608

negating some of the process and measurement noises. Similar studies are taken part in China and Senegal [24], [25], [26]. In all cases, Kalman Filter provided significant accuracy increase and proved its success.

Different from existing literature, in this study, Kalman filtering method is used for the prediction of streamflow with the help of SWAT where Kalman Filter is the predicting part in Ergene River Basin in Turkey. This river basin is located in the European part of Turkey with about 12,000 square kilometers of land having mostly very fertile agricultural fields, 1.2 million of population and seven large organized industrial zones, all exploited the surface and groundwater of this watershed. Particularly the northern part of the river basin is affected by dense industrial regions near Istanbul metropolitan. Here, the daily data of nine meteorological and three main hydrometric stations is used for the simulation studies within the frame of this paper. The prediction and analysis of stream-flow in the area is carried out via Kalman filtering method.

Organization in this paper is as follows. In section 2, we described the study area, its meteorological history and geological characteristics. In section 3, Kalman filter and its implementation to the model are explained. In section 4, the simulation results presented and discussed.

II. MATERIALS

Streamflow is affected by various natural and unnatural factors. While most of them are taken into account by physical models during the streamflow prediction stage, mathematical models tend to restrict the number of system inputs, due to the increased complexity and computational time requirements. Along with its advantages, selecting the inputs to be processed has some disadvantages. Due to removal of some terms in the equation of the model, accuracy loss that leads to uncertainty is unavoidable. In addition, removed terms become noise for the system. Depending on input selection, the equation must be adjusted with respect to the inputs and noise in order to minimize the prediction error.

A. Study area

The Ergene River Basin taken as the area of study is in the European part of Turkey. It is in the Marmara Region and located in the central part of the Thrace region between 40 39° and 42 05° north latitude and 25 59° and 28 10° east longitude, as shown in Figure 1. The total area of the Ergene River Basin is 11,020 km². The Ergene River originates from the Istranca Mountains in the northeast of the basin and travel through east-west direction by collecting various branches from North and South bank of the river. Dominant land use in the study area is cropland (76%), and then pasture and sporadic forest include (18.7%), only 5.3% of study area occupied by urban and industrial area based on prepared land use in 2012. There are more than 40 meteorological stations in the Ergene River Basin with different meteorological data periods. In addition, there are seven stream gauges in the Ergene River Basin, three of them are found in the main

river, and two of them are used in this research. The Ergene River Basin is under the influence of the terrestrial climate; the northern summers are hot and arid, and the winters are cold and hard. The Mediterranean climate is dominant in the south of the basin and the summers are hot and dry, the winters are warm and rainy. The average annual temperature in Thrace is 13°C. The highest temperature in Thrace is measured as 44.6°C in Luleburgaz. The lowest temperature in the region is -17.9°C. The distribution of precipitation within the year is geographically similar throughout the basin, but the amount of rainfall is less in regions, where industry and population growth are highest, such as in Cerkezkoy, Corlu, Luleburgaz. The average total precipitation in Thrace Region is 602 mm and the highest daily precipitation is observed in Corlu with a value of 232 mm. Annual average precipitation (for 45 years 1970-2014) calculated from meteorological stations is about 590 mm. The lowest monthly average rainfall in the basin is observed in August, whereas the highest monthly average rainfall is observed in November. Continuous daily stream flow is available for Inanli and Luleburgaz stream gauges for 35 years (1980-2014). These data are analyzed by separating the base flow, which it is approximately 6% of rainfall as the average direct runoff. Double Mass Curve analysis, applied on daily stream flows for 35 years, shows a deviation on flow regime around 1997 in both stream gauges. In addition, a clear change is observed in the base flow characteristics of the river after 1997 which coincides with the start of industrial development in the region. This base flow increment shows the amount of point source discharges to the river by industrial activities. Furthermore, in natural condition, flow of the river in summer times were approaching zero (dry), however, in recent years, there is a continuous base flow without raining upstream of the river. Because of the concentration of industrial facilities, the natural flow mechanism of the river has been disturbed due to discharge of groundwater or network water used by these facilities, and the increase of the amount of domestic wastewater discharged to the Ergene River due to rapid population growth, and as a result, the amount of flow reaches high values in the summer. For correction the effluents impact and natural streamflow prediction, a Kalman Filter model used in daily, monthly and annual time interval.

III. METHODS

After its first proposal by R.E Kalman, Kalman Filter became a subject to many studies and researchers tried to improve its performance. Calculation of Kalman Filter will be briefly explained in next section. Also, its more comprehensive explanation and derivation can be found in one of the more recent studies [27].

A. Kalman filtering

The prediction via Kalman filtering is based on two values; mathematical expectation that is calculated via equation written according to system dynamics and observed value that depends on measurements. But, due to the possible errors in

© Copyright 2020 ISSN: 2356-5608



Fig. 1. Ergene River Basin and its zones

these values based on linearization, ignoring less effective variables or measurements their reliability varies to time. This factor is tracked with Kalman Gain and weights of mathematical expectation and observed values are decided in final prediction.

Kalman Filter has a cycle structure but first cycle requires initial state and covariance information to start. after these variables provided, Kalman Filter cycle starts with the calculation of mathematical expectation and observed value as

$$x_{k+1}^{-} = Ax_k + Bu_{k+1} + w_k \tag{1}$$

and

$$z_k = Hx_k + v_k \tag{2}$$

where w and v are error matrices of the equations. These errors can be caused by external factors, linearization, ignored variables or measurement process. A and H matrices relate state to results and might change in each time step but in this study, they are assumed to be time invariant.

Kalman Filter plays a role in error negation and it is important to track error information of the system throughout the process. This information is carried by P which is the initial covariance matrix of x and updated during the process before the calculation of Kalman Gain and as preparation to next cycle with equations

$$P_{k+1}^{-} = AP_kA^T + Q \tag{3}$$

and

$$P_{k+1} = (I - K_k H) P_k \tag{4}$$

where K denotes Kalman Gain and Q is the covariance matrix of w. Before final decision, reliability variable, Kalman Gain is updated according to system elements to decide weights of two pre-predictions as

$$K_k = P_{k+1}^- H^T (H P_{k+1}^- H^T + R)^{-1}$$
 (5)

where R is the covariance matrix of v. Then, it is used in

$$x_{k+1} = \bar{x_{k+1}} + K_k (z_k - H x_{k+1})$$
 (6)

to determine whether the prediction will be close to mathematical expectation or observed value. Here, Figure 2 presents Kalman Filter's prediction cycle.

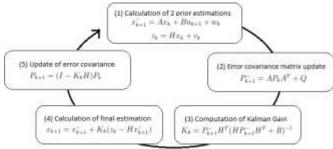


Fig. 2. Prediction cycle of Kalman Filter

B. Streamflow forecasting by Kalman Filter in Ergene River

In streamflow prediction from soil moisture to snow water numerous variable are used by physical models but to simplify them mathematical models only uses a few of them that gives most information about characteristics of river. These are mostly chosen as precipitation, evaporation and temperature.

During the implementation of Kalman Filter to Ergene River conditions of the environment are considered and variables are chosen. Since, Ergene River does not contain any big branches, changes on the main line are generally carried over to next stations. Watershed is under continental climate and that makes precipitation a major factor of this system. Due to its size, watershed is divided into 3 precipitation zones. Conversion from mm to m³/s is made by SWAT model. Since, industrialization has an increasing trend in this area, wastewater poured in the river can not be ignored. Considering these characteristics, equation for Uzunkopru station's streamflow prediction can be written as

$$Q_U(k+1) = c_1 Q_L(k) + c_2 W W_L(k) + c_3 P_Z(k)$$
 (7)

where Q_L is previous station's streamflow value, P_Z is the total precipitation data collected from three zones

$$P_Z(k) = c_4 P_{Z1}(k) + c_5 P_{Z2}(k) + c_6 P_{Z3}(k)$$
 (8)

and WW_L is wastewater affecting between two stations. For relation of prediction and measurements, it is assumed that river tends to retain its previous state which gives

$$Q_U(k+1) = Q_U(k).$$
 (9)

and

$$Q_{U}(k+1) = 1 \quad 0 \quad 0 \quad 0 \quad \Box Q_{L}(k) \quad \Box + v. \tag{11}$$

$$\Box WW_{L}(k) \quad \Box$$

$$P_{Z}(k)$$

In (9), even though next step of every input is calculated, beside Q_U (k+1), others are only temporary assignments and will be overwritten before their next use. w and v are unknown and assuming input errors are independent from each other, their covariance matrices, Q and R are diagonal. These can be written as

$$Q_{11} \quad 0 \quad 0 \quad 0$$

$$Q = \begin{bmatrix} 0 & Q_{22} & 0 & 0 & R = R_1 \\ 0 & 0 & Q_{33} & 0 & 0 \\ 0 & 0 & 0 & Q_{44} \end{bmatrix}$$
(12)

After choosing initial state of every variable, unknown ones are chosen with the optimization process that is presented in Figure 3.

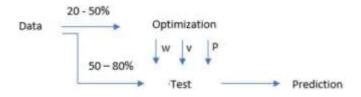


Fig. 3. Optimization process of system variables

Mesaured data is separated into two groups, one for optimization and other for testing. During optimization part constants in A matrix and covariances of w and v matrices that give best RMSE and NS are chosen. After that models are tested on test data and compared according to their RMSE and NS scores.

IV. RESULTS AND DISCUSSION

In this study, real data from 12.04.1981 to 31.12.1993 is used, where 50% of the data is utilized for optimization of constants and remaining is used for testing. To optimize Q and R matrices, constants in A matrix are chosen and with 0.5, 0.25 and 0.1 resolution, every combination of Q and R matrices are tested by predicting the data and comparing it with the real values. Also effects of the precipitation is separated into 4 days with [0 0.5 0.3 0.2] weights respectively. Effects of the wastewater around 1990s are ignorable. So, for both 6 input and 5 input systems, best combinations of Q and R matrices are chosen based on RMSE of the models. Also, 5 input model has almost the same result because of industrialization's negligible effect. Figure 3 and 4 show the predictions of these models respectively with the observed values of the Ergene River. While first model has 8.3142 RMSE and 0.8003 NS(Nash Sutcliffe Efficiency Constant), second model has 8.2756 RMSE and 0.8022 NS.

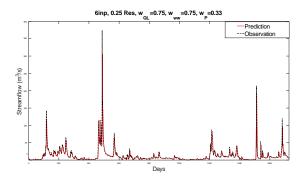


Fig. 4. Prediction of 6 input Kalman Filter model with real values, 8.3142 RMSE and 0.8003 NS

With all inputs, streamflow, precipitation and waste-water, predictions of model are close to the measured real values. Error is high only when the uncharacteristic changes occur such as flood or drought. But even in those situations Kalman Filter is able to predict the increase and decreases.

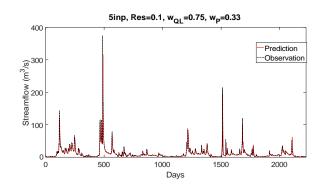


Fig. 5. Prediction of 5 input Kalman Filter model with real values, $8.2756\,$ RMSE and $0.8022\,$ NS

These two models have best results, high accuracy around river characteristics and reliable peak prediction during extreme conditions. Error covariances of this environmental model is not known and considering the length of data average values are tried to be found. For this purpose, variables of Q and R matrices from (12) are calculated with different resolutions. Table I shows the RMSE and NS performances of models that are created with different number of inputs and Q and R matrices.

TABLE I ERRORS OF MODELS WITH DIFFERENT RESOLUTION OF Q AND R MATRICES

Number of Inputs - Resolution	5	6
0.5	8.3136 RMSE, 0.8003 NS,	8.3128 RMSE, 0.8004 NS,
0.25	8.3168 RMSE, 0.8002 NS,	8.3142 RMSE, 0.8003 NS,
0.1	8.2756 RMSE, 0.8022 NS,	1

With these data and model creation choices, increased Q and R resolution and input number generally increase the

performance but these improvements are mostly underwhelming. Also, considering computational demand increase that is presented in Table II, in this case, it is preferred to avoid long computation for small gain.

TABLE II COMPUTATIONAL TIME REQUIREMENTS OF MODELS WITH DIFFERENT RESOLUTION OF ${\bf Q}$ AND ${\bf R}$ MATRICES

Number of Inputs -	5	6
Resolution		
0.5	1m30s	5m
0.25	25m	2h30m
0.1	48h	528h

As it can be seen from Table II, for 5 input models, 23 hours longer computation improves model by 0.002 NS or 0.04 RMSE. The same resolution increase for 6 input models multiplies the time requirement by 200 which at the end gives unsatisfactory improvements.

Kalman Filter is known for its successful short term predictions but models are also used to calculate 7-14 and 30days long predictions. These are calculated for every consecutive 7-14 and 30 day periods. Table III shows average errors for both one day predictions for given period(corrected) and without correcting system with observed values(uncorrected) predictions.

 $\begin{tabular}{ll} TABLE III \\ AVERAGE RMSE of models for 7-14 and 30 day predictions \\ \end{tabular}$

Period -	5 input	5 input	6 input	6 input
Model	(Corrected)	(Uncorrected)	(Corrected)	(Uncorrected)
7 days	2.4679	4.8308	2.6445	4.9811
14 days	2.8853	6.4416	3.0257	6.4571
30 days	3.5145	8.1995	3.6391	8.0850

According to results given in Table III, Kalman Filter's success drastically decreases when prediction period increases. The reason behind this is Kalman Filter makes its predictions based on previous ones and error of the model cumulates for later cycles.

Considering best models, nonlinearity of the system and Kalman Filter's restrictions are main sources of error. There are various factors affecting streamflow and majority of these effects are nonlinear. For example, Figure 6 shows Uzunkopru and Luleburgaz stations' streamflow measurements.

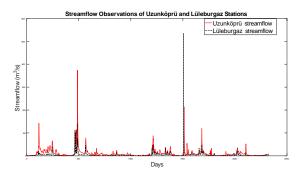


Fig. 6. Discharge of Uzunkopru and Luleburgaz stations

Even though most of the time increase at Luleburgaz station is followed by another one at Uzunkopru, it is not valid for every case whereas Kalman Filter has only one pattern and unable to adapt this nonlinearity. Similarly, any effects that cause river's discharge to exceed standard limits of the river, changes its dynamics and makes the pattern insufficient. In this case Kalman Filter answers with scaled version of previous day. Figure 7 shows an example of this problem.

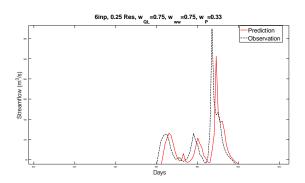


Fig. 7. Effects of river's dynamics changes

Figure 7 shows Linear Kalman Filters biggest problem in this study. Unknown characteristics lead to Kalman Filters unexpected results which are ,in most cases, scaled version of previous day. This type of error creates most of the prediction errors and once they are negated, success of Kalman Filter can be seen more clearly.

V. CONCLUSION & FUTURE WORKS

The implementation of Kalman filtering method in order to predict the stream-flow in Ergene River Basin is presented. In this study, in order to illustrate the success of proposed prediction method NS and RMSE are given and examined in detail. The successful application of Kalman filtering where the real-time data is used, proves that Kalman filtering can be utilized in order to complete the missing real-time data where it is necessary and also achieve short-term prediction for stream-flow.

Beside Linear Kalman Filter's success, its weak sides are observed such as higher error and uncharacteristic results around peaks. Also, high computational demand is seemed as another problem. In order to overcome these problems, Ensemble Kalman Filter approach can be tried. Calculation of observed value at the beginning of the Kalman Filter cycle can be switched between seasons instead of just assuming to observe the same streamflow the next day.

REFERENCES

- Limaye, A.S., Stellman, K. and Slidell, L.A., 2002. Improving flood prediction using Kalman Filter, mesoscale atmospheric model forecasts and radar-based rainfall estimates.
- [2] Noh, S.J., Tachikawa, Y., Shiiba, M. and Kim, S., 2012. Ensemble Kalman filtering and particle filtering in a lag-time window for shortterm streamflow forecasting with a distributed hydrologic model. Journal of Hydrologic Engineering, 18(12), pp.1684-1696.
- [3] Reichle, R.H., McLaughlin, D.B. and Entekhabi, D., 2002. Hydrologic data assimilation with the ensemble Kalman filter. Monthly Weather Review, 130(1), pp.103-114.
- [4] Markussen, L.M., Refsgaard, J.C., Rosbjerg D., 1985. Application of the Kalman filter to real time operation and to uncertainty analyses in hydrological modelling. Scientific procedures applied to the planning, design and management of water resources systems, 147, pp.273-282.
- [5] Ngan, P., 1985. Kalman filter and its application to flow forecasting Doctoral dissertation, University of British Columbia.
- [6] Drecourt, J.P., 2003. Kalman filtering in hydrological modeling. Horsholm, Denmark, DAIHM.
- [7] Moradkhani, H., Sorooshian, S., Gupta, H.V. and Houser, P.R., 2005. Dual state-parameter estimation of hydrological models using ensemble Kalman filter. Advances in water resources, 28(2), pp.135-147.
- [8] Clark, M.P., Rupp, D.E., Woods, R.A., Zheng, X., Ibbitt, R.P., Slater, A.G., Schmidt, J. and Uddstrom, M.J., 2008. Hydrological data assimilation with the ensemble Kalman filter: Use of streamflow observations to update states in a distributed hydrological model. Advances in water resources, 31(10), pp.1309-1324.
- [9] Noh, S.J., Tachikawa, Y., Shiiba, M. and Kim, S., 2013. Sequential data assimilation for streamflow forecasting using a distributed hydrologic model: particle filtering and ensemble Kalman filtering. Floods: from Risk to Opportunity, 357, pp.341-349.
- [10] Rasmussen, J., Madsen, H., Jensen, K.H. and Refsgaard, J.C., 2015. Data assimilation in integrated hydrological modeling using ensemble Kalman filtering: evaluating the effect of ensemble size and localization on filter performance. Hydrology and Earth System Sciences, 19(7), pp.2999-3013.
- [11] Deng, C., Liu, P., Guo, S., Li, Z. and Wang, D., 2016. Identification of hydrological model parameters variation using ensemble Kalman filter. Hydrol. Earth Syst. Sci., 20(5), pp.1-39.
- [12] Bergman, M.J. and Delleur, J.W., 1985. Kalman filter estimation and prediction of daily stream flows: I. review, algorithm, and simulation experiments. 1. JAWRA Journal of the American Water Resources Association, 21(5), pp.815-825.
- [13] Stedinger, J.R., Grygier, J. and Yin, H., 1988, June. Seasonal streamflow forecasts based upon regression. In Computerized Decision Support Systems for Water Managers (pp. 266-279). ASCE.
- [14] Garen, D.C., 1992. Improved techniques in regression-based streamflow volume forecasting. Journal of Water Resources Planning and Management, 118(6), pp.654-670.
- [15] Karunanithi, N., Grenney, W.J., Whitley, D. and Bovee, K., 1994. Neural networks for river flow prediction. Journal of computing in civil engineering, 8(2), pp.201-220.
- [16] Sun, Y., Doan, C.D., Dao, A.T., Liu, J. and Liong, S.Y., 2014. Improving numerical forecast accuracy with ensemble Kalman filter and chaos theory: case study on Ciliwung river model. Journal of hydrology, 512, pp.540-548.
- [17] Shiri, J. and Kisi, O., 2010. Short-term and long-term streamflow forecasting using a wavelet and neuro-fuzzy conjunction model. Journal of Hydrology, 394(3-4), pp.486-493.
- [18] Gu"rsoy, O". and Engin, S.N., 2019. A wavelet neural network approach to predict daily river discharge using meteorological data. Measurement and Control, p.0020294019827972.
- [19] Kalman, R.E., 1960. A new approach to linear filtering and prediction problems. Journal of basic Engineering, 82(1), pp.35-45.

- [20] Lefferts, E.J., Markley, F.L. and Shuster, M.D., 1982. Kalman filtering for spacecraft attitude estimation. Journal of Guidance, Control, and Dynamics, 5(5), pp.417-429.
- [21] Stevens, B.L., Lewis, F.L. and Johnson, E.N., 2015. Aircraft control and simulation: dynamics, controls design, and autonomous systems. John Wiley & Sons.
- [22] Brown, S.D., 1986. The Kalman filter in analytical chemistry. Analytica Chimica Acta, 181, pp.1-26.
- [23] Sun, L., Nistor, I., Seidou, O., Sambou, S., Kebe, C. and Tamba, S., 2013. Prediction of daily discharge at Bakel using multiple linear regression, Kalman filer and artificial neural networks. In CSCE 2013, 3rd Specialty Conference on Disaster Prevention and Mitigation.
- [24] Liu, Y., Wang, W., Hu, Y. and Cui, W., 2016. Improving the distributed hydrological model performance in upper Huai River basin: using streamflow observations to update the basin states via the ensemble Kalman filter. Advances in Meteorology, 2016.
- [25] Zhang, Y., Hou, J., Gu, J., Huang, C. and Li, X., 2017. SWAT-Based Hydrological Data Assimilation System (SWAT-HDAS): Description and Case Application to River Basin-Scale Hydrological Predictions. Journal of Advances in Modeling Earth Systems, 9(8), pp.2863-2882.
- [26] Sun, L., Nistor, I. and Seidou, O., 2015. Streamflow data assimilation in SWAT model using Extended Kalman Filter. Journal of Hydrology, 531, pp.671-684.
- [27] Ribeiro, M.I., 2004. Kalman and extended kalman filters: Concept, derivation and properties. Institute for Systems and Robotics, 43.