The conventional commands for the robotic system

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***Abstract*— Many researches were realized for the robotics command in order to ensure the tracking of a desired reference trajectory. This current work concentrates on computed torque controller for robotic manipulator system by applying the linear and nonlinear control to ensure the position tracking. Indeed, the approach conventional control was applied to control a nonlinear dynamic system in order to displace the terminal organ of manipulator arm of an initial position towards any desired destination. The linear control allow us to linearize our system around a fixed point so-called an equilibrium point. Then, the application of the nonlinear control allows widening the application field. Simulations are presented to show the performance of the conventional control to guarantee the boundedness of the outputs robotic systems.**

Keywords: Nonlinear systems, computed torque control, nonlinear control, position control

I. Introduction

In the last decades, the factories have been working faster than ever with the manipulator robots. The Manipulator arms have been widely used in industrial applications. Thanks to an adequate control, they accomplish the same task several times a remove day without errors and with great precision. As the manipulator arms are complex and non-linear systems, a number of theoretical and experimental studies attempt to develop robust commands based on new methods and algorithms. [1] [2] [3] [4]. By giving the complexity and nonlinearity of articulated systems, the use of mathematical tools becomes less effective in modelling and controlling such a process. However, several conventional control approaches [1] have been established to control these non-linear systems.

The question of the position control for robot manipulators is the choice of the appropriate torques so that the manipulator can follow the desired trajectory. Among these controls, there is linear control and nonlinear control. Indeed, this work proposes the realisation of a high-speed control of the robotic system. In order to have an accurate simulated model, the modelling of the robotic mechanical system 6 axis Staubli RX-60 has been done [5]. The dynamic model of three first links of six-axis Staubli RX60 robot has been developed according to the Lagrange-Euler formalism [6] [7] which translates the movement of the various articulations of the manipulator arm in order to validate the proposed controls. The linear approach will be used on the linearization of the equations of the robot's motion around an arbitrary chosen point of equilibrium. Then we implement the nonlinear control approach to extend the application field of our Robot Staubli RX-60. The present paper is organized as following. In section 2, we give a dynamic model of an industrial Staubli RX-60 wich has been done in the reference [5]. Then, the linear control is applied in section 3. Next, the nonlinear computed torque controller is designed in section 4. The conclusions are given in section 5

II. Dynamic model for control of a robotic system

The robot RX-60 is an articulated with 6 degrees of freedom. Staubli RX-60 is an anthropomorphous industrial non redundant robot with simple open structure (serial structure).

In this section, the dynamical model of the robot arm considers a relation between the joint torques/forces used by the actuators and the position, velocity and acceleration of the robot arm with respect to the time. The parameters of a dynamic model introduced here of a manipulator arm were estimated on experiments [5].

As reported Lagrange theory, a robot manipulator is described by the following equations:

$$J(\theta)\ddot{\theta} + H(\theta,\dot{\theta})\dot{\theta} + G(\theta) = U \quad (1)$$

$\theta, \dot{\theta}, \ddot{\theta}$:Vector dimension nx1, respectively, positions, velocities and accelerations joint

$J(\theta) \in \mathbb{R}^{n\times n}$: Inertia matrix, positive definite symmetric matrix

$H(\theta,\dot{\theta}) \in \mathbb{R}^{n\times 1}$: Coriolis and centripetal forces vector
$G(\theta) \in \mathbb{R}^{n\times 1}$: Gravity torques
$U \in \mathbb{R}^{n\times 1}$: Input torques

The last three joints of this manipulator which constitute the wrist are fixed at the zero(the positions $\theta_4 = \theta_5 = \theta_6 = 0$).The dynamic model for the first three joints of the Staubli RX60 robot arm is taken from [5] as follows in the relation :

$$\begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix} = \begin{bmatrix} J_{11} & J_{12} & J_{13} \\ J_{21} & J_{22} & J_{23} \\ J_{13} & J_{23} & J_{33} \end{bmatrix} \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \\ \ddot{\theta}_3 \end{bmatrix} + \begin{bmatrix} H_1(\theta,\dot{\theta}) \\ H_2(\theta,\dot{\theta}) \\ H_3(\theta,\dot{\theta}) \end{bmatrix} + \begin{bmatrix} G_1(\theta) \\ G_2(\theta) \\ G_3(\theta) \end{bmatrix} \quad (2)$$

The expressions of the elements of the matrices J, H and the vector G are taken from the reference [5].

The demonstration and the calculation of elements of the matrix of the dynamic model is presented in the reference [5].

As of Staubli RX-60 manipulator dynamic formulation, this system is nonlinear, multi-input multi-output and uncertainly

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III. LINEAR CONTROL :

The aim of modelling any process is to get the adequate command. First, we will implement a linear control based on the technique of linearization of the system's matrix writing around an equilibrium point fixed according to our choice.

The linear command is defined by the relation (3):

$$u = U_{eq} + V \quad (3)$$

The equilibrium state is represented by the following relations (4) and (5):

$$U_{eq} = G(\theta_{eq}) \quad (4)$$

$$\theta_{eq} = \theta - \varphi \quad (5)$$

With:

U_{eq} : The torque required to maintain the robot to the equilibrium position

θ_{eq} : The static equilibrium position

φ : The variation around equilibrium point

The linearization of the equation of motion around a chosen equilibrium point θ_{eq}using the first order Taylor series approximation and substituting (5) into (1), we obtain the following relation (6):

$$J(\theta_{eq})\ddot{\varphi} + G(\theta_{eq}) + \left(\frac{\partial G}{\partial \theta}\right)_{eq} \varphi = U_{eq} + V \quad (6)$$

Substituting by (4) we obtain the matrix relation (7):

$$J(\theta_{eq})\ddot{\varphi} + \left(\frac{\partial G}{\partial \theta}\right)_{eq} \varphi = V \quad (7)$$

The matrix writing of the second order equation is rewritten by the following representation (8):

$$\dot{X} = A.X + B.V \quad (8)$$

with

$$X = \begin{bmatrix} \varphi \\ \dot{\varphi} \end{bmatrix}_{6\times1} \text{ : System state vector} \quad (9)$$

$$\dot{X} = \begin{bmatrix} \dot{\varphi} \\ \ddot{\varphi} \end{bmatrix}_{6\times1} \quad (10)$$

$$A = \begin{bmatrix} \mathbb{O}_{3\times3} & \mathbb{I}_{3\times3} \\ -J^{-1}(\theta_{eq})\left(\frac{\partial G}{\partial \theta}\right)_{eq} & \mathbb{O}_{3\times3} \end{bmatrix}_{6\times6} \text{ : State Matrix} \quad (11)$$

$$B = \begin{bmatrix} \mathbb{O}_{3\times3} \\ J^{-1}(\theta_{eq}) \end{bmatrix}_{6\times3} \text{ : order Matrix} \quad (12)$$

➢ *Stability and controllability of the robotic system*

In order to develop a control of a system, we study the stability of dynamical system by calculating the eigenvalues of matrix A. The condition of stability is explained by all eigenvalues of state matrix A have negative real parts. These eigenvalues are only the roots of the characteristic polynomial showed by the relation (13):

$$\det(\lambda I - A) = \lambda^2 I + J^{-1}(\theta_{eq})\left(\frac{\partial G}{\partial \theta}\right)_{eq} = 0 \quad (13)$$

According to the above equation, the robotic system is always unstable in open loop. In order to ensure local stability around a point of equilibrium and to design a feedback controller so that the process goes from any initial state X_0, to a specified desired state X_T, in some finite time. , the controllability of the system must be verified. . We proceed to calculate the matrix Q:

$Q = (B, AB, \ldots, A^5B)$. A necessary and sufficient condition for the complete controllability is that matrix Q is full rank.

$$Q = \begin{bmatrix} \mathbb{O}_{3\times3} & J^{-1}(\theta_{eq}) \\ J^{-1}(\theta_{eq}) & \mathbb{O}_{3\times3} \end{bmatrix}_{6\times6} \quad (14)$$

The matrix Q is full rank so our system is controllable. This leads to the possibility of controlling our robotic system.

➢ *Linear control law synthesis*

We focus mainly on the design of a torque control U which ensures the displacement of our manipulator arm from an initial position $\theta_0, \dot{\theta}_0$ and $\ddot{\theta}_0$ to an equilibrium position $\theta_{eq}, \dot{\theta}_{eq}$ and $\ddot{\theta}_{eq}$.

We will ensure the stability in closed loop by the state feedback V.

$V = K.X = K_1.\varphi + K_2\dot{\varphi}$ Avec $K \in \mathfrak{R}^{3\times3}$: the position feedback gains K_1 and the velocity feedback gain K_2 are defined by the following differential equation:

$\ddot{\theta} + \Lambda_1\dot{\theta} + \Lambda_2\varphi = \mathbb{O}_{6\times6}$ Avec Λ_1 et $\Lambda_2 \in \mathfrak{R}^{3\times3}$ which present the poles imposed on the robotic system.

These are two diagonal matrices that represent respectively the sum matrix poles and the product matrix poles. So our system is written as follows in the relation (15):

$$J(\theta_{eq})\ddot{\varphi} + \left(\frac{\partial G}{\partial \theta}\right)_{eq} \varphi = K_1.\varphi + K_2\dot{\varphi} \quad (15)$$

Finally, the linear control applied to our robotic system as following (16):

$$U = U_{eq} + V = G(\theta_{eq}) + K_1.\varphi + K_2\dot{\varphi} \quad (16)$$

With:

$$K_1 = -J(\theta_{eq}).\Lambda_2 + \left(\frac{\partial G}{\partial \theta}\right)_{eq} \text{ and } K_2 = -J(\theta_{eq}).\Lambda_1 \quad (17)$$

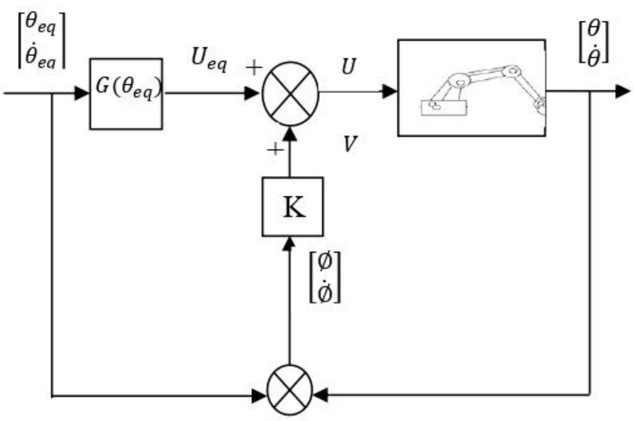

Fig. 1 Block diagram of linear control

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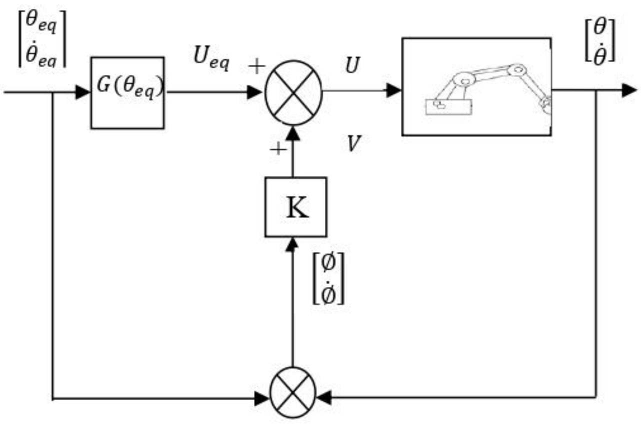

Fig. 1 Block diagram of linear control

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organ of the manipulator arm from an initial position θ_0 towards a final position θ_{des}.

➢ *Simulation of the nonlinear control*

We will validate these results by simulations on Matlab Simulink environment. First, we choose the initial positions vector in degrees $\theta_0 = [0\ 0\ 0\ 0\ 0\ 0]^T$ and the Final positions vector in degrees $\theta_{des} = [10\ 10\ 10\ 0\ 0\ 0]^T$

Then, we proceed to calculate the angular position gain matrix Λ_1 and the angular velocity gain matrix Λ_2 by placing the poles of the system to -4. Hence the values $\Lambda_1 = 8 \times \mathbb{I}_{3\times3}$ and $\Lambda_2 = 16 \times \mathbb{I}_{3\times3}$.

The simulation of the evolution of positions and angular velocities was carried out over a period of 10 seconds.

The simulation of the angular positions are showed by the figures 4 and 5.

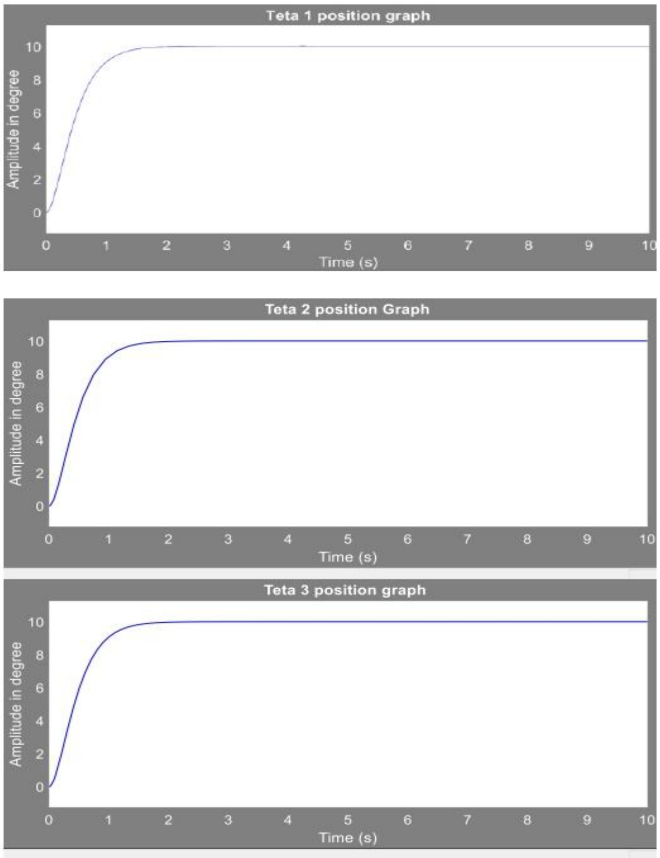

Fig. 4: Angular positions of the Staubli RX60 robot at the nonlinear control case

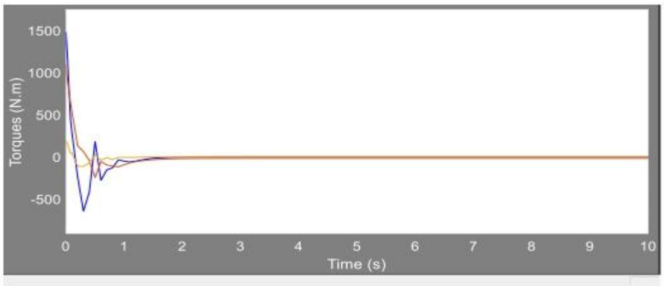

Fig. 5: Variation of the torques applied to the Staubli RX-60 provided with the nonlinear control by position and velocity feedback

To confirm the pertinence of the nonlinear control, it is proposed to attain a target in presence of uncertainty in unstructured input (e.g. disturbance). The simulation results in the presence of disturbance are given in the figures 6 and 7. The measurement noise is used the Simulink block diagram as white noise.

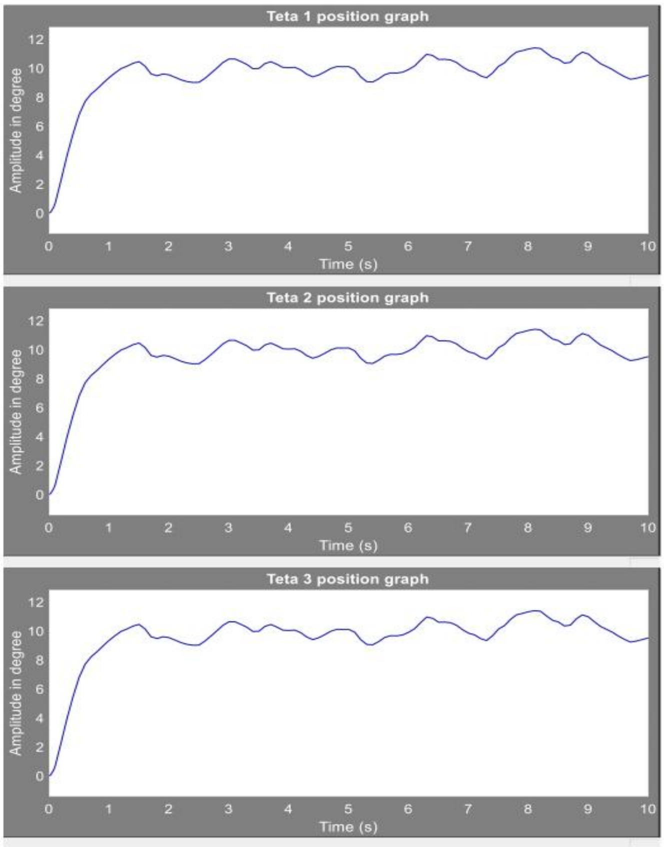

Fig. 6 ***Angular positions of the Staubli RX60 robot at the nonlinear control case in the presence of external disturbance***

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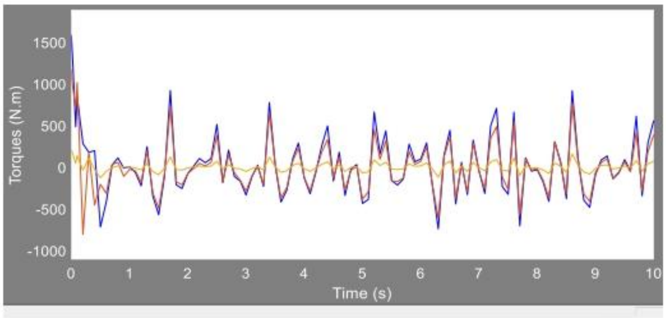

Fig. 7: Variation of the torques applied to the Staubli RX-60 in the presence of external disturbance

This command ensures the global asymptotic stability system. But the disadvantage of such control is the sensitivity against the uncertainties and disturbances. On the other hand, the modelling of a complex system is not always precise. So, we design the nonlinear feedback control of position, velocity and acceleration to improve the performance the behaviour of the robot in different circumstances.

Substitution of the equation's the reference model (18) in the equation of the dynamics of the robot (1), we get the control law:

$$U = (J(\theta) + I)\ddot{\theta} + H(\theta,\dot{\theta}) + G(\theta) + \Lambda_1\dot{\theta} + \Lambda_2(\theta - \theta_{des}) \qquad (22)$$

The figure 8 shows the control loop of the nonlinear control by position, velocity and acceleration feedback:

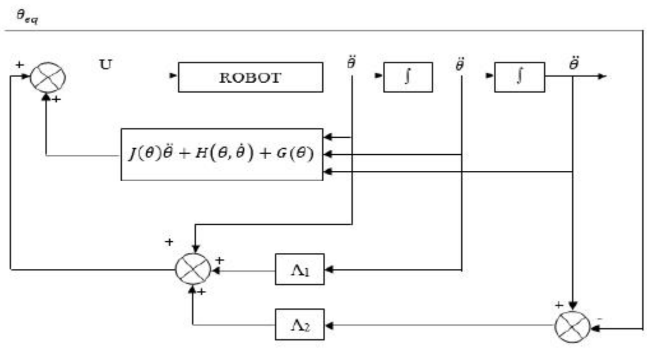

Fig. 8 Block diagram of nonlinear control by position, velocity and acceleration feedback

The same curves as the nonlinear feedback control of position and speed are obtained with and without incorporating external disturbance. So, the computed torque controller is able to hold each link at a particular angle but it is so sensible against unmodeled dynamics.

V. Conclusions

We studied nonlinear controller applied to control of robot manipulator to achieve the specified joint acceleration, velocity and position state. This strategy guarantees accurate tracking in joint space and stability. We implemented the PD-computed torque controller on a dynamic modeling in MATLAB/SIMULINK environment. This designed controller is tested by band limited white noise with a predefined 40% of relative to the input signal amplitude which the sample time is equal to 0.1. These tools become less effective and less robust against disturbances and uncertainties and / or inaccuracy. Faced with this problem, the use of unconventional learning approaches has become a necessity in order to develop a command able to tolerate the uncertainties and neutralize the effect of external disturbances.

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